AFTER-SALES SERVICES FOR DURABLE GOODS

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ABSTRACT

AFTER-SALES SERVICES FOR DURABLE GOODS

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Durable goods are products that are designed to be used for an extended period of time and require after-sales activities such as warranty, repair and maintenance. In the first part of this study, we consider a monopolistic durable goods manufacturer that also provides a base warranty and after-sales services to her customers through a designated retailer. The customers are strategic; i.e., they evaluate the total lifecycle cost of the product when making their purchasing decision. However, they can foresee the future costs only to a certain extent. In this setting, we find the manufacturer's product price decision followed by the spare parts wholesale and retail price decisions made by the manufacturer and the retailer, respectively. As a benchmark, we study the centralized system where the manufacturer and the retailer are integrated. We also study an alternative model, where the manufacturer is not active in the after-sales business and does not offer a warranty, and these services are carried out by an independent retailer. In the second part of the study, we look into downstream competition. The manufacturer and the retailer operate with similar dynamics to the first part, except that the retailer now competes with an independent workshop for after-sales services after the warranty period is over. This competition provides a cheaper alternative to the customers for owning the product, hence changing the strategic decisions of the manufacturer and the retailer. In both parts, we provide managerial insights by studying analytical equilibriums where possible, and by conducting numerical studies otherwise.

Keywords: After-Sales Services, Durable Goods, Total Cost of Ownership, Consumer Surplus, Game Theory

ÖZ

DAYANIKLI TÜKETİM MALLARI İÇİN SATIŞ SONRASI HİZMETLER

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Dayanıklı tüketim malları uzun süre kullanılmak üzere tasarlanmış ürünler olup, garanti, bakım ve onarım gibi satış sonrası aktiviteler gerektirirler. Bu çalışmanın ilk bölümünde, son müşteriye sağladığı temel garanti ve satış sonrası hizmetlerini yetkilendirilmiş bir bayi aracılığıyla yürüten tekelci bir dayanıklı tüketim malları üreticisini göz önünde bulunduruyoruz. Müşteriler stratejik davranıyor, yani satın alma kararlarını verirken ürünün yaşam boyu maliyetini değerlendiriyorlar. Ancak, gelecek maliyetleri yalnızca belli bir düzeyde öngörebiliyorlar. Bu düzen içerisinde üreticinin ürün satış fiyatı kararını, sonrasında ise üreticinin yedek parça için toptan satış fiyatı ve bayinin yedek parça için perakende satış fiyatı kararlarını buluyoruz. Kıyaslama amacıyla, bayi ve üreticinin entegre olduğu merkezi bir sistemi de çalışıyoruz. Ayrıca, üreticinin satış sonrası hizmetlerde faal olmadığı ve ürün garantisi sağlamadığı, ve satış sonrası hizmetlerin bağımsız bir servis tarafından verildiği alternatif bir modeli daha inceliyoruz. Çalışmanın ikinci bölümde, alt kademede rekabetin olduğu durumu inceliyoruz. Üretici ve bayi ilk kısımdakiyle benzer dinamiklerde çalışıyorlar, fakat bu durumda bayi garanti süresi bitiminden sonraki satış sonrası hizmetler için bağımsız bir servis ile rekabet ediyor. Bu rekabet müşterilere ürüne sahip olmak için daha ucuz bir alternatif sunuyor, dolayısıyla üreticinin ve bayinin stratejik kararlarının değişmesine sebep oluyor. Her iki kısımda da mümkün olan yerlerde analitik dengeleri inceleyerek, olmadığı durumlarda ise sayısal çalışmalar yaparak yönetsel içgörüler sunuyoruz.

Anahtar Kelimeler: Satış Sonrası Hizmetler, Dayanıklı Tüketim Malları, Toplam Yaşam Maliyeti, Tüketici Fazlası, Oyun Kuramı

To my family

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Almost everyone that I have crossed paths with since the last 15 years knows that I'm working towards my Ph.D. Well, it's finally here, with the help and support of many people. Trying to be comprehensive might get quite lengthy, so here we go, hopefully without any major omissions:

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CHAPTER 1

INTRODUCTION

Durable goods are products that are designed to satisfy a need over an extended period of time. Some common examples are automobiles, printers, home appliances, game consoles, mobile phones, clothing, etc. These products share some common properties that set them aside from consumable goods. We will first make a brief introduction to these properties and then define our scope.

1.1 Common Properties of Durable Goods

1.1.1 Aftermarket/After-sales

When one buys a product for long-term use, that is usually not the end of the story. There are generally additional goods and services the customer needs to keep buying in order to be able to get full utilization from his initial purchase, which is referred to as *after-sales* or *aftermarket* (we use the terms interchangeably). We can roughly categorize aftermarket goods and services into four groups: Operating supplies, repairs, maintenance, and complementary goods & services. Next, we describe what each of those terms entails and provide examples from everyday life.

Operating supplies are consumables or services that are required for the product to fulfill its primary use. For example, fuel is required in order to be able to travel with a car; ink and paper are needed in order to be able to operate a printer; or subscription from a mobile network provider must be purchased in order to be able to place calls on a mobile phone. *Repairs*[1] refer to activities that need to be performed in or-

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[1] Also referred to as *corrective maintenance*

der to restore a damaged product to a functioning state again. If a car's windshield is broken after being hit, the owner needs to buy a replacement part (referred to as *spare parts*), as well as paying for services to get it installed on the car. If a mobile phone's battery is dead after too many charging cycles, the owner buys a new battery and depending on the model he either installs it himself, or he needs to have it installed by an authorized service provider. Another possibility is that the product breaks down unexpectedly by no fault of the owner, where the manufacturer overtakes the responsibility of diagnosis and repair, which is referred to as *warranty*. *Maintenance*[2] refers to proactive actions that keep the product in optimal running condition, especially minimizing the effects of wear and tear from regular use, and proactively preventing breakdowns. Many mechanical equipment such as cars or air conditioners have filters to catch dust or foreign particles where a regular air or liquid flow occurs. These filters need to be checked and cleaned and/or replaced in certain intervals. A mobile phone's operating system receives regular updates to optimize performance and improve security, which usually come for free. Last, but not least, there are *complementary goods & services* that enhance product use; which can be purchased, or again might come free with the initial purchase. For example, built-in car GPS systems get outdated due to new roads being opened or traffic regulations changing, so one needs to install new maps and information. It might even be better to upgrade to a version that incorporates live traffic information and optimizes the driver's route accordingly. Or, there are only plug-in earphones that are usually included with the purchase of a mobile phone, but the recently introduced bluetooth earphones make it much easier to have hands-free calls. If one wants even more functionalities, they are also available: Watches that monitor health information and synchronize them to a mobile phone have been developed. Many examples can be added, but it is clear that several goods and services are available and even required upon the purchase of a durable product, in attaining a complete and steady value out of its use. Thus, the manufacturer of a durable good is not just focused on the product itself, but has to plan, monitor or control the aftermarket as well.

Aftermarket goods and services constitute a significant source of revenue and profit for manufacturers. As reported by Cohen et al. [28], after-sales accounts for 24% of

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[2] Also referred to as *preventive maintenance*

the revenues of a business, while accounting for 45% of its gross profits. The financials of 2020 for SAIC Motor, the biggest Chinese auto manufacturer, also confirm this finding. Their revenues from *parts, service and others* constitute 24% of the group's total revenue and 46% of its gross margin [82]. Apple's 2019 annual report shows that *services*[3] correspond to 18% of their revenues and 30% of their gross profits, having a 63.7% gross profit margin [4]. For Dell, *services* correspond to 22,5% of the total revenue and 44,8% of the total gross margin. For other industries where the manufacturer can be more captive over the after-sales business, the recent figures are even more pronounced. Otis, which produces and services escalators and elevators, has a very solid aftermarket, where 58% of their revenues were generated by the *service* segment in 2020, which corresponds to 83% of their operating profit [73]. Similarly, Xerox reports 78% of their revenues came from *post-sales* in 2020 where they have 40.3% gross margin; as opposed to *equipment sales*, where they have a significantly lower gross margin of 27.4% [101].

Manufacturers may monopolize the aftermarkets of their products for some or all of the aftermarket goods and services, which is referred to as *lock-in*. It is a natural conclusion for very specialized products, such as medical diagnostic devices, production machinery, planes, or trains, where product safety is of utmost importance. Product safety is also the first line of defense of manufacturers when they take actions to monopolize their aftermarkets by limiting access to spare parts, by refusing to provide repair information, or introducing limitations that make third party repairs essentially a negative business case [29, 79]. Alternatively, the manufacturer may also design the product in such a way that it is not compatible with other aftermarket goods or services. For example, Apple is criticized for allowing the downloading of applications to an iPhone only through their App Store [48]. Alternatively, the manufacturer may also design the product in such a way that it is not compatible with other aftermarket goods or services [37]. We refer this issue again while discussing manufacturer's design decisions and antitrust issues.

Before we conclude this category, it is also worthwhile to mention repair and maintenance contracts (RMC). Repair and maintenance contracts are essentially a quantity

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[3] The term refers to sales from the company's digital content stores and streaming services, AppleCare, licensing and other services

discount on these operations, as they are bought as a bundle at a lower price than otherwise bought separately each time. With these contracts, the customer makes an upfront discounted payment regarding the product's operating expenses, also ensuring that the product is properly taken care of.

1.1.2 Warranty

In many markets, the manufacturers also commonly offer *warranties* and don't ask the customer for additional payment for some of the after-sales costs. With this *warranty coverage*, the manufacturer rectifies any production-related defects that cause the malfunction of the product. There are five motivations identified in the literature as to why manufacturers offer warranties: insurance (risk sharing), sorting (price discrimination according to customer's risk attribute), signalling (indicator of quality due to information asymmetry), incentive (providing motivation to manufacturers to improve their quality, or at least not cheat on product quality, which would result in higher costs), and profitable bundling (manufacturer has access to cheaper service) [60]. Also, a base warranty coverage is mandated by law in most cases, since this issue is also related to the protection of the consumer.

Better warranty conditions are considered to signal a product's quality and manufacturers occasionally provide longer warranty periods than what is required by the law or what is offered by their competitors, although this means more costs for them. For example, the apparel and outdoor equipment manufacturer L.L.Bean offers a lifetime warranty for all of their products in case of defects due to materials or craftsmanship [62].

Manufacturers often outsource their warranty and repair work to third party service providers, commonly referred to as *retailers* [28, 47, 64, 80]. Outsourcing services to retailers has several advantages for a manufacturer, such as reduced investment and operational costs, utilizing localization advantages via increased proximity to customer, and reduction of inventory and receivables risks, which are also related to the manufacturer's channel decision.

Another common application is extended warranties. Such contracts are bought by the customer at an additional cost, providing longer or more extensive coverage than that offered by the manufacturer as a standard. They can be designed and sold by the manufacturer or the retailer.

1.1.3 Channel Structure

A manufacturer also needs to determine her channel structure for sales and after-sales services. In managing the after-sales business, manufacturers often outsource their warranty, repair and maintenance work to third party service providers, commonly referred to as *retailers* [28, 47, 64, 80]. Outsourcing services to retailers has several advantages for a manufacturer, such as reduced investment and operational costs, utilizing localization advantages via increased proximity to customer, and reduction of inventory and receivables risks. The retailer then needs to evaluate his investment costs and expenses vs. revenues from becoming a dealer and/or workshop. Alternatively, the manufacturer might own and operate all of her sales and service locations, or make a combination of both.

This issue is also closely related to the antitrust laws. In United States, an automotive manufacturer cannot own or operate any sales or service locations. She needs to have *franchise* dealers and workshops. A recent heated discussion on this subject was in 2013 when the electric car company Tesla opened showrooms to introduce their products and was sued by several dealers [20]. We come back this subject in Section 1.1.7.

1.1.4 Design Decisions

Manufacturer's design decisions are referred to as *durability*, *quality*, *reliability*, or *lifetime*. These determine how often the product will be replaced (i.e., planned obsolescence), or how much after-sales revenues will be generated. On the other hand, too frequent replacements or too high aftermarket costs for customers also means that the product's overall demand will be lower.

Another important design decision is compatibility level of the product with other non-OEM (Original Equipment Manufacturer) products. The non-OEM products in question might be other goods or services that either enhance or are essential for the use of the durable good. Such examples include toner for printers, service packages for regular maintenance, or applications for operating systems. Most commonly, the manufacturer's aftermarket revenues are reduced as the product's compatibility level increases. However, some of these products (such as applications) can also have a "networking effect" and increase the demand for the durable good itself.

Complementary products can be consumables that are needed to derive utility from the durable good itself, e.g., toner for printers, fuel cells for hybrid cars, etc.

1.1.5 Used Products

Once they own the products, the customers can sell them before their lifetime is over. These are called *used products*. Used products constitute a secondary market that competes with the new products. Here, three important issues arise:

Substitutability On the one hand, used products pose competition to the manufacturer's new product sales, as they provide a partial substitute for new product demand. This phenomenon is known as *cannibalization*. On the other hand, this can also be seen as an opportunity for market segmentation and offering an alternative to lower-segment customers, rather than a threat.

Time inconsistency Due to substitutability, manufacturer's price/quantity decisions for today are inconsistent with those of tomorrow, which leads to non-optimal results (e.g., competitive prices even when the firm is monopolistic)

Adverse selection Inefficiency exists in secondhand markets due to information asymmetry between sellers and buyers. This is also known as the *Lemon Problem*.

In addition to these topics, the used market value is also an important factor in the customer's buying decision. The average market price for a used product (vs. the price of a new product) affects a customer's decision to buy today. Products that lose less value in the secondary market are preferred more while buying a new product.

1.1.6 Leasing and Servicizing

Leasing is an alternative to selling where the seller keeps the property rights of the product and the buyer buys the right to use it. An attractive property of leasing is that it provides the manufacturer more power in the secondary market, as it guarantees the return of products and presents the manufacturer the option to sell them (keeping them in the market), to remanufacture/recycle them, or to dispose them. Because of this, leasing has initially been introduced as a remedy to the time inconsistency problem, so that a monopoly manufacturer would also be a monopoly in the secondary market.

Another very similar concept is called servicizing. Here, the manufacturer directly sells the services related to the use of a durable good. The major difference between leasing and servicizing is that servicizing is suitable for pooling resources. For example, you can lease trucks to a company, who would then use them to transport goods. With servicizing, you would be selling transportation services instead. This would create the opportunity to the manufacturer to combine the transportation for different companies together, reducing the overall number of trucks in use, and potentially reducing environmental impact.

1.1.7 Antitrust

Antitrust regulations are intended to ensure the process of competition for the benefit of consumers, making sure there are strong incentives for businesses to operate efficiently, keep prices down, and keep quality up. To this end, Federal Trade Commission of the United States [43] states their mission as follows: "Protecting consumers and competition by preventing anticompetitive, deceptive, and unfair business practices through law enforcement, advocacy, and education without unduly burdening legitimate business activity."

As mentioned earlier, the issue of competition for durable goods not only affects the market for the durable good itself, but also the aftermarket of it. Since the products are specialized, it is to the consumer’s benefit that these aftermarket activities are undertaken to some extent by the manufacturer, either directly or through authorized third-party workshops and dealers. However, there are also consumer advocacy groups that refer to *right to repair*, who are pushing for legislations towards manufacturers to provide enough information for the customers or third parties to successfully repair a product [78, 89].

In the United States, there are three main antitrust laws in action: Sherman Act (1890), Federal Trade Commission Act (1914), and Clayton Act (1914). As mentioned earlier, "All 50 state legislatures have enacted laws governing the sale and servicing of cars and trucks, which is usually done by franchised new-car and -truck dealerships" [71]. In Europe, the two main laws are the Competition Law and the Block Exemption Regulation for automotive products. Europe's competition law prohibits the manufacturers from taking certain measures such as enforcing dealership and workshop contracts simultaneously, or requiring the workshop to sell OEM parts only. These laws and regulations try to ensure that the consumer receives the benefits, while the manufacturer does not misuse her power against her dealers, workshops and customers.

1.1.8 Environment

The focus on environmental impact has been increasing and there are laws or initiatives regarding environmental impact during production, during usage and at the end of life. This is referred to as *Extended Producer Responsibility*. An example is the Waste from Electrical and Electronic Equipment (WEEE) directive of the European Union (EU), which mandates separate collection, reuse, recycling and/or disposal of electronic products and the materials within [39].

Another example is the goal set by the European Commission to cut greenhouse gas emissions by at least 55% by 2030, and become climate neutral by 2050 [40]. An important aspect of this goal is to reduce the emissions from commercial and passenger vehicles. So far, this was achieved through imposed emissions norms, where the manufacturers had to switch producing and selling environmentally better products based on a specified time frame. However, it is not possible to improve the emission levels from internal combustion engines anymore and the new strategy of the manufacturers is to use green energy sources such as compressed natural gas (CNG), hydrogen, or electric batteries to operate the vehicles. The main challenge is the lack of infrastructure for refilling and charging.

1.2 Scope of This Study

In this study, we consider the aftermarket of products that are specialized, require product-specific knowledge and expertise for repair and maintenance, and are sold with a warranty coverage. The customers make their purchasing decisions based on a utility function that has fixed and variable positive components, as well as considering the product price and their perceived future after-sales costs as negative components.

Based on the terminology we have introduced in the previous section, we conduct a literature review in Chapter 2. Also, following the same terminology, this study addresses the after-sales, warranty and channel decision topics. We model these topics in a novel and flexible way by considering all after-sales transactions by categorizing them into two main groups: *products* and *services*. We then define the total amount of after-sales products and services that are needed during the complete lifetime of the product. In order to easily differentiate between the durable product and the after-sales product, we refer to the after-sales products as *spare parts*, and the after-sales services as *labor*. Although the used terminology is limited mostly to repair and maintenance activities, our results and discussions apply to various after-sales-related products or services. Then, we divide the total after-sales transactions into two categories: those products and services paid by the end customer and those covered by the manufacturer; i.e., warranty.

Although warranty coverage is cited as a *service* and *protection* to the customer with rationalizations like insurance, signaling, and incentive, manufacturers do not seem reluctant to offer warranty coverage either. Thus, one wonders if warranty coverage is always in favor of the customer or always a burden for the manufacturer. In this study, we address this issue through three research questions: (1) What is the ideal warranty coverage from the customer's perspective and from the manufacturer's perspective? (2) How important is it that the customer can accurately foresee the after-sales service costs of a product? (3) What drives the manufacturer to control the after-sales market through warranties and services?

From a channel perspective, we consider a manufacturer who is the monopolistic supplier of a sophisticated durable product. In Chapter 3, we introduce our first model, which we refer to as the monopoly or the decentralized model, where the manufacturer has a retailer who is the monopolistic supplier of the after-sales services. The manufacturer sets the product retail price and the wholesale price of spare parts to the retailer and the retailer sets the retail sales price of spare parts to the end customer, where the customer makes his buying decision considering the total cost of ownership of the product, factoring both sales and after-sales related costs and benefits. We derive the equilibrium analytically and find that the manufacturer extracts all of her profit from the after-sales services of the product. Furthermore, the extent of the warranty coverage offered by the manufacturer has no effect on the equilibrium results and her profitability. It is to the manufacturer's advantage to have a less transparent after-sales market, whereas it is to the consumer's advantage to have full foresight into the future costs related to the product.

Next, we compare the monopoly model to a centralized version as a benchmark, where the manufacturer and the retailer act as a single decision maker together. We see that, though more efficient, the centralized firm still takes advantage of the limited foresight into the aftermarket, and extracts all profit from the after-sales services. Thus, in a monopoly environment, whether the retailer is integrated with the manufacturer or is an independent entity does not have an impact on how warranty affects customer welfare.

For the monopoly model, to investigate whether a base warranty coverage is preferred by the parties in question at all, we consider an alternative scenario where the durable product can be repaired or maintained by any skilled independent retailer. By comparing this scenario with the monopoly model, we evaluate whether the manufacturer would prefer to offer a warranty coverage and actively control her after-sales market (e.g., through very specific designs that can only be handled by her), or would rather stay uninvolved and let the independent retailer provide the after-sales services. We call this setting the *third party model*. Here, we evaluate the business model favored by each party (i.e., the manufacturer vs. the customer) under varying market conditions, and investigate when the preferences are aligned and when they differ.

When we compare the decentralized model with a third party alternative, we find that the manufacturer prefers to control the aftermarket business when customer foresight of future costs is low. If customers can foresee the total life cycle costs with a moderate or higher precision, the manufacturer's efficiency in after-sales services, customers' base valuation of the product, and customer heterogeneity in the market affect the manufacturer's preferences as well. Customers mostly favor the third party model, especially in markets with high heterogeneity and high product valuation. As a consequence, the two parties' preferences align occasionally, and mostly on the third party model, when the customer foresight is high enough, and especially if the manufacturer is not as efficient in after-sales services.

In Chapter 4, we introduce and study the downstream competition model, where the manufacturer is still the sole provider of the product, but the retailer competes in the aftermarket with an independent workshop that is selling and installing non-original spare parts. We are able to derive the retailer's best response, but cannot fully characterize the equilibrium analytically due to complexity. Here, we find that the downstream competition model produces results equivalent to the monopoly model if the total cost of ownership of the independent workshop alternative is too high for the customer.

In Chapter 5, we conduct a numerical analysis of the downstream competition model to gain insights on the equilibrium structure. We study the following research questions: (1) Under what conditions does the manufacturer (and the retailer) monopolize the after-sales market? Alternatively, does the independent workshop ever monopolize the after-sales market? (2) Does the manufacturer benefit from the existence of an independent workshop at all? Under what conditions? How about the consumer surplus? Is there any case where having competition in the after-sales market worsens/ does not improve the consumer surplus? When do the interests of the consumer and the manufacturer align and when do they differ? (3) What is the best warranty coverage decision from the manufacturer's perspective vs. the consumer's perspective? (4) How does the manufacturer price the product and spare parts? As a result, what is the manufacturer's main source of profits: sales or after-sales? Does she accept losses on either side? (5) How strong is double marginalization? How are the chain profits and manufacturer profits affected by the existence of the independent workshop?

Finally, Chapter 6 provides a brief conclusion and future research directions. All proofs, as well as some additional theoretical results, are given in the Appendices.

CHAPTER 2

LITERATURE REVIEW

Durable goods is linked with an extensive literature that has been studied under several topics. This study is related with four major research streams in the durable goods literature: (1) Warranty, (2) After-sales/Aftermarket and Compatibility/Complementary Products, (3) Life Cycle Costing and Total Cost of Ownership, (4) Channel structure. We give a brief overview of each below, plus some other references in the other aforementioned streams; referring the further interested reader to the reviews by Waldman [95] in the Economics field and Mantena, Tilson & Zheng [66] in the Operations Management field for further review of topics related to durable goods.

2.1 Warranty

There are several comprehensive literature reviews on the subject of warranties. Some examples are by Blischke [16], Blischke & Murthy [17], Murthy & Blischke [68, 69], and Murthy & Djamaludin [70].

Some interrelated decisions/factors that affect warranties that have been addressed in the literature are as follows:

- Customer's maintenance effort / customer demand for maintenance
- Required pre/post warranty preventive/corrective maintenance effort
- Probability of failure
- Product quality

In this context of warranties, the parameters regarding the durable nature of the product that have been looked into are given below:

- Substitutability between old and new products
- Strategic vs. myopic consumers
- Two period vs. infinite horizon

Most of the warranty literature focuses on the correct estimation of repair costs and selection of warranty policy that minimizes these costs [see 16, 17, 68, 69, 70, for relevant reviews]. However, the sub-stream that relates to our research is joint price and warranty period optimization. Lázár [60] provides a review for the empirical studies in this field which make econometric modelling of the impact of warranties on demand. The paper by Glickman & Berger [46] is one of the earliest examples of this area utilizing analytical modeling for a given warranty policy, in which the demand is represented by an exponential form, where the price reduces it and the warranty period increases it. Their model is further extended by [98], [41], and [97].

DeCroix [30] optimizes the warranty, reliability and price decisions for firms operating in an oligopoly, where consumers cannot directly observe the reliability but have an indication through the warranty (signaling). He shows that the firms are better off to offer more reliable products and longer warranties in the presence of more risk-averse consumers. Padmanabhan & Rao [74] consider the setting where the manufacturer offers extended service contracts in addition to the base warranty. They find that the manufacturer should offer a base warranty that the least risk-averse segment is willing to buy, and offer optional extended service contracts for the more risk-averse customers. They also verify their results with an empirical study. Zhou, Li & Tang [104] consider a product with a defined market entry and exit (planned obsolescence) and model jointly determining the warranty period and price dynamically.

Another stream relates the warranties with preventive/corrective maintenance and moral hazard [e.g., 65, 84]. Balachander [10] analyzes the empirical observation that a longer warranty period might be offered for a product with lower quality, explaining the circumstances of this seemingly contradictory nature of this observation with the signalling theory.

There is also a significant amount of empirical study on warranties. Some examples are as follows: Guajardo, Cohen & Netessine [50] formulate an empirical model to analyze the role of services as part of a firm's competitive strategy in the U.S. automobile industry, and the joint effect that warranty length and the quality of after-sales service have on consumer demand. Chu & Chintagunta [25] test the validity of the four warranty rationales for computer servers and automobiles in U.S. , using empirical data. They find sufficient evidence to support the insurance and sorting theories, but cannot find any correlation regarding the signalling and incentive theories.

Our study's difference from this stream lies in our approach to the problem. This literature mainly focuses on optimizing the warranty policies given the expected costs, with a very detailed representation of how these costs occur. We, however, look at warranty from a strategic perspective. Another strength of our study is that we consider the total expenses that are borne by the customer in comparison to the total expenses that are borne by the manufacturer during the product's lifetime. This allows us to focus on the utility derived by the customer from warranty, while also covering essentially the whole array of possible warranty policy and reliability parameters for the product.

2.2 Aftermarket/Aftersales

One interesting strategic design decision a manufacturer makes that relates to our study is the compatibility of her product with complementary products from other manufacturers. This compatibility decision may involve potential indirect network effects, competition, and the dominance of the after-sales market. The current literature generally focuses on the latter two aspects, mainly from an antitrust perspective. For example, Morita & Waldman [67] analytically show that a manufacturer would want to monopolize the maintenance of her durable goods, as this helps her eliminate the time inconsistency problem. Erzurumlu [37] studies a durable good manufacturer's product compatibility decisions with generic consumables provided by third-party manufacturers. A review of complementary products and network effects with an emphasis on competition can be found in Farrell & Klemperer [42]. They conclude that the public policies should be in favor of compatibility. The main contribution of our study is adding the warranty dimension to the classical comparison of manufacturer benefit vs. consumer benefit in a realistic supply chain setting: We examine two cases where the durable product is either fully compatible, or not compatible at all with third party aftermarket products.

The literature that considers the supply chain issues related to after-sales within an Operations Management context is somewhat more limited. We refer the reader to Durugbo [35] for a recent review and highlight three of these research articles that more specifically relate to our study here.

In Kurata & Nam [58], the customer's purchase decision depends on the after-sales service levels set by the manufacturer and the retailer. Basic after-sales service is given by the manufacturer, with additional maintenance available to be bought from the retailer. Kurata & Nam [59] extend on [58] by introducing uncertainty to the optimal service level preferred by the customers. Our model differs from theirs as they consider out-of-warranty maintenance as optional, and the price sensitivity of the demand is considered only partially in one of the five models studied.

Cohen & Whang [27] consider a setting where a manufacturer produces and services a product, competing with an independent service shop for the services after the warranty period. They determine the equilibrium price and service quality for the manufacturer. Our study adds to this work by considering a designated retailer undertaking the services in the downstream, and by analysis of a third party model where the manufacturer doesn't offer any warranty coverage. We also have a similar setup in our downstream competition model. Additionally, our model has a more flexible treatment of the warranty coverage.

2.2.1 Extended Warranties and Repair and Maintenance Contracts

Li *et al.* [61] study the problem where an extended warranty can be offered either directly by the manufacturer or the retailer. They look into the chosen policies and prices and compare these alternatives with respect to the system profit.

Lutz and Padmanabhan [65] consider extended warranties sold by a third party.

2.3 Channel Structure

Selection of the channel structure for durable goods involves a decision for both sales and after-sales activities. Regarding the retailer's decisions, the major research questions focused on production lead time, production capacity, end customer pricing and demand volatility. The decisions of the manufacturer usually then focuses on warranty terms and duration, product characteristics and wholesale pricing.

The first group of articles we refer to consider only the sales activities within a durable goods context by studying the problem in an analytical fashion. In this line of work, after-sales is often modeled with a service-level parameter that enhances the demand, if it is addressed at all. Taylor [87] looks into a single monopolistic manufacturer and one retailer case and analyzes when channel coordination could be achieved when the manufacturer offers the retailer rebates based on his sales effort. Decentralization has also been proposed with the goal of eliminating the time inconsistency regarding durable goods (aka the *Coase conjecture*). Desai et al. [32] study a two-period problem of a manufacturer and resolve the conflict by committing to sell to a retailer using a two-part contract. Similarly, Arya and Mittendorf [7] study two- and three-period products (as well as providing a generalization for n periods) and show that double-marginalization in itself can be used as a tool to resolve the conflict, even without a precommitment on wholesale prices. Chen & Wang [23] evaluate whether a smart phone manufacturer prefers a free or a bundled channel in the presence of a service operator. Tsay and Agrawal [93] look into a distribution channel of one manufacturer that distributes the product over two retailers with different service levels (and prices). In US, the manufacturers are banned from selling directly to the end customers by several state laws. This is often criticized (where Bodisch [18] provides a nice overview and argumentation), and recently openly challenged by Tesla [20].

The second group considers only the after-sales activities: Gill & Roberts [44] model the revenue and costs of retailers resulting from warranty repairs. Xia and Gilbert [102] study the after-sales channel structure with one manufacturer, one retailer and two partially substitutable products, where the sales is made through the retailer. The customers make their purchase decision based on the price and service level of each product. The retailer's decision is to centralize its effort on the products vs. keeping them separate, whereas the manufacturer's decision is to provide the after-sales services herself vs. delegating it to the retailer. Chen, Li & Zhou [24] analyze a manufacturer's wholesale price strategies with two competing retailers where the retailers provide warranty and decide the warranty period length. Saccani, Johansson & Perona [80] conduct a case study of the after-sales supply chain configuration for seven companies in durable goods industries. Zheng [103] looks into two research questions: Firstly, how a manufacturer in a volatile economy makes production and pricing decisions to overcome demand uncertainty and extract profit from the market. Secondly, he analyzes a two-echelon supply chain model of a manufacturer and a retailer under demand uncertainty. The main decisions for the manufacturer are her production capacity level (which results in limited supplies for the retailer) and wholesale pricing strategies. For the retailer, it is the determination of selling and leasing strategies (which affects the manufacturer's profit).

Finally, the third line of studies we refer to look at the sales and after-sales channel structure simultaneously and acknowledge that several centralized or decentralized structures are employed in real life. They then conduct empirical studies that aim to find out what kind of product-related attributes, firm-related attributes or industry/ market-related attributes lead to what kind of channel structures (see, for example, Loomba [63, 64], Goffin [47], and Nordin [72]). Loomba [63] builds three hypotheses based on a case study of two firms. Loomba [64] then extends on this study by testing the validity of these hypotheses on 393 manufacturing firms in the US computer equipment industry. Goffin [47] first provides the key elements of what we refer to as the aftermarket and he names *customer support*: installation, user training, documentation, maintenance and repair, online support, warranty, upgrades. Nordin [72] further extends on this line of research by investigating six relevant propositions from the literature concerning the firm's sales and after-sales channel structure with respect to again product-, firm-, and market-related dimensions.

We contribute to this stream of literature by studying the sales and after-sales activities in an integrated fashion through analytical modelling.

2.4 Life Cycle Costing and Total Cost of Ownership

Life Cycle Costing (LCC) deals with identifying the costs of a product throughout its complete life cycle, starting from design and ending with disposal/recycling. The application of LCC was initially developed for military and aerospace industries and their specialized equipment, which later became a tool for manufacturers to reshape the product design stage. Asiedu & Gu [8] provide a review of the subject by first defining the steps and cost components in the life cycle of a product, followed by an identification of the different cost estimation models in the literature. A relevant stream within the LCC literature deals with the life cycle cost assessment of different warranty and maintenance policies/strategies. An earlier example is by Blischke [16], followed by the more recent examples by Chattopadhyay & Rahman [21] who look into lifetime warranties, and by Wu & Longhurst [100] who focus on replacement strategies.

Total Cost of Ownership (TCO) is a subset of the LCC concept and refers to the total lifetime cost of purchasing, operating, and disposing a product. As opposed to LCC, which has a product-centric perspective and does not directly concern itself with who bears the costs (be it the manufacturer, customer, or society), TCO has a customer-centric perspective and focuses on the costs borne by the customer. The focus on TCO has been increasing over the years, first in the B2B environment, and later in B2C [81]. A recent review and a generalized model for identifying TCO is done by Saccani, Perona & Bacchetti [81]. Some other research papers worth mentioning are by Gilmore & Lave [45], Wu, Inderbitzin & Bening [99], and Dumortier et al. [34], which provide TCO analysis and comparison of old and new propulsion technologies in passenger vehicles. Both [99] and [34] observe that consumers focus too much on the high purchase price of electric vehicles and are not well-informed about the TCO information. Therefore, they advocate increasing the availability of this information to consumers, [34] additionally showing that providing the TCO information makes a positive impact on their choices.

Most of this literature in this field are either conceptual studies on how to identify and categorize the costs [as in 81], or empirical studies that look into how and why TCO affected the customer's decisions [as in 34, 45, 99]. We contribute to this stream by explicitly modeling how TCO affects the customer's buying decision, which then translates into manufacturer and retailer profits, as well as consumer surplus.

2.5 Design Decisions

Some product design decisions analyzed in the literature are as follows:

- Durability parameter (planned obsolescence vs. customer loyalty)
- New version releases (e.g., timing [83])
- Compatibility with third-party complementary products

Banker *et al.* [11] model an oligopolistic competition and analyze whether product quality improves as competition intensifies. They find that the answer depends on how the intensification of competition is defined, as well as the cost and demand structure of the industry.

2.6 Used Products

Most papers assume a monopolistic manufacturer. Esteban and Shum [38] consider an oligopoly with a used vehicle market. They take automobiles as their product and estimate the parameters of their model from the industry. They show that the presence of a secondary market affects the manufacturers' quantity decisions for today, as well as the customers' demand.

Anderson and Ginsburgh [3] analyze how a monopolist sets the new product price where a secondary market with transaction costs exists and used products are lower substitutes for new products. They show that the monopolist doesn't always necessarily price the new product so that the secondary market is eliminated, but rather can use the secondary market as a price discrimination tool to her advantage.

Kogan [57] considers a manufacturer and retailer, where the new product market is a monopoly but the used product market faces competition. The manufacturer produces goods and the retailer sells new and used products, as well as services. He shows that the secondary market acts as a means of coordinating the supply chain.

2.6.1 Adverse Selection

Rao *et al.* [77] propose regular implementation of trade-ins as a means of reducing inefficiencies arising from information asymmetry. They show that trade-in programs are more valuable for products with lower reliability and/or higher durability.

Chemmanur *et al.* [22] consider a double-sided asymmetric information case, where the manufacturer leases his products to entrepreneurs. The manufacturer has private information regarding the product's quality, whereas the entrepreneurs have private information regarding their intensity of use and the maintenance they perform. Leasing turns out to be an equilibrium for this setting. They also analyze how different contractual settings affect the outcome.

Sultan [86] models the situation where a customer can either buy or lease a new vehicle, or buy a used vehicle that is a Certified Pre-Owned (CPO) vehicle. He shows that the average quality of the traded cars in such a market is not strictly below the average quality of the non-traded cars and can be lower or higher. He then suggests that the new channels such as leasing and CPO have changed the market dynamics, reducing the adverse selection effect.

Peterson & Schneider [75] categorize a car's components in two groups: ones with higher information asymmetry (e.g. engine, transmission, etc.) and ones with lower information asymmetry (e.g., vehicle body, A/C, etc.). They base their analysis on the repair data of newly purchased used cars and show that the information asymmetry for different components is indeed different. They also argue the cases when the repair is a one-time job (a transitory problem) vs. a problem regarding the inherent quality of the vehicle. Most problems turn out to be transitory ones.

2.7 Leasing and Servicizing

2.7.1 Leasing

There is an extensive literature regarding issues related to leasing. Here, we briefly review the papers by Desai & Purohit [31], Desai & Purohit [33], Agrawal *et al.* [2], Huang *et al.* [54], Tilson *et al.* [92], Bhaskaran and Gilbert [14], Bhaskaran and Gilbert [15], Johnson and Waldman [56], Aras *et al.* [5] below.

Desai & Purohit [31] consider a monopolistic environment, where a sold product depreciates differently from a leased product. It is known that leasing is better than selling when the depreciation rates are different. But the authors show that the conclusion changes and usually a mixture of both is better when the depreciation rates are different. We based our work for this project mainly on the methodology followed in this paper. However, we were only able to address the problem when only leasing is considered.

Desai & Purohit [33] model a two-period duopoly where each manufacturer determines its quantity and fraction of leases. They show that the firms never prefer fully leasing; and the fraction of leases decrease as the substitutability between the manufacturers' products increases (as leased units compete against sold units later). They also find that a product's proportion of leases increases as its quality/durability increases.

Agrawal *et al.* [2] analyze when leasing would be both greener and more profitable than selling in the steady state, considering a monopolistic manufacturer and conducts an analysis by changing the durability. Their analysis follows from Huang *et al.* [54], who lay the ground work for the model where a monopolist's products have finite durability, new and used product markets can coexist and there are transaction costs in the used market.

Tilson, Wang & Yang [92] also extend upon Huang *et al.* [54]'s work. They consider a differentiation among customers: individual vs. corporate and model with similar settings to determine the behavior of selling and leasing prices and volumes, as well as their effects on customer welfare.

Bhaskaran & Gilbert [14] consider a monopolist manufacturer that either sells or leases its products. There is also an independent firm that produces a complementary product. They show that the manufacturer's optimal decision is to have a mix of leasing and selling, balancing the costs of overproduction and underproduction in response to the independent firm's price/quantity setting.

Bhaskaran & Gilbert [15] consider a scenario where a monopolist manufacturer sells products to her competing dealers and the dealers have the option to either sell or lease the vehicle to end customers. The main tradeoff for the manufacturer then becomes double marginalization vs. time inconsistency. They find that the manufacturer would prefer to lease the products to the end customers herself and pay a brokerage to her dealers.

Johnson & Waldman [56] provide a new model for the lease vs. sell problem where the main cost of leasing is moral hazard, while its returns are reduced adverse selection and reduced transaction costs. They derive five propositions from their model regarding relations between different parameters and their outcomes. They then statistically test their propositions on real life data, showing that the model explains some interesting empirical findings, such as leasing becoming more popular over time and among high-income users, and off-lease cars sell better than used cars.

Aras, Güllü & Yürülmez [5] model an operational-level decision, where the firm leases remanufacturable products. The aim is to meet the demand, while optimizing the total profits (revenues minus material, manufacturing, holding and shortage costs) by determining the amount and pricing of external purchases of used products, as well as the pricing for the leases.

2.7.2 Servicizing

Baines *et al.* [9] provide a literature review on the subject of servicizing (also called *servitization*). They identify the major research challenge in the field as developing work that can help practitioners. They identify the major challenge for the manufacturers as understanding how customers will value their services.

2.8 Antitrust

Most of the aftermarket discussion in Law builds around aftermarket monopolization, which is signified by the infamous Kodak Supreme Court case in 1992, where Kodak refused to supply spare parts to independent service providers. Here, the pivotal question is whether the market for the product (usually referred to as the *primary market*) can be considered separate from its aftermarket. In the Kodak case, they were considered as a single market and Kodak was not found guilty as they had little market share in the primary market. The research then splits into to major streams; The first group discusses that aftermarket monopoly due to consumer lock-in is harmful and provides reasonings why; whereas the second group discusses that aftermarket monopoly helps eliminate some inefficiencies.

Ardiyok [6] reviews the history and current status regarding competition laws relevant to aftermarket sales for durable goods in the U.S., Europe and Turkey. He then looks into the Block Exemption Regulation and conducts an empirical study to assess its effectiveness.

Bauer [12] discusses aftermarket competition issues from a law perspective. He identifies some settings with aftermarket issues, how firms handle these, what the result is, and how they can be treated by the law.

Bennett *et al.* [13] argue that competition can encourage firms to engage in unethical and even unlawful behavior in order to gain and/or keep customers. They use data from vehicle emission testing facilities to test their hypothesis.

The paper by Bodisch [18] is an advocacy paper which suggests that the ban on auto manufacturers to sell directly to the customers in the U.S. should be lifted. We see that both parties that are for and against this ban use the "economy" argument; where the group that is for the ban focuses on the number of people employed by the dealers, and the group that is against the ban focuses on potential cost savings for the manufacturer, which then would be reflected to the end customers.

Urban *et al.* [94] discuss that an overly regulated market for used vehicles that impose a fixed selling price between a manufacturer and a dealer is actually bad for consumers, under the concept of retailing for used vehicles. Their findings are supported with statistical results from a questionnaire they conducted.

Waldman [96] addresses the following issues: optimal antitrust policy for durable-goods mergers; practices that eliminate secondhand markets; tying in markets characterized by upgrades and switching costs; and antitrust policy for aftermarket monopolization in durable-goods markets. His approach is similar to the one in Waldman [95], the major difference being that this paper focuses on antitrust issues in durable goods markets, rather than the overall picture.

Gonçavales [49] looks into whether the introduction of the block exemption regulation in 2003 resulted in the targeted level of competition in the automotive industry and concludes that the effects observed so far are not at the desired level. Her analysis is mainly based on interviews and interpretation of statistics regarding the market.

2.8.1 Compatibility and Complementary Products

An interesting strategic design decision a manufacturer makes is regarding compatibility of the durable good with complementary products available from other manufacturers, hence considering both competition and potential positive networking effects (increasing demand and/or utility). A review of complementary products and network effects with an emphasis on competition can be found in Farrell and Klemperer [42]. They conclude that the public policies should be in favor of compatibility.

Morita and Waldman [67] analytically show that a manufacturer would want to monopolize the maintenance of her durable goods, as this helps her eliminate the time inconsistency problem.

As mentioned before, Bhaskaran and Gilbert [14] also consider leasing vs. selling in the presence of a third-party complementary goods manufacturer. Similarly, Erzurumlu [37] analyzes a firm's strategic decisions regarding product compatibility and production quantities when competitive consumables enter the market.

The literature on durable goods with complementary products usually focuses on competition and monopolization of aftermarkets. Research regarding interaction with other strategic decisions is not very extensive. Mantena *et al.* [66] also point out this issue, stating that durable goods that especially involve information have a large number complements and how the producers of those should coordinate innovation is an important issue to consider.

2.9 Environment

Plambeck and Wang [76] look into the manufacturer's new product introduction and pricing strategies in the existence of regulation for collection of end-of-life electronic products (referred to as *e-waste*).

Thomas [91] investigates the relationship between the reuse of products and the demand for new products. Her findings indicate that if the product has a positive secondhand price, increased secondhand sales in these markets can correspond to increase in new product markets, increasing total material consumption. For products with no secondhand price, the increase in demand originates from the customers who did not buy the new product.

As mentioned before, Agrawal *et al* [2] look into the leasing vs. selling problem from an environmental perspective. Their findings are in line with those of Intlekofer *et al.* [55]'s, who provide two case studies. Their results indicate that products with high use impacts and improving technology can benefit from leases, whereas products with high manufacturing impacts and no improving technology do not.

Souza [85] provides a recent and critical review of the literature for closed-loop supply chains. He mainly discusses remanufacturing, but also mentions leasing and trade-ins as a manufacturer's means of purchasing used vehicles.

Gilmore and Lave [45] make a statistical analysis of auction data to compare the resale values and total costs of ownership (mainly based on fuel usage) of gasoline, hybrid and diesel passenger cars and trucks. The analysis shows that the hybrid and diesel vehicles lose less value than gasoline vehicles and hybrid vehicles have a higher resale value when the gasoline prices went higher.

Agrawal and Toktay [1] identify potential areas of research for closed loop supply chains, based on a discussion of current industry practices. One area they specify is really interesting, which is called a "product-service system." It is defined as "a product and a service combined in a system to deliver consumer needs and reduce environmental impact, typically by displacing new production or increasing usage efficiency." This is essentially a broader concept of leasing. Some of the research questions they propose for this area have already been addressed, but the topic remains interesting, as well as the other potential research questions they address.

CHAPTER 3

MONOPOLY MODEL

In this chapter, we consider a monopolistic manufacturer (she) that produces a durable good and provides after-sales services through an authorized retailer (he). When making purchase decisions, customers do not only consider the price of the product. They also take into account, to the extent of transparency in the market, necessary costs involved with using the product after the purchase; i.e., the total cost of ownership (TCO). We consider all after-sales-related expenses and refer to all after-sales products as *spare parts* and all after-sales services as *labor*.

The manufacturer offers a base warranty that comes free with the product and outsources the warranty-related repairs to the retailer, where she provides him with spare parts free of charge and fully reimburses the associated labor costs. The retailer buys all non-warranty-related spare parts from the manufacturer. The retailer himself is a monopoly in after-sales services and the customers need to buy their non-warranty repair and maintenance needs from him during the lifetime of the product. We call this base monopoly model “decentralized" and refer to it with superscript D.

The calculation of repair costs is a very complex problem, which is relevant for both managerial decisions and accounting purposes of the manufacturer. A relatively simple modelling approach for estimating these costs is multiplying the unit cost of failure with the expected number of failures within a given time interval [90]. Failures would then trigger repair activities. Here, we extend the same methodology to the cost calculation for operating supplies (e.g., a new cartridge for an inkjet printer), maintenance activities (e.g., oil change for a car), and complementary goods and services; as all of these purchases need to be made regularly. We refer to all these potential costs and purchases under the term “repair and maintenance activities." Thus, we represent the total expected number of repair and maintenance activities during the lifetime of the product as f. The number of repair and maintenance activities whose costs are covered by the manufacturer are denoted by f_m (which we consider as the manufacturer's warranty coverage), and the number of those paid by the customer are denoted by f_e, where $f = f_m + f_e$. The expected cost per each repair or maintenance request is assumed to be constant throughout the lifetime of the product.

A number of different factors can be incorporated into the parameters f_e and f_m: Firstly, if a repair and maintenance package or an extended warranty package is included in the manufacturer's default option, those can be reflected by higher f_m and lower f_e values. Secondly, no explicit discounting is made for the future costs while calculating the TCO in the model. This adjustment can be reflected by reducing the values of f_e, f_m, and hence f accordingly.

Each after-sales transaction, regardless of whether it is covered by the manufacturer or paid by the customer, is composed of two factors: Labor (after-sales services) and spare parts (after-sales products). On average, one unit of spare parts (which corresponds to the average amount of spare parts used per transaction) requires α units of labor in order to be installed on the product. The customer pays s_o for each unit of spare parts and l_r for each unit of labor sold by the retailer. The retailer sets the spare parts selling price, s_o, whereas the labor cost l_r is exogenously determined by the market standards.[1]

Customers make the purchase decision based on their net utility, $U_\theta^D(p, s_o)$, as detailed in Equation 3.1.

$$U_\theta^D(p, s_o) = \begin{cases} 0, & \\ \qquad \text{if the customer of type } \theta \text{ does not buy,} & \\ \upsilon + \theta - p - \beta f_e(s_o + \alpha l_r), & \\ \qquad\qquad \text{if the customer of type } \theta \text{ buys.} & \end{cases} \tag{3.1}$$

The utility decreases as TCO increases (i.e., as the labor and parts prices increase and/or the warranty coverage decreases). Here, β is a market-specific gauge on

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[1] For high-value goods like automobiles or construction equipment, the labor required for each "job" in after sales services is standardized. This is also reflected in TCO estimations provided by manufacturers [e.g., 26] or platforms such as Kelley Blue Book and Edmunds.com [36].

transparency regarding future costs associated with a product and customer's level of strategic thinking. For example, $\beta = 0$ shows that only immediate expenses are considered, without any regard or access to TCO information. We assume $0 < \beta \leq 1$ in our model. A market with customers that have low (high) foresight into the future costs has a low (high) β, which we refer to as a market with "myopic" ("strategic") customers.

Customer's valuation of the product has four components: the base valuation υ that is constant for all customers, θ that is the customer's individual valuation attached to the product, p that is the retail price of the product, and $\beta f_e(s_o + \alpha l_r)$ that is the total foreseen repair and maintenance cost that will be paid by the customer. υ and θ are utility-increasing components: All customers derive a fixed utility from a product and prefer owning one at no cost ($\upsilon > 0$). Differently, customers are heterogeneous in their additional valuation; we assume that θ is uniformly distributed on $[0, b]$ as set forth by Hotelling [52], where $b > 0$ and the total market size is 1. Overall, a customer with a high θ is willing to pay more for the product. The other two components, namely p and $\beta f_e(s_o + \alpha l_r)$ are the cost terms (which are also fixed for all customers), hence reducing the utility. The reservation utility is assumed 0.

We consider a game where the manufacturer (the leader) is followed by the retailer. Please note that we inherently assume the product itself comes into existence first, followed by its spare parts; and the pricing decisions then follow the same sequence. The decisions and the sequence of events are as below.

1. Manufacturer sets the product retail price, p
2. Manufacturer sets the spare parts wholesale price to the retailer, ω_o
3. Retailer sets the spare parts retail price to the end customer, s_o

A depiction of the financial flow is given in Figure 3.1 and an overview of the parameters and decision variables is given in Table 3.1.

The retailer's profit function is given in Equation 3.2. He earns a profit from warranty-related labor work for all products sold, charged to the manufacturer.[2] The second

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[2] Note that the retailer charges the same price to the manufacturer and the customer for labor.

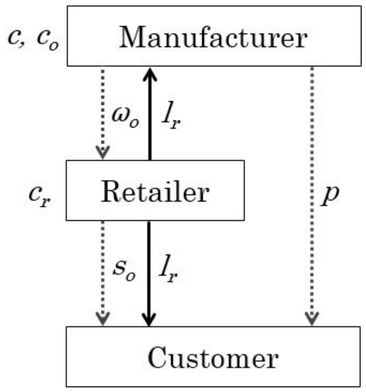

Figure 3.1: Financial Flow of the Decentralized Model

profit stream for the retailer is sales of non-warranty labor and spare parts to the customers, where the unit margin for the parts is found by subtracting the manufacturer's wholesale price ω_o from the retailer's selling price s_o. Each unit profit component is multiplied by the total number of purchasing customers, $q^D(p, s_o)$.

$$\Pi_r^D(p, \omega_o, s_o) = (f\alpha(l_r - c_r) + f_e(s_o - \omega_o))\, q^D(p, s_o) \tag{3.2}$$

The manufacturer's profit function is given in Equation 3.3. She sells each unit for the retail price p with a margin $p - c$, but she incurs the costs of spare parts and labor, $c_o + \alpha l_r$ under warranty. The second profit stream is the sales of spare parts to the retailer with a wholesale price of ω_o for non-warranty repair and maintenance services.

$$\Pi_m^D(p, \omega_o, s_o) = (p - c - f_m(c_o + \alpha l_r) + f_e(\omega_o - c_o))\, q^D(p, s_o) \tag{3.3}$$

Consumer surplus is calculated with the generic function $\int_{\hat{\theta}}^{b} U_\theta^D(p, s_o)$, where $\hat{\theta}$ represents the last customer that purchases the product. However, since the customers underestimate the future costs by a factor of β, but actually receive a *real* utility of $v - p + \theta - f_e(s_o + \alpha l_r)$, we consider this function when estimating the consumer surplus.

$$CS^D(p, \omega_o, s_o) = \int_{\hat{\theta}}^{b} \frac{1}{b}(v - p + \theta - f_e(s_o + \alpha l_r))d\theta \tag{3.4}$$

Table 3.1: List of Parameters and Decision Variables for the Decentralized Model

Parameters	
f_m	Expected number of repair and maintenance needs paid by the manufacturer (under warranty)
f_e	Expected number of repair and maintenance needs paid by the customer (non-warranty)
f	Total expected number of repair and maintenance needs during product's lifetime ($f = f_m + f_e$)
c	Manufacturer's unit product cost
α	Units of labor required per unit of spare parts installed in a workshop order
β	Customer's ability to foresee/forecast future costs ($0 < \beta \leq 1$)
c_o	Manufacturer's cost of spare parts
c_r	Retailer's cost of labor for one unit of labor
l_r	Price of retailer's labor sales to the customers and the manufacturer
v	Customer's base valuation of the product ($v > 0$)
θ	Customer's additional valuation of the product ($\theta \sim U[0, b]$)

Decision Variables	
s_o	(Retailer) Retail price of spare parts to the end customers
ω_o	(Manufacturer) Wholesale price of spare parts to the retailer
p	(Manufacturer) Retail price of the product

3.1 Analysis

For the analysis of the decentralized model, we find the equilibrium with backward induction. For a given pair of s_o and p, the product sales quantity ($q^D(p, s_o)$) is realized as follows:

$$q^D(p, s_o) = \begin{cases} q_0^D = 1, & \text{if } s_o \leq \frac{v-p}{\beta f_e} - \alpha l_r, \\ q_1^D = 1 - \frac{\beta f_e(s_o+\alpha l_r)-v+p}{b}, & \text{if } \frac{v-p}{\beta f_e} - \alpha l_r < s_o \leq \frac{v-p+b}{\beta f_e} - \alpha l_r, \\ q_2^D = 0, & \text{if } s_o > \frac{v-p+b}{\beta f_e} - \alpha l_r. \end{cases} \quad (3.5)$$

In the sales quantity expression above, we observe that the required labor component, αl_r, comprises a deduction term in the threshold values. In other words, the thresholds are set essentially to cover the customer's total repair cost per visit; i.e., spare parts plus the required labor, $s_o + \alpha l_r$. If this total cost is lower than $\frac{v-p}{\beta f_e}$, then all customers buy the product, hence the market is fully covered. We can refer to this value as the customer's minimum willingness-to-pay for a repair. On the other extreme, no one buys if the repair cost is more expensive than $\frac{v-p+b}{\beta f_e}$. Similar to the other threshold, we can refer to this value as the customer's maximum willingness-to-pay for a repair. The range between full market coverage and no market coverage is defined by the market heterogeneity in valuation, b, but divided by the parameters β and f_e. Therefore, the partial market coverage range for the total repair price, $s_o + \alpha l_r$, is larger if the customers have a broad range in valuation (high b), or when they are myopic (low β), or when the manufacturer covers fewer number of repairs (low f_e). Another point to be noted is that f_m, hence the warranty coverage, only indirectly affects the sales; manifested in the function above through f_e.

Because the demand function is piecewise linear, all profit and surplus functions follow similarly. From this point onwards, when a profit or surplus function has a subscript of 0, 1, or 2, we refer to the piece function valid in the interval for full, partial, and no market coverage, respectively. This notation is especially relevant for the Appendix and the proofs within.

Taking the consequent sales quantity into account, we can characterize the retailer's spare parts price as follows:

Proposition 1 *In the decentralized model, the retailer's best response given the manufacturer's product retail price and the spare parts wholesale price decision, $s_o^*(p, \omega_o)$,* *is as follows:*

$$s_o^*(p,\omega_o) = \begin{cases} \frac{v-p}{\beta f_e} - \alpha l_r, \\ \qquad\qquad\qquad \text{\textit{if}}\ \omega_o \leq \frac{v-p-b}{\beta f_e} + \frac{f_m\alpha(l_r-c_r)}{f_e} - \alpha c_r, \\ \frac{1}{2}\left(\omega_o + \alpha c_r + \frac{v-p+b}{\beta f_e} - \frac{f_m\alpha(l_r-c_r)}{f_e}\right) - \alpha l_r, \\ \text{\textit{if}}\ \frac{v-p-b}{\beta f_e} + \frac{f_m\alpha(l_r-c_r)}{f_e} - \alpha c_r < \omega_o \leq \frac{v-p+b}{\beta f_e} + \frac{f_m\alpha(l_r-c_r)}{f_e} - \alpha c_r, \\ \left[\frac{v-p+b}{\beta f_e} - \alpha l_r\ ,\ \infty\right), \\ \qquad\qquad\qquad \text{\textit{if}}\ \omega_o > \frac{v-p+b}{\beta f_e} + \frac{f_m\alpha(l_r-c_r)}{f_e} - \alpha c_r. \end{cases}$$

The retailer, in a similar way to what we observed for the customer demand in Equation 3.5, essentially sets a total price per repair, $s_o + \alpha l_r$. The cost of labor c_r is deducted from the threshold values as well, defining the boundaries on the total cost per repair, $\omega_o + \alpha c_r$.

The retailer rejects to serve any customer in the market when the manufacturer's wholesale price is higher than a certain threshold. Essentially, he makes sure that no customer purchases when his total cost per repair; i.e., $\omega_o + \alpha c_r$, is higher than the maximum price he can extract from a repair. Interestingly, he takes both types of revenues into account in this calculation: (1) the value that the customer will be willing to pay per (non-warranty) repair, and (2) the labor expenses he expects to charge the manufacturer for his services under the warranty coverage. This second stream is generated in the warranty-related services, and influences his spare parts price decision for the non-warranty services; and hence takes the form $\frac{f_m\alpha(l_r-c_r)}{f_e}$. Thus, the retailer will tolerate higher wholesale prices if the warranty coverage or labor margin is high enough.

When the wholesale price is low enough; i.e., lower than $\frac{v-p-b}{\beta f_e} + \frac{f_m\alpha(l_r-c_r)}{f_e} - \alpha c_r$, the retailer finds serving everyone in the market profitable. In this case, the retailer sets the pricing such that he ensures a profit of $\frac{b}{\beta f_e}$ per non-warranty repair, hence a total profit of $\frac{b}{\beta}$ over the product's lifetime. Note that, the price he charges is free of warranty-related labor; he prefers to extract the full willingness-to-pay of the customer with the lowest valuation.

When we observe the retail price in the partial market coverage case, we see that it contains three components: the customer's maximum willingess-to-pay, $\frac{v-p+b}{\beta f_e}$, the cost of a repair for the retailer, $\omega_o + \alpha c_r$, and the warranty-related profit the retailer receives from the manufacturer, $\frac{f_m \alpha (l_r - c_r)}{f_e}$, all averaged out by dividing into two. This is a result of the tradeoff between demand and margin for the retailer.

Note also that the retailer's spare parts price increases as β decreases; i.e., as customers become more myopic in purchase decisions. Hence, the less a customer is aware of TCO, the more he would be charged.

Proposition 2 *In the decentralized model, for a given value of p, the spare part wholesale price $\omega_o^*(p)$ set by the manufacturer, and the consequent retail price $s_o^*(p)$ are as follows:*

$$\omega_o^*(p) = \begin{cases} \frac{v-p-b}{\beta f_e} + \frac{f_m \alpha (l_r - c_r)}{f_e} - \alpha c_r, & \\ \qquad \textit{if } f(c_o + \alpha c_r) - (p-c) \leq \frac{v-p-3b}{\beta}, & \\ \frac{v-p+b}{2\beta f_e} + \frac{f(c_o+\alpha c_r)-(p-c)}{2f_e} + \frac{f_m \alpha (l_r - c_r)}{f_e} - \alpha c_r & \\ \qquad \textit{if } \frac{v-p-3b}{\beta} < f(c_o + \alpha c_r) - (p-c) \leq \frac{v-p+b}{\beta}, & \\ \left[\frac{v-p+b}{\beta f_e} + \frac{f_m \alpha (l_r - c_r)}{f_e} - \alpha c_r \; , \infty \right), & \\ \qquad \textit{if } f(c_o + \alpha c_r) - (p-c) > \frac{v-p+b}{\beta}. & \end{cases}$$

$$s_o^*(p) = \begin{cases} \frac{v-p}{\beta f_e} - \alpha l_r, & \\ \qquad \textit{if } f(c_o + \alpha c_r) - (p-c) \leq \frac{v-p-3b}{\beta}, & \\ \frac{3(v-p+b)}{4\beta f_e} + \frac{f(c_o+\alpha c_r)-(p-c)}{4f_e} - \alpha l_r, & \\ \qquad \textit{if } \frac{v-p-3b}{\beta} < f(c_o + \alpha c_r) - (p-c) \leq \frac{v-p+b}{\beta}, & \\ \left[\frac{v-p+b}{\beta f_e} - \alpha l_r \; , \infty \right), & \\ \qquad \textit{if } f(c_o + \alpha c_r) - (p-c) > \frac{v-p+b}{\beta}. & \end{cases}$$

We observe that, just as the retailer sets the total repair price $s_o + \alpha l_r$, the manufacturer determines the total cost of a repair to the retailer, $\omega_o + \alpha c_r$.

The thresholds are driven by the term $f(c_o + \alpha c_r) - (p - c)$. This expression represents the cost of repairs over the lifetime of the product (including warranty repairs), $f(c_o + \alpha c_r)$, minus the manufacturer's profit from the sales of the product. Intuitively, the after-sales services should generate an income greater than this amount in order for the sale of the product to be viable for everyone in the system. Thus, we see that the manufacturer prefers not to sell at all when the extra cost she must incur for after-sales services is higher than the maximum willingness-to-pay for after-sales services in the market; i.e., $\frac{v-p+b}{\beta}$.

The threshold for full market coverage is similarly determined based on how the cost-related term $f(c_o + \alpha c_r) - (p - c)$ compares to the utility-related term $\frac{v-p-3b}{\beta}$. We see that the gap between the customer's maximum willingness-to-pay and the minimum cost of repairs is higher (a factor of $4b$ instead of $2b$ as compared to the retailer's best response) to convince both partners to determine full coverage of the market; i.e., a consequence of double marginalization in the chain.

When we look at the manufacturer's wholesale price decision, we see that she charges the retailer completely for the earnings he receives from the manufacturer for the warranty work $\left(\frac{f_m \alpha (l_r - c_r)}{f_e}\right)$, as this expression is added to ω_o^* in all three cases. Thus, the manufacturer extracts all her expected warranty-related payments through spare parts sales from the retailer.

Now, we look into how the product retail price p is determined by the manufacturer, given that her wholesale price ($\omega_o^*(p)$) and the retailer's spare parts price decision ($s_o^*(p, \omega_o^*(p))$) will follow.

Proposition 3 *The equilibrium product price set by the manufacturer (p^*) in the decentralized model is the minimum possible value for p, which is 0.*

The equilibrium results of the decentralized model are summarized in Table 3.2. The manufacturer would want to set p as low as possible, which is 0, based on our model assumptions. Note that, this also implies that the manufacturer sells each product with a loss, as $c > 0$. Thus, the manufacturer prefers to extract all the value not at the time of purchase, but throughout the life cycle of the product. This result occurs due to the two essential properties of the model: Firstly, the customer can fully evaluate p at the time of purchase, but can only partially assess the future costs, due to the foresight factor β. Secondly, the customer is committed to purchasing the after-sales services; i.e., *locked-in*.

Since we study a stylized monopoly model with complete lock-in, which does not take into account factors such as capital investment, time value of money, and etc., our results may *partially* shed light on what happens in practice. Directionally, these findings explain the manufacturers' tendency or future plans to recoup a significant part of their profits from the aftermarket in the industry; i.e., as in the examples of

Table 3.2: Equilibrium of the Decentralized Model, where $F = f(c_o + \alpha c_r) + c$

	$\upsilon + b - \beta F \ \geq \ 4b$	$0 \ \leq \ \upsilon + b - \beta F \ < \ 4b$	$\upsilon + b - \beta F \ < \ 0$
p^*	0	0	0
ω_o^*	$\frac{\upsilon - b + \beta f \alpha (l_r - c_r)}{\beta f_e} - \alpha c_r$	$\frac{\upsilon + b + \beta F + 2\beta f_m \alpha (l_r - c_r)}{2\beta f_e} - \alpha c_r$	$\frac{\upsilon + b + \beta f \alpha (l_r - c_r)}{\beta f_e} - \alpha c_r$
s_o^*	$\frac{\upsilon}{\beta f_e} - \alpha l_r$	$\frac{3(\upsilon + b) + \beta F}{4\beta f_e} - \alpha l_r$	$\frac{\upsilon + b}{\beta f_e} - \alpha l_r$
$q^D(p^*, s_o^*)$	1	$\frac{\upsilon + b - \beta F}{4b}$	0
$\Pi_r^D(p^*, \omega_o^*, s_o^*)$	$\frac{b}{\beta}$	$\frac{(\upsilon + b - \beta F)^2}{16\beta b}$	0
$\Pi_m^D(p^*, \omega_o^*, s_o^*)$	$\frac{\upsilon - b - \beta F}{\beta}$	$\frac{(\upsilon + b - \beta F)^2}{8\beta b}$	0
$CS^D(p^*, \omega_o^*, s_o^*)$	$\frac{b}{2} - \frac{\upsilon(1-\beta)}{\beta}$	$\frac{(\upsilon + b - \beta F)^2}{32b} - \frac{(1-\beta)(\upsilon + b - \beta F)(4(\upsilon + b) - (\upsilon + b - \beta F))}{16\beta b}$	0

Otis, SAIC Motor, or Xerox. Whether as a consequence of a strategic price decision as we find here, or due to potential disruptions in the primary market, we see disparity in profitability between the product sales and the aftermarket for some other companies. For example, Boeing, in 2019 and 2021, suffered from product-related losses of around \$6.5 million, \$2-\$2.5 million of which it could compensate with profits from services [88]. This strategy is also frequently attributed to the inkjet printer and razor markets where the manufacturers are said to sell the printers and razor handles at very low prices, whereas selling the printer ink cartridges/refills and razor blades at high prices [19, 53].

We next discuss a critical result regarding the warranty coverage decisions.

Corollary 1 *Equilibrium profits of the retailer and the manufacturer, and the consumer surplus in equilibrium depend only on the total number of after-sales services throughout the product's lifetime, f; and not on the quantity of these paid by the customer, f_e, or the portion of them paid by the customer, $\frac{f_e}{f}$.*

In equilibrium, the customer essentially pays a fixed amount for after-sales services, where the price per repair then depends on the number of times he receives service, *independent of the warranty coverage* offered. No matter what the extent of warranty coverage (i.e., f_e vs. f_m) is, the manufacturer can adjust the non-warranty service prices to extract the same total value, being sure that the customer will purchase them. From a regulatory perspective, if it is known that the after-sales market is a monopoly, it is then insignificant whether a mandatory minimum warranty coverage is imposed on the manufacturer or not, as the consumer surplus is not affected by it.

On a similar note, it is also *not* to the manufacturer's advantage if she reduces the reliability of the product and increases the total number of repairs needed (f), expecting to extract more value from the locked-in customers for after-sales services. Instead, this additional cost gets shared and paid by all parties in the system, without an advantage to anyone. When the market is fully covered, the retailer's profit and the consumer surplus are even independent of f, whereas the manufacturer fully incurs the total cost of after-sales services.

Interestingly, the unit labor price, l_r plays no role in the profits and the consumer surplus. This can be attributed to the result that the retailer essentially prices the total unit repair cost, and then deduces the labor component in setting the spare parts price.

As β is an important parameter that might be manipulated by efforts of the manufacturer, or through legal requirements imposed by authorities for disclosure, we next analyze how each party's performance is affected by it.

Corollary 2 *The manufacturer and the retailer are better off with a β value as low as possible. Conversely, the consumer surplus shows that a β value as high as possible is to the consumer's advantage.*

Therefore, we do verify the notion that customers would rather have an environment with higher transparency and awareness, whereas the retailer and the manufacturer would rather have an environment with myopic customers.

3.2 Centralized Model

In this section, we study the centralized version of the base model in order to evaluate the efficiency loss due to decentralization in the chain.

In the centralized model, the total profit of the manufacturer and the retailer is maximized by a single decision maker, where the wholesale spare parts price ω_o becomes irrelevant and the decision variables are p and s_o. The terms l_r and c_r, consistent with the decentralized model, represent the labor price and cost in the aftersales services in the centralized model. Here we again study the two decisions in sequence: First p is determined, followed by s_o.

The utility function, hence the demand with respect to prices stays the same, as stated in Equation 3.5. The profit function of the system, $\Pi^C(p, s_o)$ is given as follows:

$$\Pi^C(p, s_o) = (p - c - f_m(c_o + \alpha c_r) + f_e(s_o - c_o + \alpha(l_r - c_r)))\, q^C(p, s_o) \qquad (3.6)$$

As the individual analysis steps and the outcomes are quite similar to the decentralized model, these details are provided in Appendix A.1 and a side-by-side comparison of the equilibrium with the decentralized model is given in Table 3.3[3]. The differences between the centralized and the decentralized model mainly arise due to the double marginalization in the decentralized model and the resulting efficiency loss. Firstly, the centralized model is more likely to result in full market coverage. This is achieved by a spare parts price that is always lower than or equal to that of the decentralized model, reaching more customers, mainly because of lack of double marginalization. The whole market is served under the centralized model when $\upsilon \geq \beta(f(c_o + \alpha c_r) + c) + b$ whereas the condition $\upsilon \geq \beta(f(c_o + \alpha c_r) + c) + 3b$ must hold under the decentralized model. Thus, the base valuation of the product (υ) must be significantly higher than the customer heterogeneity in the market (b) to warrant full coverage under the decentralized model compared to the centralized system. Secondly, the centralized model always results in equal or better total profit and consumer surplus. Interestingly, when both models achieve full market coverage with a sufficiently high base valuation of the product itself, i.e., $\upsilon \geq \beta(f(c_o + \alpha c_r) + c) + 3b$, the models have the exact same outcome, without any efficiency loss. As another parallel to the base model, Corollary 1 and 2 still hold.

3.3 Third Party Model

It is usually considered that a monopolistic after-sales market is a lucrative business opportunity for a manufacturer. Therefore, one might assume that a manufacturer would always like to be the sole provider for the after-sales services of its products. In this section, we challenge this assumption and investigate if there are any cases where the manufacturer is not interested in being actively involved in the after-sales business of her products.

In the third party model, the manufacturer is still the monopolistic supplier of a product; but she does not offer a warranty and is not involved in the after-sales business. Customers still require after-sales services, which are provided by an independent workshop or retailer. This model may occur when the manufacturer intentionally designs and produces a product that is highly compatible with the active spare parts

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[3] The last column where $\upsilon < \beta F - b$ is skipped, as there is no sales and all profit and surplus functions are zero in both models.

Table 3.3: Comparison of the Decentralized Model Equilibrium with the Centralized Model Solution, where $F = f(c_o + \alpha c_r) + c$

$Conditions$	$\upsilon \geq \beta F{+}3b$	$\beta F{+}3b > \upsilon \geq \beta F{+}b$	$\beta F{+}b > \upsilon \geq \beta F{-}b$
		Decentralized Model	
p^*	0	0	0
ω_o^*	$\frac{\upsilon-b+\beta f\alpha(l_r-c_r)}{\beta f e}-\alpha c_r$	$\frac{\upsilon+b+\beta F+2\beta f_m\alpha(l_r-c_r)}{2\beta f e}-\alpha c_r$	$\frac{\upsilon+b+\beta F+2\beta f_m\alpha(l_r-c_r)}{2\beta f e}-\alpha c_r$
s_o^*	$\frac{\upsilon}{\beta f e}-\alpha l_r$	$\frac{3(\upsilon+b)+\beta F}{4\beta f e}-\alpha l_r$	$\frac{3(\upsilon+b)+\beta F}{4\beta f e}-\alpha l_r$
$q^D(p^*,s_o^*)$	1	$\frac{\upsilon+b-\beta F}{4b}$	$\frac{\upsilon+b-\beta F}{4b}$
$\Pi_r^D(p^*,\omega_o^*,s_o^*)$	$\frac{b}{\beta}$	$\frac{(\upsilon+b-\beta F)^2}{16\beta b}$	$\frac{(\upsilon+b-\beta F)^2}{16\beta b}$
$\Pi_m^D(p^*,\omega_o^*,s_o^*)$	$\frac{\upsilon-b-\beta F}{\beta}$	$\frac{(\upsilon+b-\beta F)^2}{8\beta b}$	$\frac{(\upsilon+b-\beta F)^2}{8\beta b}$
$\Pi_{r+m}^D(p^*,\omega_o^*,s_o^*)$	$\frac{\upsilon-\beta F}{\beta}$	$\frac{3(\upsilon+b-\beta F)^2}{16\beta b}$	$\frac{3(\upsilon+b-\beta F)^2}{16\beta b}$
$CS^D(p^*,\omega_o^*,s_o^*)$	$\frac{b}{2}-\frac{\upsilon(1-\beta)}{\beta}$	$\frac{(2-\beta)(\upsilon+b-\beta F)^2}{32\beta}-\frac{(1-\beta)(\upsilon+b)(\upsilon+b-\beta F)}{4\beta b}$	$\frac{(2-\beta)(\upsilon+b-\beta F)^2}{32\beta}-\frac{(1-\beta)(\upsilon+b)(\upsilon+b-\beta F)}{4\beta b}$
		Centralized Model	
p^*	0	0	0
s_o^*	$\frac{\upsilon}{\beta f e}-\alpha l_r$	$\frac{\upsilon}{\beta f e}-\alpha l_r$	$\frac{\upsilon+b+\beta F}{2\beta f e}-\alpha l_r$
$q^{MC}(p^*,s_o^*)$	1	1	$\frac{\upsilon+b-\beta F}{2b}$
$\Pi^{MC}(p^*,s_o^*)$	$\frac{\upsilon-\beta F}{\beta}$	$\frac{\upsilon-\beta F}{\beta}$	$\frac{(\upsilon+b-\beta F)^2}{4\beta b}$
$CS^{MC}(p^*,s_o^*)$	$\frac{b}{2}-\frac{(1-\beta)\upsilon}{\beta}$	$\frac{b}{2}-\frac{(1-\beta)\upsilon}{\beta}$	$\frac{(2-\beta)(\upsilon+b-\beta F)^2}{8\beta b}-\frac{(1-\beta)(\upsilon+b)(\upsilon+b-\beta F)}{2\beta b}$

and technology in the independent after-sales market. The “compatibility" may be achieved with the past generations of products offered by the manufacturer or other firms with an independent retail network presence[4].

The only decision variable in this variation is the product’s retail sales price, p, which is set by the manufacturer. As opposed to the decentralized model, the independent retailer’s sales price for the spare parts, s_t, and the labor, l_t, are assumed exogenous. A depiction of the financial flow is given in Figure 3.2.

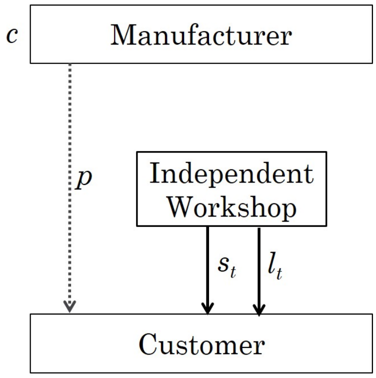

Figure 3.2: Financial Flow of the Third Party Model

Under this setting, the utility captured by a customer of type $\theta \in [0, b]$ will be as follows:

$$U_\theta^T(p) = \begin{cases} 0, & \\ \qquad \text{if the customer of type } \theta \text{ does not buy,} & \\ v - p + \theta - \beta f(s_t + \alpha l_t), & \\ \qquad\qquad \text{if the customer of type } \theta \text{ buys.} & \end{cases} \tag{3.7}$$

The manufacturer’s sole source of revenue is from the sales of her product; she obtains a profit of $p - c$ from each unit sold, as depicted in Equation 3.8 below.

$$\Pi_m^T(p) = (p - c)q^T(p) \tag{3.8}$$

Then, the manufacturer’s product price decision is as described in Proposition 4 below.

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[4] GM preferred to use the same pool of spare parts for many of its brands except for Saturn with unique characteristics and requirements (Hanna [51]). Though the company’s motivation behind this decision is not known, this is a good indicator that the third party model could occur if the market conditions and the manufacturer’s incentives allowed.

Proposition 4 *The optimal product retail price set by the manufacturer (p^*) under the third party model is as follows:*

$$p^* = \begin{cases} v - \beta f(s_t + \alpha l_t), & \\ \qquad\qquad\qquad\qquad\qquad\qquad \textit{if}\; b + c + \beta f(s_t + \alpha l_t) \leq v, & \\ \frac{1}{2}\left(v + b + c - \beta f(s_t + \alpha l_t)\right), & \\ \qquad\qquad \textit{if}\; -b + c + \beta f(s_t + \alpha l_t) \leq v < b + c + \beta f(s_t + \alpha l_t) & \\ \left[v + b - \beta f(s_t + \alpha l_t)\;, \infty\right), & \\ \qquad\qquad\qquad\qquad\qquad\qquad \textit{if}\; v < -b + c + \beta f(s_t + \alpha l_t). & \end{cases}$$

The results of the third party model are given in Table 3.4. Comparing them with the equilibrium results in Table 3.2, we characterize when the manufacturer is better off being a monopoly in the after-sales market (the decentralized model, D) when compared with not being involved in the after-sales market (the third party model, T); and how this choice aligns with the total consumer surplus. In other words, we evaluate whether the manufacturer always prefers controlling the after-sales business and the customer always favors the third party model, or whether there are cases where their interests are aligned.

Table 3.4: Results of the Third Party Model, where $S = f(s_t + \alpha l_t)$

	$v - \beta S \geq b + c$	$b + c > v - \beta S \geq c - b$	$v - \beta S < c - b$
p^*	$v - \beta S$	$\frac{1}{2}\left(v + b + c - \beta S)\right)$	$v + b - \beta S$
$q^T(p^*)$	1	$\frac{v+b-c-\beta S}{2b}$	0
$\Pi_m^T(p^*)$	$v - c - \beta S$	$\frac{(v+b-c-\beta S)^2}{4b}$	0
$CS^T(p^*)$	$\frac{b-2(1-\beta)S}{2}$	$\frac{(v+b-c-\beta S)^2}{8b} - \frac{(1-\beta)S(v+b-c-\beta S)}{2b}$	0

Corollary 3 *The manufacturer's preferences over the two business models are as follows:*

(i) When she serves the whole market under both models in equilibrium (i.e., $v \geq 3b + \beta[f(c_o + \alpha c_r) + c]$ and $v \geq b + c + \beta[f(s_t + \alpha l_t)]$), $\Pi_m^D > \Pi_m^T$ if and only if

$$\frac{v(1-\beta) - b}{\beta} > f(c_o + \alpha c_r) - \beta f(s_t + \alpha l_t)$$

(ii) *When she partially serves the market under both models in equilibrium (i.e.,* $-b+\beta[f(c_o+\alpha c_r)+c] \leq v < 3b+\beta[f(c_o+\alpha c_r)+c]$ *and* $-b+c+\beta[f(s_t+\alpha l_t)] \leq v < b+c+\beta[f(s_t+\alpha l_t)]$*),* $\Pi_m^D > \Pi_m^T$ *if and only if*

$$(v+b)(\sqrt{2\beta}-1) < c(\sqrt{2\beta}-\beta)+\beta f[\sqrt{2\beta}(s_t+\alpha l_t)-(c_o+\alpha c_r)]$$

Under both models, the manufacturer serves the whole market when the base product valuation (v) is high enough compared to the heterogeneity of customers in additional valuation (b), and the costs of production and after-sales services. Manufacturer prefers full coverage when the willingness-to-pay of the high-type customers is not attractive enough to sacrifice the low-valuation customers and the opportunity to charge them by utilizing the base product valuation (v). We observe that the manufacturer is more likely to favor the decentralized model over the third party model when this tradeoff is even stronger.[5] More importantly, the manufacturer prefers to control the after-sales market when customers are myopic (i.e., β is low). When she controls the after-sales market, the manufacturer gives the product away for free and extracts all her profit from after-sales transactions, taking advantage of the limited foresight of the customers at the time of purchase. Under the third party model, however, she has to set a certain price for the product (base valuation reduced by the estimated after-sales costs), and that is her sole source of profit. That's why the decentralized model becomes more profitable when the customers are myopic, and less profitable otherwise. When β takes moderate values, she prefers the decentralized model if she is sufficiently efficient in after-sales services compared to the independent retailer option.

The manufacturer will partially serve the market under both models when the base product valuation (v) takes moderate values compared to market heterogeneity (b) and the costs of production and after-sales services. Then she sells to the high-valuation customers only, and charges them accordingly. Under these cases, the manufacturer becomes more likely to favor the decentralized model as customers become more myopic (i.e., β decreases) and unit production cost c increases. When customer foresight is low, the manufacturer effectively extracts value through the after-sales services in

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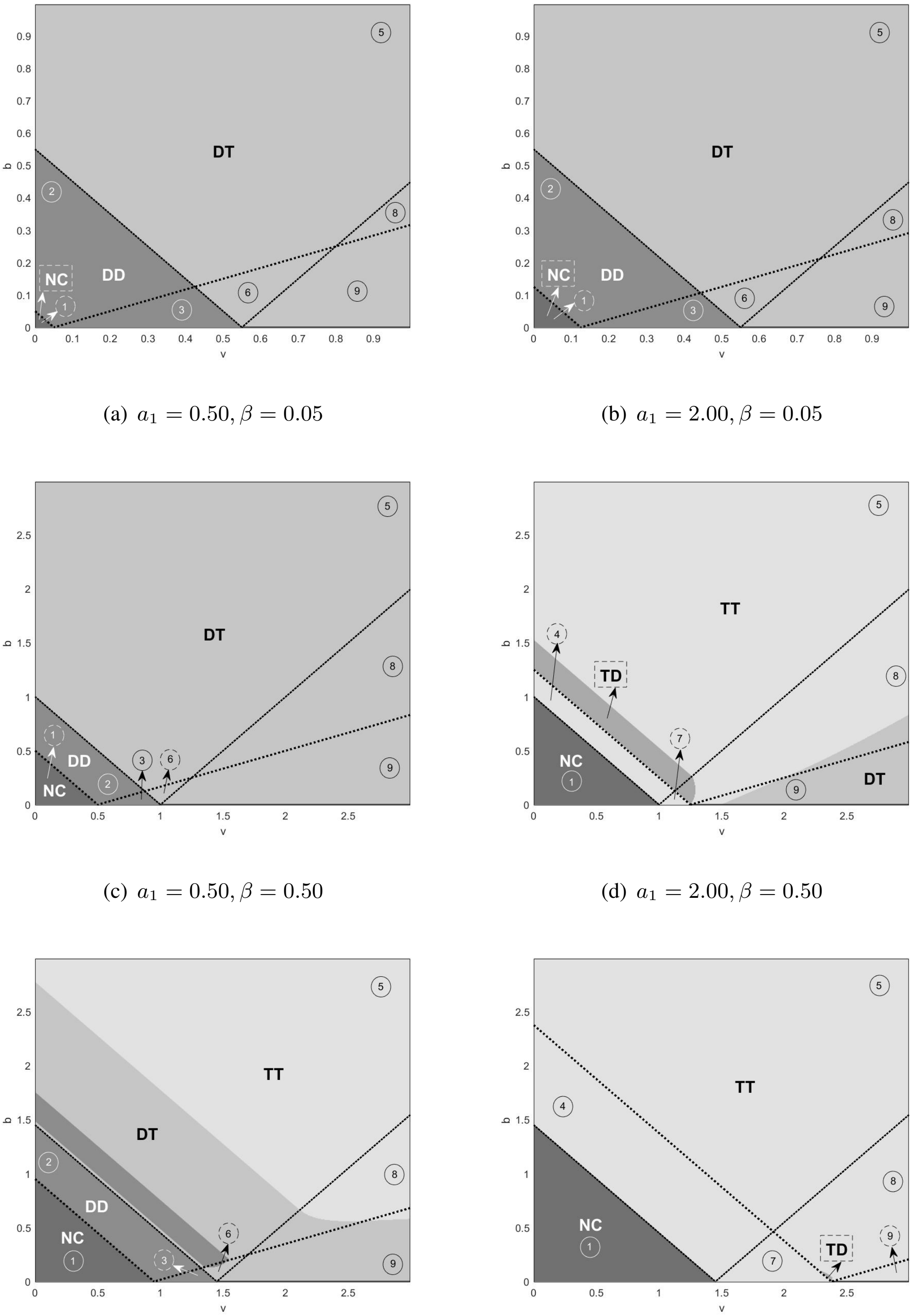
[5] The relative magnitude of v compared to b becomes important also due to the double marginalization in the chain. The manufacturer, to convince the retailer to set a low enough price and fully serve the market, has to set a wholesale price low enough (which is decreasing in b). In the third party model, however, there is no interaction between the manufacturer and the independent workshop.

the decentralized system, compared to relying on the product price as the only source of revenue under the third party model. Hence the decentralized system becomes even more favorable as the product valuation potential $(v + b)$ increases. When customers are strategic (i.e., β is high), the decentralized model loses effectiveness, and can dominate the third party model only when it shows moderate performance as well. This arises when the product valuation potential $(v + b)$ is low enough and the manufacturer has the efficiency advantage in after-sales services. Total product valuation reduced by the estimated after-sales costs represents the highest value that the manufacturer can capture from a customer under the third party model. When this value is low, the market coverage as well as the margin potential of the manufacturer decreases, and the relative importance of the limited customer foresight (β) and unit cost (c) in the comparison of the two models increases. That's why a low total valuation potential $(v + b)$ favors the decentralized model when customers are strategic.

The market coverage under the two models may point different directions, depending on the relative efficiency of the manufacturer in after-sales services, unit production cost, market heterogeneity, and customer foresight. When $v \geq 3b+\beta[f(c_o+\alpha c_r)+c]$ and $-b+c+\beta[f(s_t+\alpha l_t)] \leq v < b+c+\beta[f(s_t+\alpha l_t)]$, all customers are served under the decentralized model whereas the third party model results in partial coverage. For this case to arise, the condition $c(1 - \beta) + \beta f[(s_t + \alpha l_t) - (c_o + \alpha c_r)] > 2b$ must be satisfied. This is more likely to happen if market heterogeneity (b) is relatively low, and unit production cost (c) is relatively high. When she controls the aftermarket, the manufacturer extracts all her revenue from after-sales transactions, and can tolerate a higher production cost by leveraging the myopic behavior of the customers. Under the third party model, however, she loses that flexibility since she has to rely on the product price to cover all her costs. Note that, if the third party retailer is the more efficient one in after-sales services (i.e., $(s_t + \alpha l_t) - (c_o + \alpha c_r) < 0$), then this case may arise in markets with low foresight (β) and low number of expected repair and maintenance activities (f). These represent the market types where the manufacturer does not suffer much from her disadvantage in after-sales operations and can easily extract revenue from myopic customers.

To investigate how the manufacturer's preferences align with the consumer welfare, we generate numerical examples using a wide range of parameter values. In Figure 3.3, we identify the preferred business model (i.e., decentralized vs. third party) in terms of the manufacturer profit and the consumer surplus (*CS*). Each zone is represented with two letters; the first letter (*D* or *T*) represents the model that produces the higher profit for the manufacturer, and the second letter (*D* or *T*) identifies the model that achieves the higher consumer surplus. For example, *TD* denotes that the manufacturer prefers the third party model whereas the decentralized model is beneficial for the customers since it produces a higher consumer surplus. Additionally, *NC* represents market characteristics where no customer is served under either of the business models. The plots also include divisions into regions that indicate the market coverage combinations of each model, the perimeters of which are indicated by dashed lines, and the legend for which is given in Table 3.5. The figure provides six distinct observations with respect to low ($\beta = 0.05$), medium ($\beta = 0.50$), and high ($\beta = 0.95$) customer foresight values; combined with high and low relative costs of lifetime repairs in the decentralized model. To be more precise, we define $a_1 := \frac{f(c_o+\alpha c_r)}{f(s_t+\alpha l_t)}$ as a gauge for the cost-efficiency of the decentralized model compared to the third party counterpart. Essentially, it is a ratio of the lifetime cost of repairs under the decentralized model ($f(c_o + \alpha c_r)$) to the lifetime price of repairs under the third party model ($f(s_t + \alpha l_t)$). We study $a_1 = 2$ and $a_1 = 0.5$ to represent the high and low settings, respectively. For each plot, the fixed utility derived from the product v is in the x-axis, and the customer heterogeneity in product valuation b is in the y-axis. Please also note that plots in Figures 3.3(a)-3.3(b) show a range of $v = 1$ in the x-axis and $b = 1$ in the y-axis for proper labeling and display of different regions, whereas Figures 3.3(c)-3.3(f) cover a wider range of $v = 3$ in the x-axis and $b = 3$ in the y-axis.

From Figure 3.3, we observe that neither of the business models serve the market (i.e., no coverage) when the product valuations v and b are low enough. This coincides with the no coverage conditions $v+b-\beta[f(c_o+\alpha c_r)+c] < 0$ and $v+b-c-\beta[f(s_t+\alpha l_t)] < 0$ from Tables 3.2 and 3.4 for the decentralized and third party models, respectively. This no coverage zone expands further as β increases; as customers can anticipate the future costs better, it becomes difficult to remain profitable for the manufacturer and generate sales at the same time. However, it happens that the no coverage region for one model is bigger than the other, hence the model with the smaller region becomes



(a) $a_1 = 0.50, \beta = 0.05$

(b) $a_1 = 2.00, \beta = 0.05$

(c) $a_1 = 0.50, \beta = 0.50$

(d) $a_1 = 2.00, \beta = 0.50$

(e) $a_1 = 0.50, \beta = 0.95$

(f) $a_1 = 2.00, \beta = 0.95$

Figure 3.3: Preferred Business Model with respect to the Manufacturer's Profit and Real Consumer Surplus (respectively), where $a_1 = \frac{f(c_o+\alpha c_r)}{f(s_t+\alpha l_t)}$, and the parameters are set to $f(s_t + \alpha l_t) = 1$ and $c = 0.5$

Table 3.5: Region Descriptions for Figure 3.3 with respect to Market Coverage

Region	Decentralized	Third Party
1	No coverage	No coverage
2	Partial coverage	No coverage
3	Full coverage	No coverage
4	No coverage	Partial coverage
5	Partial coverage	Partial coverage
6	Full coverage	Partial coverage
7	No coverage	Full coverage
8	Partial coverage	Full coverage
9	Full coverage	Full coverage

the only feasible alternative. Here, neither of the models have a strict advantage over the other and the result depends on the parameter combinations. For example, as a_1 increases, the manufacturer's after-sales costs increase, and hence it becomes more likely for the decentralized model to fail in serving the market. We can also observe this in the graphs: Figures 3.3(a)-3.3(c) and 3.3(e) show a larger area where the third party model has no coverage, but the decentralized model has partial or full coverage (regions 2 and 3). Figures 3.3(d) and 3.3(f) showcase the reverse situation where the decentralized model has a larger area of no coverage, but the third party model has partial or full coverage (regions 4 and 7).

Our observations about the manufacturer's preference over the two business models mainly echo our results in Corollary 3.[6] Her preference is mainly driven by the customer's ability to forecast future costs; i.e., β, the product valuation and heterogeneity in the market; i.e., v and b, and the manufacturer's relative efficiency in after-sales services; i.e., a_1. She always prefers a monopoly environment with myopic customers and mostly a third party model with strategic customers. Her relative cost-efficiency, a_1, somewhat delays this switch of preference when it takes a lower value (compare Figures 3.3(c) and 3.3(d)). As both the manufacturer's cost disadvantage and customer foresight become significant, she wants to completely stay out of the after-sales business, and leave it to the independent retailer, which is the more efficient

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[6] Corollary 3 provides analytical insight into region 5 for partial coverage in both models and 9 for full coverage in both models. The plots additionally include regions 6 and 8 that show combinations where one model has partial coverage and the other has full coverage; however, we see that the customer and manufacturer preferences are not especially affected by the different market coverages shown by these regions.

alternative. Note that, Figure 3.3(d) showcases full coverage domination of the decentralized model, as discussed in part (i) of Corollary 3. In Figure 3.3(e), we also observe partial coverage domination, i.e., an instance of part (ii) of the corollary in the left side of the *DT*, and *DD* regions; and an example for part (i) in the right of the *DT* region. Lastly, the manufacturer's preference of the decentralized model over the third party alternative may arise in any number of nonzero market coverage scenarios, even when the decentralized model partially covers the market when the third party model produces full coverage, depending on the parameters.

In our experiments, we observe that it is not always to the customer's advantage if a minimum warranty coverage (hence availability of after-sales services by the manufacturer) is imposed on a monopolist manufacturer as a legal requirement (e.g., through consumer protection laws). The third party model is predominantly advantageous from the consumer perspective, especially in markets with high product valuation (v) and high heterogeneity (b). Under the decentralized model, when the customer foresight of future costs (β) is low, the manufacturer takes advantage of the myopic market with high after-sales prices. Under the third party model, however, she is only left with the product retail price as a lever, and customers can completely evaluate it at the time of purchase. Thus, the consumer surplus is always higher with the third party model under these cases. Customers are better off under the decentralized model in rare cases only, when the maximum product valuation ($v + b$) is below a certain threshold and the customers are strategic (β is high)[7]. In these cases, the decentralized model becomes favorable since it is the lesser evil. Both models enforce very limited market coverage, possibly with negative total surplus. In fact, out of the three regions we observe the customer preference of the decentralized model, in only one of them the manufacturer prefers the decentralized model as well. Interestingly, the manufacturer does not even have to be cost-efficient compared to the independent retailer for customers to prefer the decentralized model; i.e., a_1 may be low or high.

The preferences based on the manufacturer's profit and the total consumer surplus align occasionally. This alignment occurs mostly on the third party model, when the customer foresight (β) is high enough, and especially if the manufacturer is the costly

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[7] The DD regions in Figures 3.3(a), 3.3(b) and 3.3(c) indicate cases where the third party model has no coverage. Hence the 'D' preference of customers is because there is no other viable option. That's why we do not discuss them for comparison purposes.

option in the after-sales business (high a_1). We see only one alignment on the decentralized model; when the customer foresight is high enough and the manufacturer has the cost advantage (see Figure 3.3(e)). Here, the customer being strategic does not allow over-pricing on the manufacturer's side, and she still prefers to offer warranty to the market since she is the efficient alternative in the after-sales business. Thus, both parties are better off when the manufacturer offers the after-sales services in her own monopoly channel. Note that, this alignment occurs only when the maximum product valuation ($v + b$) is at a certain range and low enough. As this value gets higher, or the base product utility is high enough with low customer heterogeneity, the manufacturer manages to extract more value from the customers than the cost advantage she can offer in the after-sales services. Thus, the third party model becomes favorable from the consumer perspective. As a result, even when the manufacturer is the efficient provider in after-sales services, her offering a warranty is rarely the preferable option for the customer.

Alignment of preferences, whether it is on the decentralized or the third party model, only occurs when customers are strategic enough (i.e., β is moderate to high). Thus, above anything else, the first priority of policymakers should be to enforce/encourage visibility in the aftermarket. Requiring regular publication of after-sales costs over the whole life cycle of a product and its enforced communication by the manufacturer to potential customers are policy levers that could be utilized to achieve high visibility and make customers more strategic. In markets with visibility, stringent after-market regulations should still be carefully implemented. Especially when the product valuation (v) and customer heterogeneity (b) are high, an enforcement on the availability of after-sales services and warranty over the manufacturer could hurt both the manufacturer profit and the consumer surplus. Only when the total valuation (v and b) is in a certain range, the manufacturer is the efficient alternative, and consumers are highly strategic, the manufacturer's monopoly after-sales services is favored by both sides. Thus, policymakers should make sure that all three conditions are met before enforcing manufacturer-related aftermarket obligations. Lastly, in markets with very low total valuation (i.e., both v and b), and especially when customers are myopic, a manufacturer's active role in the aftermarket should be encouraged since that is the only viable option for the market.

3.4 Extension: Heterogeneity in Customer Foresight

So far, we have treated customer foresight into future costs as a market-related parameter. In this section, we study an extension where the market is composed of two distinct segments regarding customer foresight: λ fraction of the customers have low foresight (β_L), who are referred to as “myopic;" $1 - \lambda$ fraction of the customers have high foresight (β_H), who are referred to as “strategic;" and where $0 < \beta_L < \beta_H < 1$. In each segment, customer type θ is uniformly distributed on $[0, b]$. Besides this change, all decisions, their sequence, and their decision makers are the same as those of the decentralized model. We first used backward induction to characterize the equilibrium in this game analytically. The retailer’s best response, i.e., $s_o^*(p, \omega_o)$, is fully characterized and available in Appendix A.4. However, the analysis of the manufacturer’s wholesale price decision gets too complicated due to breakdown on several intervals and conditions that require multiple comparisons between objective function values. Therefore, we have conducted a numerical study for 3,600 instances with the parameters provided in Table 3.6, using the analytical solution for $s_o^*(p, \omega_o)$ and with full enumeration over p and ω_o in order to fully characterize the equilibrium.

Table 3.6: Parameter Values in Heterogeneity in Customer Foresight Experiments

v	$\{1, 2, 3, 4\}$	f	1
b	$\{1, 2, 3, 4\}$	α	0.5
β_L	$\{0.05, 0.15, 0.25, 0.35, 0.4\}$	l_r	0.5
β_H	$\{0.6, 0.65, 0.75, 0.85, 0.95\}$	c_r	0.25
λ	{0.1,0.5,0.9}	c	0.5
f_e	{0.25,0.5,0.75}	c_o	0.375

When customers are heterogeneous in their ability to foresee future costs, we mainly see that the manufacturer and the retailer follow a pricing strategy that chooses to focus on the myopic customer segment. The strategic segment has no demand in 2,514 (70%) of the instances; producing identical pricing, demand and profits as the decentralized model equilibrium with $\beta = \beta_L$. In the remaining 1,086 instances where the strategic segment is partially covered, the spare parts retail price s_o is always lower than that of the decentralized model equilibrium with $\beta = \beta_L$. Within 738 instances out of the 1,086 instances where the strategic segment has positive demand, the myopic segment is fully covered, hence it is safe to say that the myopic segment is still the primary target and the strategic segment is addressed only for the additional potential it offers. In the remaining 348 instances where both segments are partially captured, the pricing is done with the aim of addressing both segments and striking a balance between them. The manufacturer dominantly prefers this strategy when either the myopic segment is quite small in size and yet is moderately able to foresee costs, or the segments are in equal share but the strategic segment's ability to foresee costs is relatively lower.[8]

The equilibria for the case $v = 4$, $b = 4$, and $f_e = 0.5$ for selected β_L and β_H values are provided for exemplary purposes in Table 3.7.[9] The same table also provides the corresponding decentralized model equilibria, where we use the weighted average of β_L and β_H, i.e., $\hat{\beta} = \lambda\beta_L + (1 - \lambda)\beta_H$, as the β counterpart for the decentralized model.

In a market with customers heterogeneous in foresight, the manufacturer and the retailer may be content with just targeting the myopic segment in many cases. This implies a lower market coverage, and higher spare parts wholesale and retail prices when compared with the decentralized model counterpart. Other than this, we see that almost all results of the decentralized model also hold when the customers are heterogeneous in their foresight of future costs, albeit with a few caveats. First of all, the manufacturer makes the majority of her revenues and profits from after-sales rather than product sales, except for a limited number of cases where the product price is positive (362 out of 3,600). The product price is still below cost in 174 of these 362 instances in total.[10] Here, the manufacturer prefers setting a nonzero product price in order to effectively manage the myopic and strategic segments with high and partial coverages, respectively. In these cases, the spare parts wholesale price as a lever is not sufficient in the decentralized chain where the retailer determines the final market coverage by setting the spare parts retail price. Thus, the manufacturer

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[8] In 158 of these 348 instances, $\lambda = 0.1$ is combined with higher levels of β_L (0.35 and 0.40); and in 111 instances, $\lambda = 0.5$ is combined with lower levels of β_H (0.60 and 0.65).

[9] The equilibrium results across all tested v and b values over a given β_L, β_H, λ, and f_e combination can be found in Table A.4.

[10] Selected instances that produce a positive product price ($p > 0$) are given in Table A.5.

Table 3.7: Heterogeneity in Customer Foresight Model Equilibrium Results ($\upsilon = 4, b = 4, f_e = 0.5$), where q_L and q_H represent the market coverage of the myopic and strategic segments, respectively; and $\hat{q} = \lambda q_L + (1 - \lambda) q_H$

Heterogeneous Beta Model														**Decentralized Model**					
λ	β_L	β_H	$\hat{\beta}$	p^*	ω_o^*	s_o^*	q_L^*	q_H^*	$\hat{q}^*$	Π_r^*	Π_m^*	$\frac{\Pi_m^*}{\Pi_{total}^*}\%$	CS^*	ω_o^*	s_o^*	q^*	Π_r^*	Π_m^*	CS^*
0.9	0.05	0.60	0.105	0	161.07	240.28	50%	0%	0.45	17.76	35.55	67%	-50.63	77.19	114.54	0.49	9.28	18.55	-24.86
0.9	0.05	0.75	0.120	0	161.07	240.28	50%	0%	0.45	17.76	35.55	67%	-50.63	67.67	100.25	0.49	8.09	16.17	-21.29
0.9	0.05	0.95	0.140	0	161.07	240.28	50%	0%	0.45	17.76	35.55	67%	-50.63	58.14	85.96	0.49	6.90	13.79	-17.73
0.9	0.25	0.75	0.300	0	32.98	48.24	48%	0%	0.44	3.38	6.76	67%	-7.51	27.67	40.25	0.48	3.09	6.18	-6.36
0.9	0.25	0.95	0.320	0	32.98	48.24	48%	0%	0.44	3.38	6.76	67%	-7.51	26.00	37.75	0.48	2.88	5.76	-5.74
0.5	0.05	0.60	0.325	0	161.07	240.28	50%	0%	0.25	9.87	19.75	67%	-28.13	25.62	37.17	0.48	2.83	5.66	-5.60
0.9	0.40	0.60	0.420	0	20.99	30.24	48%	0%	0.43	2.03	4.06	67%	-3.51	20.05	28.82	0.47	2.14	4.28	-3.55
0.5	0.25	0.60	0.425	0	32.98	48.24	48%	0%	0.24	1.88	3.75	67%	-4.17	19.82	28.49	0.47	2.11	4.22	-3.46
0.9	0.40	0.75	0.435	0	20.99	30.24	48%	0%	0.43	2.03	4.06	67%	-3.51	19.39	27.84	0.47	2.06	4.11	-3.30
0.5	0.05	0.95	0.500	0	161.07	240.28	50%	0%	0.25	9.87	19.75	67%	-28.13	17.00	24.25	0.47	1.76	3.52	-2.43
0.5	0.25	0.75	0.500	0	32.98	48.24	48%	0%	0.24	1.88	3.75	67%	-4.17	17.00	24.25	0.47	1.76	3.52	-2.43
0.5	0.40	0.60	0.500	0	16.99	24.24	78%	16%	0.47	1.76	3.52	67%	-2.62	17.00	24.25	0.47	1.76	3.52	-2.43
0.1	0.25	0.60	0.565	0	15.06	21.35	100%	38%	0.44	1.45	2.88	67%	-1.70	15.16	21.49	0.46	1.53	3.06	-1.77
0.5	0.40	0.75	0.575	0.352	9.87	17.99	100%	20%	0.60	2.52	2.58	51%	-1.93	14.91	21.12	0.46	1.50	3.00	-1.68
0.1	0.40	0.60	0.580	0	14.80	20.94	94%	41%	0.46	1.48	2.97	67%	-1.68	14.79	20.94	0.46	1.48	2.97	-1.63
0.5	0.40	0.95	0.675	0	20.99	30.24	48%	0%	0.24	1.13	2.26	67%	-1.95	12.85	18.03	0.46	1.24	2.48	-0.94
0.1	0.05	0.75	0.680	0	161.07	240.28	50%	0%	0.05	1.97	3.95	67%	-5.63	12.76	17.90	0.46	1.23	2.46	-0.91
0.1	0.25	0.75	0.700	0	12.25	17.13	100%	37%	0.43	1.11	2.22	67%	-0.75	12.43	17.39	0.46	1.19	2.38	-0.79
0.1	0.05	0.95	0.860	0	161.07	240.28	50%	0%	0.05	1.97	3.95	67%	-5.63	10.30	14.20	0.45	0.93	1.85	-0.05
0.1	0.25	0.95	0.880	1.112	7.25	11.10	100%	37%	0.44	0.89	1.63	65%	0.08	10.09	13.89	0.45	0.90	1.80	0.02
0.1	0.40	0.95	0.895	0	9.89	13.59	100%	36%	0.42	0.83	1.66	67%	0.03	9.94	13.66	0.44	0.88	1.76	0.07

may prefer to use the product price as a second lever and generate higher profits for herself. Secondly, the manufacturer captures 67$\%$ of the total chain profit, in 2,977 (83$\%$) of the tested instances, consistent with the results of the decentralized model. In 5$\%$ of the instances, the manufacturer's share in profit is between 69$\%$-75$\%$, which correspond to full coverage of the myopic segment and no or very limited coverage of the strategic segment ($0 < q_H \leq 0.375$), still producing identical s_o values to the decentralized model with $\beta = \beta_L$. In 12$\%$ of the instances, the manufacturer's share in profit is between 40$\%$-65$\%$. Thus, the heterogeneous structure of the market may shift some power to the retailer compared to the base model, although in a limited fashion. Thirdly, as the average customer foresight increases, the spare parts wholesale and retail prices, as well as the manufacturer and retailer profits decrease, and the consumer surplus increases; except for rare cases with high disparity in consumer foresight.

Finally, we have also studied how different levels of warranty coverage, i.e., $f_m/f\ \%$, impacts the equilibrium in a heterogeneous market, the results of which are reported in Table 3.8. We see that the spare parts wholesale and retail prices increase as the warranty coverage decreases. In parallel to our findings in the decentralized model, the profits and the consumer surplus are quite robust to the extent of warranty coverage offered by the manufacturer.

3.5 Conclusion

In this study, we have analyzed three alternative models for the after-sales service channel structure of a durable goods manufacturer when the customers evaluate the total cost of ownership of a product when they make their purchasing decision. For the cases where the manufacturer and her authorized retailer are the monopolistic after-sales service providers, we have identified optimal pricing of the product and its after-sales services.

For the decentralized model, we have shown that the manufacturer extracts all of her profit from the after-sales services of the product. This has two implications: (1) having guaranteed the after-sales revenues and profits, the manufacturer gives away the

Table 3.8: Heterogeneity in Customer Foresight Model Equilibrium with respect to $f_m/f\ \%$ ($\upsilon = 4, b = 4$)

$\hat{\beta} = 0.5$	$\beta_L = 0.05, \beta_H = 0.95, \lambda = 0.5$								$\beta_L = 0.40, \beta_H = 0.60, \lambda = 0.5$							
$f_m/f\ \%$	p^*	ω_o^*	s_o^*	q_L^*	q_H^*	Π_r^*	Π_m^*	CS^*	p^*	ω_o^*	s_o^*	q_L^*	q_H^*	Π_r^*	Π_m^*	CS^*
10%	0	89.36	133.38	50%	0%	9.87	19.75	-28.13	0	9.33	13.36	78%	16%	1.76	3.52	-2.62
20%	0	100.53	150.06	50%	0%	9.87	19.75	-28.13	0	10.53	15.06	77%	16%	1.76	3.52	-2.62
30%	0	114.90	171.52	50%	0%	9.88	19.75	-28.14	0	12.08	17.25	77%	16%	1.76	3.52	-2.62
40%	0	134.04	200.13	50%	0%	9.89	19.75	-28.15	0	14.13	20.17	77%	16%	1.76	3.52	-2.62
50%	0	161.07	240.28	50%	0%	9.87	19.75	-28.13	0	16.99	24.24	78%	16%	1.76	3.52	-2.62
60%	0	201.33	300.39	50%	0%	9.87	19.75	-28.13	0	21.30	30.37	78%	16%	1.76	3.52	-2.62
70%	0	268.44	400.56	50%	0%	9.88	19.75	-28.14	0	28.49	40.58	78%	16%	1.76	3.52	-2.62
80%	0	402.67	600.90	50%	0%	9.89	19.75	-28.14	0	42.87	61.00	78%	16%	1.76	3.52	-2.62
90%	0	806.40	1202.45	50%	0%	9.87	19.75	-28.12	0	86.00	122.25	77%	16%	1.76	3.52	-2.62

product itself for free; (2) the extent of the warranty coverage offered by the manufacturer does not have any effect on the equilibrium results, because the customer essentially pays for it throughout the remaining after-sales services she receives. This result is mainly due to the lack of full foresight of the after-sales costs for the customer. It is to the manufacturer's advantage to have a less transparent after-sales market, whereas it is to the consumer's advantage to have full transparency.

Secondly, we have analyzed the centralized model and shown that the decentralized model has equal or less profits due to double marginalization. There is efficiency loss when the market is partially covered, but the pricing strategy of the centralized decision-maker is parallel to that of the manufacturer in the decentralized system. Thus, the efficiency captured does not help the customer much; consumer surplus is still independent of the warranty coverage offered, and suffers due to less-than-perfect foresight of future costs.

Finally, we have analyzed the third party model where the after-sales services are provided by an independent retailer and investigated when the manufacturer would prefer being a monopolistic provider of after-sales services. We have found that the manufacturer prefers to control the after-sales market when customers are myopic. When customer foresight is moderate to high, manufacturer's efficiency in after-sales services, the base product valuation in the market and customer heterogeneity in valuation affect the manufacturer's preferences as well. In these cases, the manufacturer favors the decentralized system if she is sufficiently efficient in after-sales services as compared to the independent retailer option. She is also more inclined to monopolize the after-sales market in a high-base-valuation low-heterogeneity market, when compared with a high-heterogeneity counterpart. When customer heterogeneity in valuation is comparable or more dominant than the base product valuation, the manufacturer will prefer to stay out of the after-sales market unless the highest willingness-to-pay in the market is limited and she has the efficiency advantage in the after-sales market. From the customer perspective, we see that third party is the model that is mostly favored, especially in markets with high product valuation and high heterogeneity. Decentralized model may be preferable in rare cases only, when both models perform badly with very limited market coverage. As a consequence, we observe that the manufacturer's and the consumer's preferences align occasionally, and mostly on the third party model, when customers are strategic, and especially if the manufacturer is not efficient in after-sales services.

As an extension, we studied a model where the market is composed of two distinct segments: a fraction of the customers are myopic with low foresight of future costs, and the rest are strategic with high foresight of future costs. In this model, we mainly see that the manufacturer and the retailer follow a pricing strategy that chooses to focus on the myopic customer segment, and address the strategic segment only when they can capture the additional potential it offers without significantly lowering the prices. We also found that most results of the decentralized model also hold when the customers are heterogeneous in their foresight of future costs: First of all, the manufacturer makes the majority of her revenues and profits from after-sales rather than product sales, except for a few instances where the product price is positive. Secondly, the manufacturer captures two-thirds of the total chain profit in the majority of the instances, consistent with the results of the decentralized model. Thirdly, as the average customer foresight increases, the spare parts wholesale and retail prices, as well as the manufacturer and retailer profits decrease, and the consumer surplus increases; except for rare cases with high disparity in consumer foresight. Finally, we found that the profits and consumer surplus are not sensitive to different levels of warranty coverage offered by the manufacturer.

Based on our findings and insights, we would advise a manufacturer to increase the reliability of her product and reduce her after-sales costs. We also expect her to avoid measures that would actively increase her after-sales cost transparency. We would advise the regulatory authorities and non-governmental organizations to focus their effort on increasing the transparency of after-sales costs, to consequently increase the consumer surplus in the market. Additionally, they should be aware that when both the sales and the after-sales markets are a monopoly, a law-mandated warranty coverage is irrelevant in any case. From a regulatory authority's perspective, one should be careful when mandating a warranty coverage and forcing the manufacturer to participate in the after-sales market, especially when the sales market is a monopoly. In majority of the cases, that could work to the disadvantage of the consumer. The competitive structure in the after-sales market also needs to be taken into account, along with other market characteristics, since it will influence the effect of a regulation. As a final remark, we also would like to point out that enforcing a decentralized system through an independent retailer for providing after-sales services as opposed to a fully integrated retail channel does not change the manufacturer's main strategy. In both decentralized and centralized versions, customers still face steep after-sales costs once locked in with the product and the warranty coverage has no effect on profits or consumer surplus.

CHAPTER 4

DOWNSTREAM COMPETITION MODEL

In this model, an independent (third-party) workshop competes with the retailer for after-sales services, offering non-original spare parts for the repair and maintenance activities. The non-original spare parts are perfect substitutes for the original spare parts and tend to have lower prices, but the customers perceive them as lower quality replacements, because they do not carry the manufacturer's brand. We call this model "Downstream Competition" and refer to it with superscript C.

We mainly conduct an analytical study of two models in this chapter: downstream competition in a decentralized and a centralized setting. In the next chapter, we complement the analytical results in this chapter with numerical experiments.

The utility function for the customers under the downstream competition model, $U_\theta^C(p, s_o)$, is given in Equation 4.1. In addition to the basic dynamics described in the previous chapter, we now have an additional parameter, $\delta \in (0, 1)$, which indicates how much the service received in an independent workshop with non-original spare parts are perceived by the customers as similar quality service received in the retailer with original spare parts. Higher δ figures indicate that the customer's perception of the service in the independent workshop with non-original spare parts is higher, where $\delta = 1$ would indicate they are considered identical.

In this model, the total product demand, $q^C(p, s_o)$, consists of two parts: Customers that will use the after-sales services provided by the retailer with original spare parts ($q_r^C(p, s_o)$), and customers that will use the after-sales services provided by the independent workshop with non-original spare parts ($q_i^C(p, s_o)$). Each customer makes their decision regarding which strategy to follow based on their utility function at the point of purchase, and applies this decision throughout the lifetime of the product.

$$U_\theta^C(p, s_o) = \begin{cases} 0 & \\ \quad \text{if the customer of type } \theta \text{ does not buy} & \\ v - p + \theta - \beta f_e(s_o + \alpha l_r) & \\ \text{if the customer of type } \theta \text{ buys and gets serviced in} & \\ \quad \text{the retailer with original parts} & \\ v - p + \delta\theta - \beta f_e(s_n + \alpha l_i) & \\ \text{if the customer of type } \theta \text{ buys and gets serviced in} & \\ \text{the independent workshop with non-original parts} & \end{cases} \tag{4.1}$$

$$q^C(p, s_o) = q_r^C(p, s_o) + q_i^C(p, s_o) \tag{4.2}$$

The retailer's profit function, $\Pi_r^C(p, \omega_o, s_o)$, is given in Equation 4.3. The retailer earns a profit from warranty-related labor work for all products sold, which is obtained by subtracting the unit labor cost c_r from the unit labor price l_r.[1] The second profit stream for the retailer is sales of non-warranty labor and original parts to the customers, where the unit profit for the parts is found by subtracting the manufacturer's wholesale price ω_o from the retailer's selling price s_o. In the monopoly model, the second profit stream applies to all products sold; whereas in the downstream competition model it only applies to the customers that choose the retailer for non-warranty work. Each unit profit component is multiplied by the relevant total demand, $q^C(p, s_o)$, or $q_r^C(p, s_o)$.

$$\Pi_r^C(p, \omega_o, s_o) = f_m \alpha (l_r - c_r) q^C(p, s_o) + f_e \left(\alpha(l_r - c_r) + s_o - \omega_o\right) q_r^C(p, s_o) \tag{4.3}$$

The independent workshop's profit function, $\Pi_i^C(p, s_o)$, is given in Equation 4.4. The independent workshop earns a profit from sales of non-warranty labor and non-original parts to the customers. Unit profit for the labor is found by subtracting the unit labor cost c_i from the unit labor sales price l_i. Unit profit for non-original parts

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[1] Note that the retailer charges the same price to the manufacturer and the customer for labor.

is given by subtracting the purchase price ω_n from the sales price s_n.

$$\Pi_i^C(p, s_o) = f_e \left(\alpha(l_i - c_i) + s_n - \omega_n\right) q_i^C(p, s_o) \tag{4.4}$$

The manufacturer's profit function, $\Pi_m^C(p, \omega_o, s_o)$, is given in Equation 4.5. The manufacturer obtains a profit of $p - c$ from the sales of each unit, which is reduced by the warranty period costs of original spare parts and labor, $c_o + \alpha l_r$, that are incurred or paid to the retailer. The second profit stream is the sales of original spare parts to the retailer with a wholesale price of ω_o and a cost of c_o. The manufacturer is not involved in the business for non-original spare parts.

$$\Pi_m^C(p, \omega_o, s_o) = \left(p - c - f_m(\alpha l_r + c_o)\right) q^C(p, s_o) + f_e(\omega_o - c_o) q_r^C(p, s_o) \tag{4.5}$$

Consumer surplus is calculated with the generic function $\int_{\hat{\theta}_i}^{\hat{\theta}_r} U_i^D(\theta, p, s_o) + \int_{\hat{\theta}_r}^{b} U_r^D(\theta, p, s_o)$, where $\hat{\theta}_i$ represents the last customer that buys the product and gets serviced by the independent workshop and $\hat{\theta}_r$ represents the last customer that buys the product and gets serviced by the retailer. However, since the customers underestimate the future costs by a factor of β, but actually receive a lower *real* utility, we consider the following function when estimating the consumer surplus.

$$\begin{aligned} CS^C(p, \omega_o, s_o) = &\int_{\hat{\theta}_i}^{\hat{\theta}_r} \frac{1}{b}(v - p + \theta - f_e(s_n + \alpha l_i))d\theta \\ &+ \int_{\hat{\theta}_r}^{b} \frac{1}{b}(v - p + \theta - f_e(s_o + \alpha l_r))d\theta \end{aligned} \tag{4.6}$$

We consider a game where the manufacturer (the leader) is followed by the retailer. The decisions and the sequence of events are as below. All other parameters are assumed exogenous.

1. Manufacturer sets the product retail price to the end customer, p
2. Manufacturer sets the wholesale price of original spare parts to the retailer, ω_o
3. Retailer sets the retail price of original spare parts to the end customer, s_o

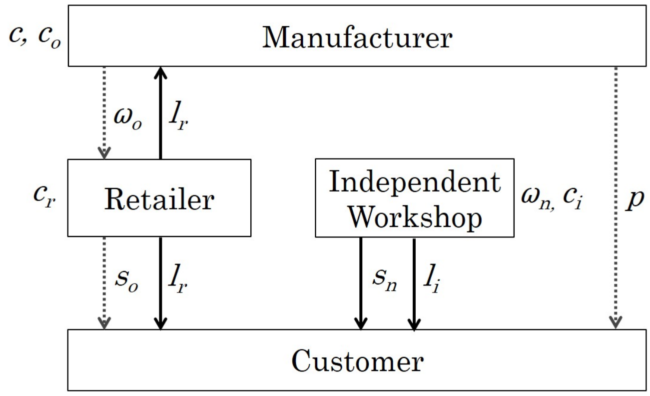


Figure 4.1: Financial Flow of the Downstream Competition Model

An overview of the parameters and decision variables is given in Table 4.1. A depiction of the financial flow is given in Figure 4.1. All proofs are given in Appendix D.

4.1 Analysis of the Downstream Competition Model

In this section, we attempt deriving the analytical solution to the equilibrium of the downstream competition model by backward induction. We first start by deriving the demand based on the utility function.

Lemma 1 *If* $\theta_2 > \theta_1$ *and* $U_r^C(\theta_1, p, s_o) > U_i^C(\theta_1, p, s_o)$*, then* $U_r^C(\theta_2, p, s_o) > U_i^C(\theta_2, p, s_o)$.

Lemma 1 shows an important step towards deriving the demand function. It states that if a customer with lower variable valuation (θ_1) prefers the retailer, then another customer with higher valuation than him (θ_2 with $\theta_2 > \theta_1$) prefers the retailer, as well. Next, we derive the demand function as follows:

Lemma 2 *For a given* p *and* s_o*, the total demand,* $q^C(p, s_o)$*, the division of demand between retailer (with original spare parts) and independent workshop (with non-*

Table 4.1: List of Parameters and Decision Variables for the Downstream Competition Model

Parameters	
f_m	Expected number of repair and maintenance needs paid by the manufacturer
f_e	Expected number of repair and maintenance needs paid by the customer
f	Total expected number of repair and maintenance needs during product's lifetime ($f = f_m + f_e$)
p	Customer end price of the product
c	Manufacturer's cost of product
α	Units of labor required per unit of spare parts installed in a workshop order
β	Customer's level of strategic thinking ($\beta > 0$)
θ	Customer's utility parameter on after-sales service quality (for both services and spare parts)
c_o	Manufacturer's cost of original spare parts
c_r	Retailer's cost of labor for one workshop order
c_i	Independent workshop's cost of labor for one workshop order
l_r	Average price of retailer's labor sales to the customers and the manufacturer
l_i	Average price of independent workshop's labor sales to the customers ($l_i < l_r$)
ω_n	Independent workshop's cost of non-original spare parts
s_n	Independent workshop's retail price of non-original spare parts to the end customers
δ	Similarity of the independent workshop service with non-original parts to the retailer service with original parts ($0 < \delta < 1$)

Decision Variables	
s_o	(Retailer) Average retail price of spare parts to the end customers
ω_o	(Manufacturer) Average wholesale price of spare parts to the retailer
p	(Manufacturer) Retail price of the product

original spare parts), $q_r^C(p, s_o)$ and $q_i^C(p, s_o)$, are given as follows:

$$
\begin{aligned}
&(q_i^C(p, s_o), q_r^C(p, s_o)) \\
&= \begin{cases}
(0, 1), \\
\qquad\qquad \textit{if } s_o - s_n + \alpha(l_r - l_i) < 0 \textit{ and } 0 < s_o + \alpha l_r < \frac{\upsilon - p}{\beta f_e}, \\
\left(0, 1 - \frac{\beta f_e(s_o + \alpha l_r) - \upsilon + p}{b}\right), \\
\textit{if } s_n + \alpha l_i - \delta(s_o + \alpha l_r) > \frac{(1-\delta)(\upsilon - p)}{\beta f_e} \textit{ and } \frac{\upsilon - p}{\beta f_e} \leq (s_o + \alpha l_r) \leq \frac{b + \upsilon - p}{\beta f_e}, \\
(0, 0), \\
\qquad\qquad \textit{if } s_n + \alpha l_i > \frac{b\delta + \upsilon - p}{\beta f_e} \textit{ and } s_o + \alpha l_r > \frac{b + \upsilon - p}{\beta f_e}, \\
\left(1 - \frac{\beta f_e(s_n + \alpha l_i) - \upsilon + p}{b\delta}, 0\right), \\
\qquad\qquad \textit{if } \frac{\upsilon - p}{\beta f_e} \leq s_n + \alpha l_i \leq \frac{b\delta + \upsilon - p}{\beta f_e} \textit{ and } s_o - s_n + \alpha(l_r - l_i) > \frac{b(1-\delta)}{\beta f_e}, \\
(1, 0), \\
\qquad\qquad \textit{if } 0 < s_n + \alpha l_i < \frac{\upsilon - p}{\beta f_e} \textit{ and } s_o - s_n + \alpha(l_r - l_i) > \frac{b(1-\delta)}{\beta f_e}, \\
\left(\frac{\beta f_e(s_o - s_n + \alpha(l_r - l_i))}{b(1-\delta)}, 1 - \frac{\beta f_e(s_o - s_n + \alpha(l_r - l_i))}{b(1-\delta)}\right), \\
\qquad\qquad \textit{if } 0 < s_n + \alpha l_i < \frac{\upsilon - p}{\beta f_e} \textit{ and } 0 \leq s_o - s_n + \alpha(l_r - l_i) \leq \frac{b(1-\delta)}{\beta f_e}, \\
\left(\frac{\beta f_e(s_o - s_n + \alpha(l_r - l_i))}{b(1-\delta)} - \frac{\beta f_e(s_n + \alpha l_i) - \upsilon + p}{b\delta}, 1 - \frac{\beta f_e(s_o - s_n + \alpha(l_r - l_i))}{b(1-\delta)}\right), \\
\qquad\qquad \textit{if } s_n + \alpha l_i \geq \frac{\upsilon - p}{\beta f_e} \textit{ and } s_n + \alpha l_i - \delta(s_o + \alpha l_r) \leq \frac{(1-\delta)(\upsilon - p)}{\beta f_e} \\
\qquad\qquad\qquad \textit{and } s_o - s_n + \alpha(l_r - l_i) \leq \frac{b(1-\delta)}{\beta f_e}.
\end{cases}
\end{aligned}
$$

A graphical depiction of the demand function can be seen in Figure 4.2. A more detailed version of the same graph that shows how the thresholds are derived is available in Figure **??** in Appendix D. We observe that there are three main areas of interest:

- $p < \upsilon - \beta f_e(s_n + \alpha l_i)$, or equivalently, $p + \beta f_e(s_n + \alpha l_i) < \upsilon$ is the area where the perceived total cost of ownership (TCO) when the independent workshop is chosen is below the base valuation of any customer. Therefore, it is ensured that the market is fully covered and all customers purchase the product regardless of the retailer's original spare part pricing. However, the split of the market amongst the retailer and the independent workshop depends on the retailer's original spare part pricing:
  - The market is fully covered by the retailer if the average repair cost of the independent workshop is more expensive than the retailer, *i.e.*, $s_o + \alpha l_r < s_n + \alpha l_i$ (region (-3)).

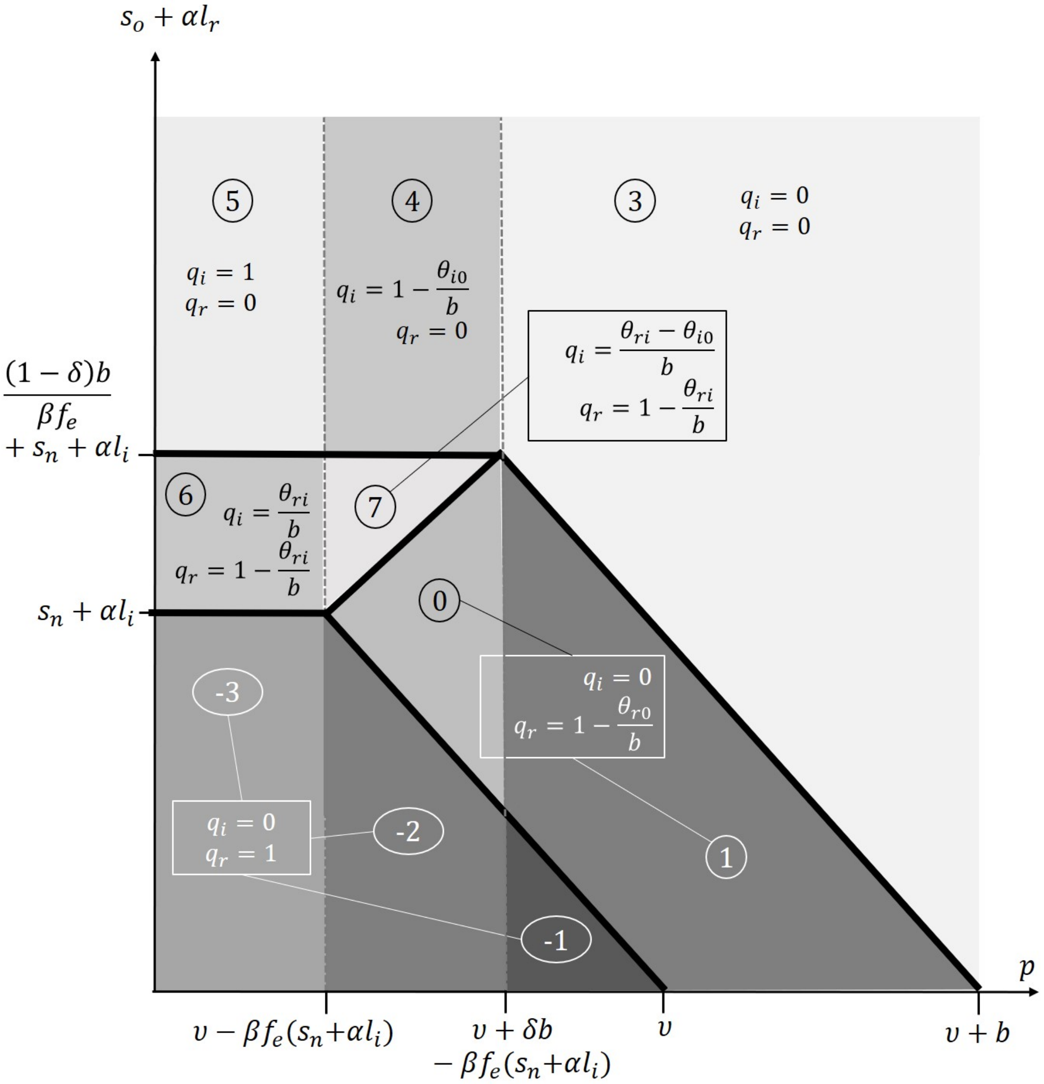


Figure 4.2: Demand function for different s_o and p combinations where $\theta_{ri} = \frac{\beta f_e((s_o+\alpha l_r)-(s_n+\alpha l_i))}{1-\delta}$, $\theta_{r0} = \beta f_e(s_o + \alpha l_r) - (\upsilon - p)$, and $\theta_{i0} = \frac{\beta f_e(s_n+\alpha l_i)-(\upsilon-p)}{\delta}$

- The market is fully covered and non-warranty after-sales services are solely provided by the independent workshop if the average repair cost of the retailer is $\frac{b(1-\delta)}{\beta f_e}$ units more expensive than that of the independent workshop, *i.e.,* $s_o + \alpha l_r > s_n + \alpha l_i + \frac{b(1-\delta)}{\beta f_e}$ (region ⑤). Note that this difference of $\frac{(1-\delta)b}{\beta f_e}$ corresponds to what the customer with the highest variable valuation, b, would be willing to pay extra for the retailer's services that are perceived to be $(1-\delta)$ times better (division by f_e ensuring even split across the number of repairs and division by β reflecting underestimation of future costs). When the retailer's price difference to the independent workshop is above this value, no customer would be willing to purchase any services from the retailer with original spare parts.
- When the retailer's price is somewhere in between the above described regions, the market is divided between the retailer and the independent workshop, the shares depending on the price difference between the retailer and the independent workshop (region ⑥).

- $p > v+\delta b-\beta f_e(s_n+\alpha l_i)$, or equivalently, $p+\beta f_e(s_n+\alpha l_i) > v+\delta b$ is the opposite area where the perceived TCO when the independent workshop is chosen is below the maximum valuation of any customer, which is the base valuation plus the highest possible variable valuation, b, multiplied by the similarity of the independent workshop alternative, δ. In this case, the customer will never prefer the independent retailer under any circumstances. Therefore, the demand for the independent workshop is zero and the model analytically reduces to the monopoly model. The retailer's spare parts price, s_o, then determines the level of market coverage, which can be analyzed in three regions:
  - The market is fully covered where $s_o + \alpha l_r < \frac{v-p}{\beta f_e}$ (region (-1)).
  - There is partial coverage of the market in between (region ①).
  - There is no demand where $s_o + \alpha l_r > \frac{v-p+b}{\beta f_e}$ (region ③).

- The area in between the above two described areas where $p + \beta f_e(s_n + \alpha l_i)$ is neither too low, nor too high is the one that displays most diversity in terms of market coverage and split between the independent workshop and the retailer. It is made up of four regions:

  - The market is fully covered by the retailer where $s_o + \alpha l_r < \frac{v-p}{\beta f_e}$ (region ⊖2).
  - The market is partially covered by the retailer only, with no demand for the independent workshop, where $s_o + \alpha l_r > \frac{v-p}{\beta f_e}$ (region ⓪). Here, $s_n + \alpha l_i$ is still too high as compared to $s_o + \alpha l_r$ to get any share and increase the market coverage.
  - The market is partially covered by the independent workshop only, with no demand for the retailer, where $s_o + \alpha l_r > s_n + \alpha l_i + \frac{b(1-\delta)}{\beta f_e}$ (region ④).
  - The remaining sub-region is the most complicated one with partial demand for both the retailer and the independent workshop, summing up to still partial coverage of the market (region ⑦).

Next, we analyze the retailer's best response, s_o, which is the first step to the backward induction.

4.1.1 The Retailer's Best Response, s_o

For the analysis of the retailer's best response, we separately handle the three areas categorized by the level of the perceived TCO when the customer buys and uses the independent workshop with non-original spare parts for after-sales services, $p + \beta f_e(s_n + \alpha l_i)$. Since this term frequently arises, we name it TCO_i. The naming and notation for the areas are then set as follows:

- The area where $TCO_i > v+\delta b$, i.e., $p > v+\delta b-\beta f_e(s_n+\alpha l_i)$, is considered the "High TCO_i" area, and is to be denoted by superscript CH. This region has the same analytical analysis and results as the Monopoly Model, so only the final results are given, omitting the detailed derivations where possible. Consistent with the x-axis of Figure 4.2, we also refer to this area as the "High p" area.

- The area where $TCO_i < v$, i.e., $p < v - \beta f_e(s_n + \alpha l_i)$, is considered the "Low TCO_i" area, to be denoted by superscript CL. Similarly it is also referred to as the "Low p" area.

- The area where $\upsilon < TCO_i < \upsilon + \delta b$, i.e., $\upsilon - \beta f_e(s_n + \alpha l_i) < p < \upsilon + \delta b - \beta f_e(s_n + \alpha l_i)$, is considered the "Medium TCO_i" area, or equivalently as the "Medium p" area, to be denoted by superscript CM.

First, let us have a look into the high TCO_i area and repeat the previous result from the Monopoly Model:

Proposition 5 *Under the high TCO_i area of downstream competition model where $p > \upsilon + \delta b - \beta f_e(s_n + \alpha l_i)$, the retailer's best response given the manufacturer's product retail price and the spare parts wholesale price decision, $s_o^*(p, \omega_o)$, is as follows:*

$$s_o^*(p,\omega_o) = \begin{cases} \frac{\upsilon-p}{\beta f_e} - \alpha l_r, & \\ \hfill \textit{if}\, \omega_o \leq \frac{\upsilon-p-b}{\beta f_e} + \frac{f_m\alpha(l_r-c_r)}{f_e} - \alpha c_r, & \\ \frac{1}{2}\left(\omega_o + \alpha c_r + \frac{\upsilon-p+b}{\beta f_e} - \frac{f_m\alpha(l_r-c_r)}{f_e}\right) - \alpha l_r, & \\ \textit{if}\, \frac{\upsilon-p-b}{\beta f_e} + \frac{f_m\alpha(l_r-c_r)}{f_e} - \alpha c_r < \omega_o \leq \frac{\upsilon-p+b}{\beta f_e} + \frac{f_m\alpha(l_r-c_r)}{f_e} - \alpha c_r, & \\ \left[\frac{\upsilon-p+b}{\beta f_e} - \alpha l_r\ , \infty\right), & \\ \hfill \textit{if}\, \omega_o > \frac{\upsilon-p+b}{\beta f_e} + \frac{f_m\alpha(l_r-c_r)}{f_e} - \alpha c_r. & \end{cases}$$

The interpretation of these results have already been made in the first chapter and are not repeated here.

Next, we analyze the low TCO_i region, where the independent workshop is highly competitive, and the market is always fully covered (either by the independent workshop, or the retailer, or both).

Lemma 3 *The following statements are true regarding the profit function of the retailer, $\Pi_r^{CL}(\omega_o, s_o)$, within the low TCO_i region of $p < \upsilon - \beta f_e(s_n + \alpha l_i)$,*

$$\Pi_r^{DL}(p,\omega_o,s_o) = \begin{cases} \Pi^{CL}_{r(-3)}(p,\omega_o,s_o) = f_m\alpha(l_r - c_r) + f_e(\alpha(l_r - c_r) + s_o - \omega_o), & \\ \hfill \textit{if}\, s_o < s_n + \alpha l_i - \alpha l_r, & \\ \Pi^{CL}_{r(6)}(p,\omega_o,s_o) = f_m\alpha(l_r - c_r) + f_e(\alpha(l_r - c_r) + s_o - \omega_o) & \\ \hfill \times\left(1 - \frac{\beta f_e(s_o - s_n + \alpha(l_r - l_i))}{b(1-\delta)}\right), & \\ \hfill \textit{if}\, s_n + \alpha l_i - \alpha l_r \leq s_o \leq \frac{b(1-\delta)}{\beta f_e} + s_n + \alpha l_i - \alpha l_r, & \\ \Pi^{CL}_{r(5)}(p,\omega_o,s_o) = f_m\alpha(l_r - c_r), & \\ \hfill \textit{if}\, s_o > \frac{b(1-\delta)}{\beta f_e} + s_n + \alpha l_i - \alpha l_r\ : & \end{cases}$$

1. $\Pi_r^{CL}(p, \omega_o, s_o)$ *is continuous in* s_o.

2. $\Pi_{r(-3)}^{CL}(p, \omega_o, s_o)$ *is increasing in* s_o.

3. $\Pi_{r(6)}^{CL}(p, \omega_o, s_o)$ *is concave in* s_o.

Proposition 6 *The retailer's best response given the manufacturer's spare parts wholesale price and product retail price decisions,* $s_o^*(p, \omega_o)$*, within the low* TCO_i *region of* $p < v - \beta f_e(s_n + \alpha l_i)$ *is as follows:*

$$s_o^*(p, \omega_o) = \begin{cases} s_n + \alpha l_i - \alpha l_r, \\ \qquad\qquad\qquad\qquad \textit{if}\, \omega_o < s_n + \alpha l_i - \frac{b(1-\delta)}{\beta f_e} - \alpha c_r, \\ \frac{b(1-\delta)}{\beta f_e} + \frac{\omega_o + s_n + \alpha(c_r + l_i)}{2} - \alpha l_r, \\ \quad \textit{if}\, s_n + \alpha l_i - \frac{b(1-\delta)}{\beta f_e} - \alpha c_r \leq \omega_o \leq s_n + \alpha l_i + \frac{b(1-\delta)}{\beta f_e} - \alpha c_r, \\ \left[s_n + \alpha l_i + \frac{b(1-\delta)}{\beta f_e} - \alpha l_r \ , \infty \right), \\ \qquad\qquad\qquad\qquad \textit{if}\, \omega_o > s_n + \alpha l_i + \frac{b(1-\delta)}{\beta f_e} - \alpha c_r. \end{cases}$$

In the low TCO_i area, the retailer's pricing is mainly steered by the independent workshop's prices and the wholesale price. We see neither v, nor p play a part in the best response prices or the boundaries that they are valid (apart from setting the area we operate in the first place, as $p < v - \beta f_e(s_n + \alpha l_i)$). We see that the retailer sets his total repair price ($s_o + \alpha l_r$) equal to that of the independent workshop ($s_n + \alpha l_i$) when he chooses to cover the aftermarket completely by himself. When we check the boundaries of this region, we see that the retailer ensures a profit of at least $b(1 - \delta)$, hence his profit increases if the customers are more heterogeneous in the market and/or they see the independent workshop as a less worthy substitute.

On the other extreme case of no market coverage, the spare part wholesale price is at least $\frac{b(1-\delta)}{\beta f_e}$ higher than the independent workshop's total repair price, which exceeds any price the retailer could possibly charge the customer. Therefore, his best response is to set the price as high as this amount and choose not to serve in the customer-paid after-sales business, letting the independent workshop fully take over. As $s_n + \alpha l_r$ is sufficiently low and the market is fully covered, he still makes profit from the labor component of the warranty-covered repairs paid by the manufacturer.

Unlike the high TCO_i area, we see that the retailer doesn't explicitly sacrifice his warranty-coverage profits of $f_m \alpha(l_r - c_r)$ in order to attract the customers. The main issue here is that as the market is always fully covered regardless of the price the retailer sets, the warranty profits are always ensured for the retailer. Then what matters is whether $\omega_o + \alpha c_r$ is sufficiently low in order to motivate the retailer into competing for the customer-paid after-sales business, or letting the independent workshop take care of it.

Next, we analyze the medium TCO_i area, where the independent workshop is averagely competitive, and we observe the highest variety of possible outcomes.

Lemma 4 *The following statements are true regarding the profit function of the retailer,* $\Pi_r^{CM}(p, \omega_o, s_o)$*, within the medium* TCO_i *area of* $v - \beta f_e(s_n + \alpha l_i) < p < b\delta + v - \beta f_e(s_n + \alpha l_i)$*,*

$$\Pi_r^{CM}(p, \omega_o, s_o)$$
$$= \begin{cases} \Pi_{r(-2)}^{CM}(p, \omega_o, s_o) = f_m \alpha(l_r - c_r) + f_e(\alpha(l_r - c_r) + s_o - \omega_o), \\ \hfill \textit{if } s_o < \frac{v-p}{\beta f_e} - \alpha l_r, \\ \Pi_{r(0)}^{CM}(p, \omega_o, s_o) = (f_m \alpha(l_r - c_r) + f_e(\alpha(l_r - c_r) + s_o - \omega_o)) \\ \hfill \times \left(1 - \frac{\beta f_e(s_o + \alpha l_r) - v - p}{b}\right), \\ \hfill \textit{if } s_o < \frac{s_n + \alpha l_i}{\delta} - \frac{(1-\delta)(v-p)}{\delta \beta f_e} - \alpha l_r \textit{ and } s_o \geq \frac{v-p}{\beta f_e} - \alpha l_r, \\ \Pi_{r(7)}^{CM}(p, \omega_o, s_o) = f_m \alpha(l_r - c_r)\left(1 - \frac{\beta f_e(s_n + \alpha l_i) - v - p}{b\delta}\right) \\ \hfill + f_e(\alpha(l_r - c_r) + s_o - \omega_o)\left(1 - \frac{\beta f_e(s_o - s_n + \alpha(l_r - l_i))}{b(1-\delta)}\right), \\ \textit{if } s_o > \frac{s_n + \alpha l_i}{\delta} - \frac{(1-\delta)(v-p)}{\delta \beta f_e} - \alpha l_r \textit{ and } s_o \leq \frac{b(1-\delta)}{\beta f_e} + s_n + \alpha l_i - \alpha l_r, \\ \Pi_{r(4)}^{CM}(p, \omega_o, s_o) = f_m \alpha(l_r - c_r)\left(1 - \frac{\beta f_e(s_n + \alpha l_i) - v - p}{b\delta}\right), \\ \hfill \textit{if } s_o > \frac{b(1-\delta)}{\beta f_e} + s_n + \alpha l_i - \alpha l_r\textit{:} \end{cases}$$

1. $\Pi_r^{CM}(p, \omega_o, s_o)$ *is continuous in* s_o*.*
2. $\Pi_{r(-2)}^{CM}(p, \omega_o, s_o)$ *is increasing in* s_o*.*
3. $\Pi_{r(0)}^{CM}(p, \omega_o, s_o)$ *is concave in* s_o*.*
4. $\Pi_{r(7)}^{CM}(p, \omega_o, s_o)$ *is concave in* s_o*.*

Next, we derive the retailer's best response in the medium TCO_i region in Proposition 7. But before moving on to the proposition itself, let us explain how complicated the analytical solutions can get in this model. The complexity mainly arises from having two concave pieces in Π_r^{CM}. The cases these concave pieces appear in Proposition 7 is visualized in Figure 4.3. As it can also be seen there, these pieces can behave in an increasing, decreasing or concave manner depending on where their peak values are as compared to the boundaries of the interval they are valid within. Because we have two of these concave pieces, the alternative conditions they create also interact with each other and result in more cases to explore.

Here is how the intervals interact with each other and the complexity develops: The first concave piece, $\Pi_{r(0)}^{CM}$, creates two boundaries for ω_o: $A = \frac{\upsilon - p - b + \beta f_m \alpha (l_r - c_r)}{\beta f_e} - \alpha c_r$ and $B = \frac{2\beta f_e (s_n + \alpha l_i) + \delta \beta f_m \alpha (l_r - c_r) - (2-\delta)(\upsilon - p) - \delta b}{\delta \beta f_e} - \alpha c_r$, where $A < B$, and the piece is

- decreasing when $\omega_o < A$,
- concave when $A < \omega_o < B$,
- increasing when $\omega_o > B$.

Similarly, the second concave piece ($\Pi_{r(7)}^{CM}$) creates two boundaries for ω_o: $C = \frac{\beta f_e (2-\delta)(s_n + \alpha l_i) - (1-\delta)(\delta b + 2(\upsilon - p))}{\delta \beta f_e} - \alpha c_r$ and $D = \frac{\beta f_e (s_n + \alpha l_i) + (1-\delta) b}{\beta f_e} - \alpha c_r$, where $C < D$.[2] Then, we need to look into how these intervals interact with each other (e.g., $A < B < C < D$, $C < A < B < D$, $A < C < B < D$) and see which are valid and which best responses they correspond to. Since four of the nine alternatives for best responses require an objective function comparison between two potential peaks, this also creates sub-cases within each case. In short, we have two complications that increase the number of potential cases: (1) Not knowing how the boundaries on ω_o are ordered and having the need to study all possible alternatives; (2) Having more than one alternative peak in some alternatives due to piecewise continuity and having the necessity to compare objective function values in order to be able to determine the s_o value that maximizes the retailer's profit function (as illustrated in Figure 4.3). We provide the details of the derivations in the proof for the upcoming Proposition 7 and two summary tables that follow it in Appendix D, which show the details for the resulting 23 sub-cases.

---

[2] As we have done in the other areas, we represent all conditions that need to be satisfied for the best response s_o to be valid within as lower and upper bounds on ω_o. This is needed for consistency in representation and in preparation for the next step of backward induction, the manufacturer's best response on ω_o.

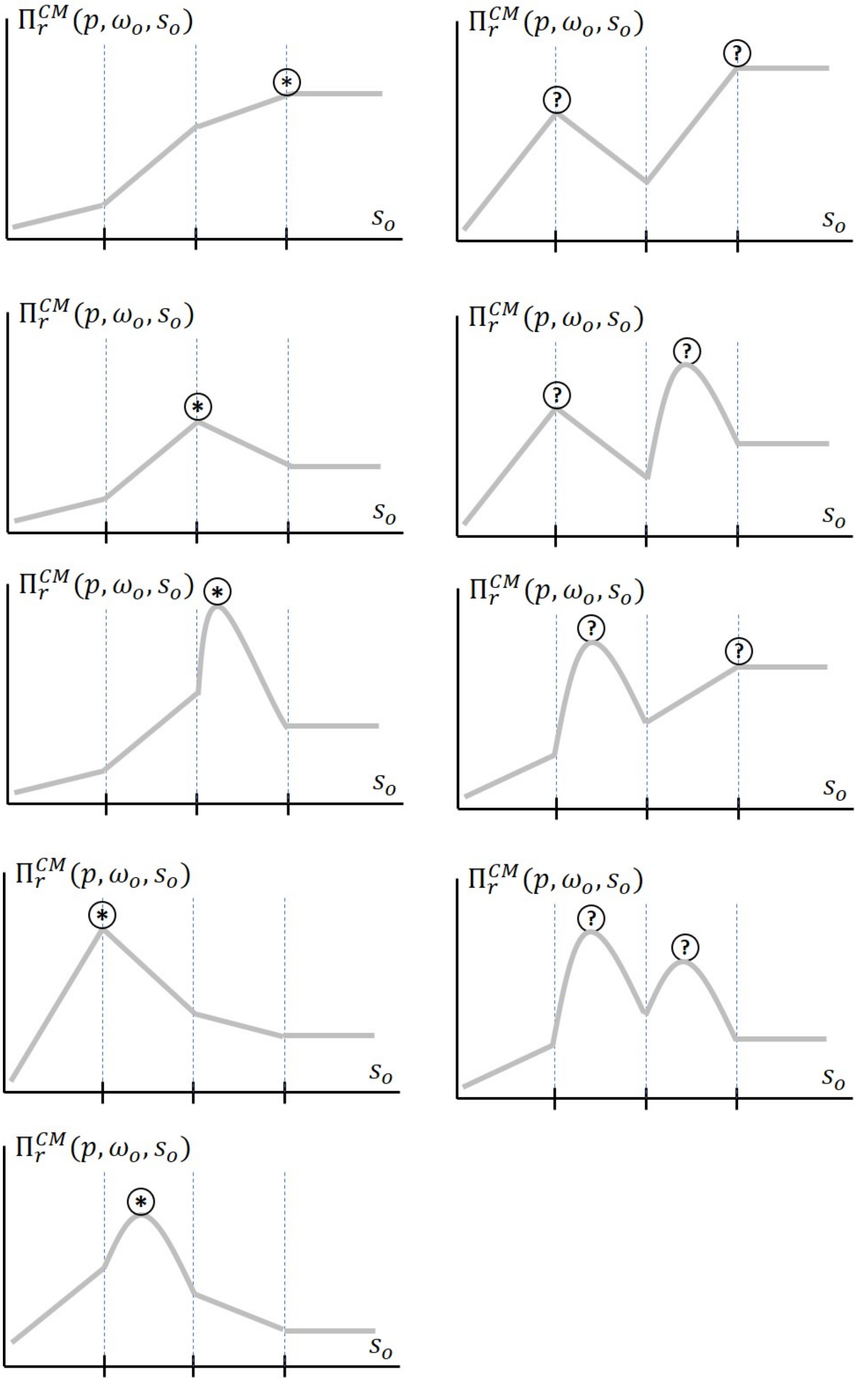


Figure 4.3: Graphical visualisation of how the retailer's best response is formed in the medium TCO_i area of the downstream competition model

Proposition 7 *The retailer's best response given the manufacturer's wholesale price decision, $s_o^*(p, \omega_o)$, within the medium s_n region of $\frac{\upsilon-p}{\beta f_e} < s_n + \alpha l_i < \frac{b\delta+\upsilon-p}{\beta f_e}$ is as follows:*

$$
s_o^*(\omega_o) =
$$

$$
\left\{
\begin{array}{l}
\frac{\beta f_e(s_n+\alpha l_i)+b(1-\delta)}{\beta f_e} - \alpha l_r, \\
\hfill \textit{if}\ \omega_o + \alpha c_r > \frac{2\beta f_e(s_n+\alpha l_i)+\delta\beta f_m\alpha(l_r-c_r)-(2-\delta)(\upsilon-p)-\delta b}{\delta\beta f_e} \\
\hfill \textit{and}\ \omega_o > \frac{\beta f_e(s_n+\alpha l_i)+(1-\delta)b}{\beta f_e} - \alpha c_r, \\
\frac{\beta f_e(s_n+\alpha l_i)-(1-\delta)(\upsilon-p)}{\delta\beta f_e} - \alpha l_r, \\
\hfill \textit{if}\ \omega_o + \alpha c_r > \frac{2\beta f_e(s_n+\alpha l_i)+\delta\beta f_m\alpha(l_r-c_r)-(2-\delta)(\upsilon-p)-\delta b}{\delta\beta f_e} \\
\hfill \textit{and}\ \omega_o + \alpha c_r < \frac{\beta f_e(2-\delta)(s_n+\alpha l_i)-(1-\delta)(\delta b+2(\upsilon-p))}{\delta\beta f_e}, \\
\frac{b(1-\delta)+\beta f_e(\omega_o+\alpha c_r+s_n+\alpha l_i)}{2\beta f_e} - \alpha l_r, \\
\hfill \textit{if}\ \omega_o + \alpha c_r > \frac{2\beta f_e(s_n+\alpha l_i)+\delta\beta f_m\alpha(l_r-c_r)-(2-\delta)(\upsilon-p)-\delta b}{\delta\beta f_e} \\
\hfill \textit{and}\ \frac{\beta f_e(s_n+\alpha l_i)+(1-\delta)b}{\beta f_e} > \omega_o + \alpha c_r > \frac{\beta f_e(2-\delta)(s_n+\alpha l_i)-(1-\delta)(\delta b+2(\upsilon-p))}{\delta\beta f_e}, \\
\frac{\upsilon-p}{\beta f_e} - \alpha l_r, \\
\hfill \textit{if}\ \omega_o + \alpha c_r < \frac{\upsilon-p-b+\beta f_m\alpha(l_r-c_r)}{\beta f_e} \\
\hfill \textit{and}\ \omega_o + \alpha c_r < \frac{\beta f_e(2-\delta)(s_n+\alpha l_i)-(1-\delta)(\delta b+2(\upsilon-p))}{\delta\beta f_e}, \\
\frac{\upsilon-p+b+\beta f_e(\omega_o+\alpha c_r)-\beta f_m\alpha(l_r-c_r)}{2\beta f_e} - \alpha l_r, \\
\textit{if}\ \frac{2\beta f_e(s_n+\alpha l_i)+\delta\beta f_m\alpha(l_r-c_r)-(2-\delta)(\upsilon-p)-\delta b}{\delta\beta f_e} > \omega_o + \alpha c_r > \frac{\upsilon-p-b+\beta f_m\alpha(l_r-c_r)}{\beta f_e} \\
\hfill \textit{and}\ \omega_o + \alpha c_r < \frac{\beta f_e(2-\delta)(s_n+\alpha l_i)-(1-\delta)(\delta b+2(\upsilon-p))}{\delta\beta f_e}, \\
\frac{\upsilon-p}{\beta f_e} - \alpha l_r, \\
\hfill \textit{if}\ \omega_o + \alpha c_r < \frac{\upsilon-p-b+\beta f_m\alpha(l_r-c_r)}{\beta f_e} \\
\hfill \textit{and}\ \omega_o + \alpha c_r > \frac{\beta f_e(s_n+\alpha l_i)+(1-\delta)b}{\beta f_e} \\
\hfill \textit{and}\ \frac{\upsilon-p+\beta(f_m\alpha(l_r-c_r)-f_e(\omega_o+\alpha c_r))}{\beta} > \frac{f_m\alpha(l_r-c_r)\beta(\upsilon-p+b\delta-\beta f_e(s_n+\alpha l_i))}{b\delta}, \\
\frac{\beta f_e(s_n+\alpha l_i)+b(1-\delta)}{\beta f_e} - \alpha l_r, \\
\hfill \textit{if}\ \omega_o + \alpha c_r < \frac{\upsilon-p-b+\beta f_m\alpha(l_r-c_r)}{\beta f_e} \\
\hfill \textit{and}\ \omega_o + \alpha c_r > \frac{\beta f_e(s_n+\alpha l_i)+(1-\delta)b}{\beta f_e} \\
\hfill \textit{and}\ \frac{\upsilon-p+\beta(f_m\alpha(l_r-c_r)-f_e(\omega_o+\alpha c_r))}{\beta} < \frac{f_m\alpha(l_r-c_r)\beta(\upsilon-p+b\delta-\beta f_e(s_n+\alpha l_i))}{b\delta}, \\
\frac{\beta f_e(s_n+\alpha l_i)+b(1-\delta)}{\beta f_e} - \alpha l_r, \\
\textit{if}\ \frac{2\beta f_e(s_n+\alpha l_i)+\delta\beta f_m\alpha(l_r-c_r)-(2-\delta)(\upsilon-p)-\delta b}{\delta\beta f_e} > \omega_o + \alpha c_r > \frac{\upsilon-p-b+\beta f_m\alpha(l_r-c_r)}{\beta f_e} \\
\hfill \textit{and}\ \omega_o + \alpha c_r > \frac{\beta f_e(s_n+\alpha l_i)+(1-\delta)b}{\beta f_e} \\
\hfill \textit{and}\ \frac{(\upsilon-p+b-\beta(f_e(\omega_o+\alpha c_r)-f_m\alpha(l_r-c_r)))^2}{4\beta b} < \frac{f_m\alpha(l_r-c_r)\beta(\upsilon-p+b\delta-\beta f_e(s_n+\alpha l_i))}{b\delta},
\end{array}
\right.
$$

$$
s_o^*(\omega_o) =
$$

$$
\begin{cases}
\frac{\upsilon-p}{\beta f_e} - \alpha l_r, \\
\qquad \textit{if}\ \omega_o + \alpha c_r < \frac{\upsilon-p-b+\beta f_m\alpha(l_r-c_r)}{\beta f_e} \\
\qquad \textit{and}\ \frac{\beta f_e(s_n+\alpha l_i)+(1-\delta)b}{\beta f_e} > \omega_o + \alpha c_r > \frac{\beta f_e(2-\delta)(s_n+\alpha l_i)-(1-\delta)(\delta b+2(\upsilon-p))}{\delta\beta f_e} \\
\qquad \textit{and}\ \frac{\upsilon-p+\beta(f_m\alpha(l_r-c_r)-f_e(\omega_o+\alpha c_r))}{\beta} > \\
\qquad \frac{f_m\alpha(l_r-c_r)(\upsilon-p-\beta f_e(s_n+\alpha l_i)+b\delta)}{\delta b} + \frac{\alpha c_r\beta f_e-b(1-\delta)-\beta f_e(s_n+\alpha l_i-\omega_o)^2}{4\beta b(1-\delta)}, \\
\frac{b(1-\delta)+\beta f_e(c_o+\alpha c_r+s_n+\alpha l_i)}{2\beta f_e} - \alpha l_r, \\
\qquad \textit{if}\ \omega_o + \alpha c_r < \frac{\upsilon-p-b+\beta f_m\alpha(l_r-c_r)}{\beta f_e} \\
\qquad \textit{and}\ \frac{\beta f_e(s_n+\alpha l_i)+(1-\delta)b}{\beta f_e} > \omega_o + \alpha c_r > \frac{\beta f_e(2-\delta)(s_n+\alpha l_i)-(1-\delta)(\delta b+2(\upsilon-p))}{\delta\beta f_e} \\
\qquad \textit{and}\ \frac{\upsilon-p+\beta(f_m\alpha(l_r-c_r)-f_e(\omega_o+\alpha c_r))}{\beta} \\
\qquad < \frac{f_m\alpha(l_r-c_r)(\upsilon-p-\beta f_e(s_n+\alpha l_i)+b\delta)}{\delta b} + \frac{\alpha c_r\beta f_e-b(1-\delta)-\beta f_e(s_n+\alpha l_i-\omega_o)^2}{4\beta b(1-\delta)}, \\
\frac{\upsilon-p+b+\beta f_e(\omega_o+\alpha c_r)-\beta f_m\alpha(l_r-c_r)}{2\beta f_e} - \alpha l_r, \\
\textit{if}\ \frac{2\beta f_e(s_n+\alpha l_i)+\delta\beta f_m\alpha(l_r-c_r)-(2-\delta)(\upsilon-p)-\delta b}{\delta\beta f_e} > \omega_o + \alpha c_r > \frac{\upsilon-p-b+\beta f_m\alpha(l_r-c_r)}{\beta f_e} \\
\qquad \textit{and}\ \omega_o + \alpha c_r > \frac{\beta f_e(s_n+\alpha l_i)+(1-\delta)b}{\beta f_e} \\
\qquad \textit{and}\ \frac{(\upsilon-p+b-\beta(f_e(\omega_o+\alpha c_r)-f_m\alpha(l_r-c_r)))^2}{4\beta b} > \frac{f_m\alpha(l_r-c_r)\beta(\upsilon-p+b\delta-\beta f_e(s_n+\alpha l_i))}{b\delta}, \\
\frac{\upsilon-p+b+\beta f_e(\omega_o+\alpha c_r)-\beta f_m\alpha(l_r-c_r)}{2\beta f_e} - \alpha l_r, \\
\textit{if}\ \frac{2\beta f_e(s_n+\alpha l_i)+\delta\beta f_m\alpha(l_r-c_r)-(2-\delta)(\upsilon-p)-\delta b}{\delta\beta f_e} > \omega_o + \alpha c_r > \frac{\upsilon-p-b+\beta f_m\alpha(l_r-c_r)}{\beta f_e} \\
\qquad \textit{and}\ \frac{\beta f_e(s_n+\alpha l_i)+(1-\delta)b}{\beta f_e} > \omega_o + \alpha c_r > \frac{\beta f_e(2-\delta)(s_n+\alpha l_i)-(1-\delta)(\delta b+2(\upsilon-p))}{\delta\beta f_e} \\
\qquad \textit{and}\ \frac{(\upsilon-p+b-\beta(f_e(\omega_o+\alpha c_r)-f_m\alpha(l_r-c_r)))^2}{4\beta b} > \\
\qquad \frac{f_m\alpha(l_r-c_r)(\upsilon-p-\beta f_e(s_n+\alpha l_i)+b\delta)}{\delta b} + \frac{\alpha c_r\beta f_e-b(1-\delta)-\beta f_e(s_n+\alpha l_i-\omega_o)^2}{4\beta b(1-\delta)}, \\
\frac{b(1-\delta)+\beta f_e(c_o+\alpha c_r+s_n+\alpha l_i)}{2\beta f_e} - \alpha l_r, \\
\textit{if}\ \frac{2\beta f_e(s_n+\alpha l_i)+\delta\beta f_m\alpha(l_r-c_r)-(2-\delta)(\upsilon-p)-\delta b}{\delta\beta f_e} > \omega_o + \alpha c_r > \frac{\upsilon-p-b+\beta f_m\alpha(l_r-c_r)}{\beta f_e} \\
\qquad \textit{and}\ \frac{\beta f_e(s_n+\alpha l_i)+(1-\delta)b}{\beta f_e} > \omega_o + \alpha c_r > \frac{\beta f_e(2-\delta)(s_n+\alpha l_i)-(1-\delta)(\delta b+2(\upsilon-p))}{\delta\beta f_e} \\
\qquad \textit{and}\ \frac{(\upsilon-p+b-\beta(f_e(\omega_o+\alpha c_r)-f_m\alpha(l_r-c_r)))^2}{4\beta b} < \\
\qquad \frac{f_m\alpha(l_r-c_r)(\upsilon-p-\beta f_e(s_n+\alpha l_i)+b\delta)}{\delta b} + \frac{\alpha c_r\beta f_e-b(1-\delta)-\beta f_e(s_n+\alpha l_i-\omega_o)^2}{4\beta b(1-\delta)}.
\end{cases}
$$

A summary of Proposition 7 and the finding of the best response for each interval is also provided in Tables D.1-D.2 in Appendix D. Due to the complexity, we leave the analytical analysis at this stage and do not continue further. You can also find the analysis of the simplest one of the six possible cases in Appendix B.1.

In order to see if we can simplify the cases, we have also studied a change in the sequence of events. In this alternative, the manufacturer determines ω_o before p. Although it produces somewhat simpler results as compared to this version, it is still quite complicated and results in 17 different cases. The details are given in Appendix B.2 for the interested reader.

4.1.2 The Manufacturer's Best Response, ω_o

As the next step of backward induction, we study the manufacturer's best response of the spare parts wholesale price, ω_o, in this section. Similar to the previous step, we analyze the areas separately based on how high TCO_i, *i.e.,* the level of $p + \beta f_e(s_n + \alpha l_i)$. We provide analytical results for the high and low TCO_i areas and leave the analysis of the medium TCO_i area to the numerical experiments due to complexity.

First, let us have a look into the high TCO_i area and repeat the previous result from the Monopoly Model:

Proposition 8 *Under the high TCO_i area of downstream competition model where $p > b\delta + v - \beta f_e(s_n + \alpha l_i)$, for a given value of p, the original spare part wholesale price ω_o^* set by the manufacturer, and the consequent retail price s_o^* are as follows:*

$$\omega_o^*(p) = \begin{cases} \frac{v-p-b+\beta f_m \alpha(l_r-c_r)}{\beta f_e} - \alpha c_r, \\ \qquad\qquad \text{if } \beta(f(c_o+\alpha c_r)-p+c) < v-p-3b, \\ \frac{v-p+b+\beta(f(c_o-\alpha c_r)-p+c)+2\beta f_m \alpha(l_r-c_r)}{2\beta f_e} - \alpha c_r, \\ \qquad\qquad \text{if } v-p-3b \leq \beta(f(c_o+\alpha c_r)-p+c) \leq v-p+b, \\ \left[\frac{v-p+b+\beta f_m \alpha(l_r-c_r)}{\beta f_e} - \alpha c_r\ ,\ \infty\right), \\ \qquad\qquad \text{if } \beta(f(c_o+\alpha c_r)-p+c) > v-p+b. \end{cases}$$

$$s_o^*(p) = \begin{cases} \frac{v-p}{\beta f_e} - \alpha l_r, \\ \qquad\qquad \text{if } \beta(f(c_o+\alpha c_r)-p+c) < v-p-3b, \\ \frac{3(v-p+b)+\beta(f(c_o+\alpha c_r)-p+c)}{4\beta f_e} - \alpha l_r, \\ \qquad\qquad \text{if } v-p-3b \leq \beta(f(c_o+\alpha c_r)-p+c) \leq v-p+b, \\ \left[\frac{v-p+b}{\beta f_e} - \alpha l_r\ ,\ \infty\right), \\ \qquad\qquad \text{if } \beta(f(c_o+\alpha c_r)-p+c) > v-p+b. \end{cases}$$

The interpretation of these results have already been made in the previous chapter and are not repeated here. In Table 4.2, we report the corresponding profit functions and the consumer surplus.

Next, we derive the manufacturer's best response for the low TCO_i area.

Table 4.2: Profit and consumer surplus levels for a given product retail price under the high TCO_i area of downstream competition model where $p > b\delta + \upsilon - \beta f_e(s_n + \alpha l_i)$, and $F = f(c_o + \alpha c_r) + c$

	$\upsilon - p - \beta(F-p) \geq 3b$ Full Coverage	$-b \leq \upsilon - p - \beta(F-p) < 3b$ Partial Coverage	$\upsilon - p - \beta(F-p) < -b$ No Coverage
$\Pi_r^{CM}(p,\omega_o^*(p),s_o^*(p))$	$\frac{b}{\beta}$	$\frac{(\upsilon-p+b-\beta(F-p))^2}{16\beta b}$	0
$\Pi_m^{CM}(p,\omega_o^*(p),s_o^*(p))$	$\frac{\upsilon-p-b-\beta(F-p)}{\beta}$	$\frac{(\upsilon-p+b-\beta(F-p))^2}{8\beta b}$	0
$CS^{CM}(p,\omega_o^*(p),s_o^*(p))$	$\frac{b}{2} - \frac{(\upsilon-p)(1-\beta)}{\beta}$	$\frac{(2-\beta)(\upsilon-p+b-\beta(F-p))^2}{32\beta b} -$ $\frac{(1-\beta)(\upsilon-p+b)(\upsilon-p+b-\beta(F-p))}{4\beta b}$	0

Lemma 5 *The following statements are true regarding the manufacturer's profit function given the retailer's best response,* $\Pi_m^{CL}(p, \omega_o, s_o^*(p, \omega_o))$*, within the low* TCO_i *area of* $p < \upsilon - \beta f_e(s_n + \alpha l_i)$*,*

$$\Pi_m^{CL}(p, \omega_o, s_o^*(p, \omega_o)) =$$

$$\begin{cases} \Pi_{m(-3)}^{CL}(p, \omega_o) = p - c - f_m(c_o + \alpha l_r) + f_e(\omega_o - c_o), \\ \qquad\qquad \text{if } \omega_o < s_n + \alpha l_i - \frac{b(1-\delta)}{\beta f_e} - \alpha c_r, \\ \Pi_{m(6)}^{CL}(p, \omega_o) = \frac{f_e(\omega_o - c_o)}{2} + p - c - f_m(c_o + \alpha l_r) + \frac{\beta f_e^2(\omega_o - c_o)(s_n + \alpha l_i - \omega_o - \alpha c_r)}{2b(1-\delta)}, \\ \qquad\qquad \text{if } s_n + \alpha l_i - \frac{b(1-\delta)}{\beta f_e} - \alpha c_r \leq \omega_o \leq s_n + \alpha l_i + \frac{b(1-\delta)}{\beta f_e} - \alpha c_r, \\ \Pi_{m(5)}^{CL}(p, \omega_o) = p - c - f_m(c_o + \alpha l_r), \\ \qquad\qquad \text{if } \omega_o > s_n + \alpha l_i + \frac{b(1-\delta)}{\beta f_e} - \alpha c_r : \end{cases}$$

1. $\Pi_m^{CL}(p, \omega_o, s_o^*(p, \omega_o))$ *is continuous in* ω_o*.*

2. $\Pi_{m(-3)}^{CL}(p, \omega_o, s_o^*(p, \omega_o))$ *is increasing in* ω_o*.*

3. $\Pi_{m(6)}^{CL}(p, \omega_o, s_o^*(p, \omega_o))$ *is concave in* ω_o*.*

Proposition 9 *The equilibrium retail price of the retailer* (s_o^*) *and wholesale price of the manufacturer* (ω_o^*) *for original spare parts, within the low* TCO_i *area of* $p <$

$v - \beta f_e(s_n + \alpha l_i)$, under the downstream competition model are given as follows:

$$s_o^*(p) = \begin{cases} s_n + \alpha l_i - \alpha l_r, & \\ & \textit{if}\, c_o + \alpha c_r < s_n + \alpha l_i - \frac{3b(1-\delta)}{2\beta f_e}, \\ \frac{3b(1-\delta)+\beta f_e(c_o+\alpha c_r)+3\beta f_e(s_n+\alpha li)}{4\beta f_e} - \alpha l_r, & \\ & \textit{if}\, s_n + \alpha l_i - \frac{3b(1-\delta)}{2\beta f_e} \leq c_o + \alpha c_r \leq s_n + \alpha l_i, \\ \Big[s_n + \alpha l_i + \frac{b(1-\delta)}{\beta f_e} - \alpha l_r \ , \infty), & \\ & \textit{if}\, c_o + \alpha c_r > s_n + \alpha l_i. \end{cases}$$

$$\omega_o^*(p) = \begin{cases} s_n + \alpha l_i - \frac{b(1-\delta)}{\beta f_e} - \alpha c_r, & \\ & \textit{if}\, c_o + \alpha c_r < s_n + \alpha l_i - \frac{3b(1-\delta)}{2\beta f_e}, \\ \frac{(b(1-\delta)+\beta f_e(c_o+\alpha c_r+s_n+\alpha l_i)}{2\beta f_e} - \alpha c_r, & \\ & \textit{if}\, s_n + \alpha l_i - \frac{3b(1-\delta)}{2\beta f_e} \leq c_o + \alpha c_r \leq s_n + \alpha l_i, \\ \Big[s_n + \alpha l_i + \frac{b(1-\delta)}{\beta f_e} - \alpha c_r \ , \infty\Big), & \\ & \textit{if}\, c_o + \alpha c_r > s_n + \alpha l_i. \end{cases}$$

4.1.3 Conclusion

For the downstream competition model, we were able to fully derive the retailer's best response for the original spare part retail price, s_o, and partially derive the manufacturer's best response for the original spare part wholesale price, ω_o. Therefore, we stop here, skipping the derivation of the product price, p, hence the final step of the equilibrium. We analyze how the equilibrium behaves in a numerical study, which is presented in the next chapter.

4.2 Centralized Version of the Downstream Competition Model

In this section, we study the centralized version of the base model in order to evaluate the efficiency loss due to decentralization in the chain.

In the centralized model, the total profit of the manufacturer and the retailer is maximized by a single decision maker, where the wholesale spare parts price ω_o becomes irrelevant and the decision variables are p and s_o. Here we again study the two decisions in sequence: First p is determined, followed by s_o.

Table 4.3: Manufacturer's best response and corresponding results for the downstream competition model in the low TCO_i area $((p < \upsilon - \beta f_e(s_n + \alpha l_i))$

	$c_o + \alpha c_r \ > \ s_n + \alpha l_i + \frac{b(1-\delta)}{\beta f_e}$	$s_n + \alpha l_i - \frac{3b(1-\delta)}{\beta f_e} \ \leq \ c_o + \alpha c_r \ \leq \ s_n + \alpha l_i + \frac{b(1-\delta)}{\beta f_e}$	$c_o + \alpha c_r \ < \ s_n + \alpha l_i - \frac{3b(1-\delta)}{\beta f_e}$
	Full Coverage	Partial Coverage	No Coverage
$\omega_o^*(p)$	$s_n + \alpha l_i + \frac{b(1-\delta)}{\beta f_e} - \alpha c_r$	$\frac{(b(1-\delta)+\beta f_e(c_o+\alpha c_r+sn+\alpha l_i)}{2\beta f_e} - \alpha c_r$	$s_n + \alpha l_i - \frac{b(1-\delta)}{\beta f_e} - \alpha c_r$
$s_o^*(p)$	$s_n + \alpha l_i + \frac{b(1-\delta)}{\beta f_e} - \alpha l_r$	$\frac{3b(1-\delta)+\beta f_e(c_o+\alpha c_r)+3\beta f_e(s_n+\alpha l_i)}{4\beta f_e} - \alpha l_r$	$s_n + \alpha l_i - \alpha l_r$
$q_i^{CL}(p, s_o^*(p))$	1	$\frac{3b(1-\delta)-\beta f_e(s_n+\alpha l_i-c_o-\alpha c_r)}{4b(1-\delta)}$	0
$q_r^{CL}(p, s_o^*(p))$	0	$\frac{b(1-\delta)+\beta f_e(s_n+\alpha l_i-c_o-\alpha c_r)}{4b(1-\delta)}$	1
$q^{CL}(p, s_o^*(p))$	1	1	1
$\Pi_i^{CL}(p, \omega_o^*(p), s_o^*(p))$	$f_e(s_n - \omega_n + \alpha(l_i - c_i))$	$\frac{(3b(1-\delta)-\beta f_e(s_n+\alpha l_i-c_o-\alpha c_r))f_e(s_n-\omega_n+\alpha(l_i-c_i))}{4b(1-\delta)}$	0
$\Pi_r^{CL}(p, \omega_o^*(p), s_o^*(p))$	$f_m\alpha(l_r - c_r)$	$f_m\alpha(l_r - c_r) - \frac{f_e(c_o+\alpha c_r-s_n-\alpha l_i)}{4} + \frac{(\beta f_e(c_o+\alpha c_r-s_n-\alpha l_i)+b(1-\delta))^2}{16\beta b(1-\delta)}$	$\frac{b(1-\delta)+\beta f_m\alpha(l_r-c_r)}{\beta}$
$\Pi_m^{CL}(p, \omega_o^*(p), s_o^*(p))$	$p - c - f_m(c_o + \alpha l_r)$	$p - c - f_m(c_o + \alpha c_r) - \frac{f_e(c_o+\alpha c_r-s_n-\alpha l_i)}{2} + \frac{(\beta f_e(c_o+\alpha c_r-s_n-\alpha l_i)+b(1-\delta))^2}{8\beta b(1-\delta)}$	$(p - c - f_m(c_o + \alpha c_r) - f_e(c_o + \alpha c_r - s_n - \alpha l_i) - \frac{b(1-\delta)}{\beta}$
$CS(p, \omega_o^*(p), s_o^*(p))$	$\frac{b^2\delta+2b(\upsilon-p-\beta f_e(s_n+\alpha l_i))}{2}$	$\frac{b^2\delta+2b(\upsilon-p)}{2} - \frac{7b\beta f_e(s_n+\alpha l_i)+b\beta f_e(c_o+\alpha c_r)}{8} + \frac{(\beta f_e(c_o+\alpha c_r-s_n-\alpha l_i)+b(1-\delta))^2}{32(1-\delta)}$	$\frac{b^2+2b(\upsilon-p-\beta f_e(s_n+\alpha l_i))}{2}$

The system's profit function is given in Equation 4.7. We use superscript CC to denote that this is the **c**entralized version of the downstream **c**ompetition model. The manufacturer provides warranty to all items sold, but earns after-sales profits only from the $q_r^C(p, s_o)$ portion of the demand.

$$\Pi^{CC}(p, \omega_o, s_o) = (p - c - f_m(\alpha c_r + c_o))\, q^C(p, s_o) + f_e(s_o - c_o) q_r^C(p, s_o) \quad (4.7)$$

The manufacturer first sets the product price, p, followed by the spare parts retail price, s_o.

4.2.1 Best Centralized Spare Part Retail Price, s_o, for a Given Product Price, p

Similar to the structure we followed for the first part with no centralization, we conduct our analysis separately for each of the three TCO_i areas: low (L), medium (M), and high (H). The valid area is added to next to the model superscript, e.g., CCL for the low TCO_i area of the centralized downstream competition model.

We start with the analysis of the low TCO_i area, i.e., $p < \beta f_e(s_n + \alpha l_i) + v$:

Lemma 6 *The following statements are true regarding the profit function of the system, $\Pi^{CCL}(p, s_o)$, within the low TCO_i area of $p < v - \beta f_e(s_n + \alpha l_i)$,*

$$\Pi^{CCL}(p, s_o) = \begin{cases} \Pi^{CCL}_{(-3)}(p, s_o) = (p - c - f_m(c_o + \alpha c_r)) + f_e(s_o - c_o + \alpha(l_r - c_r)), \\ \qquad \text{if } s_o < s_n + \alpha l_i - \alpha l_r, \\ \Pi^{CCL}_{(6)}(p, s_o) = p - c - f_m(c_o + \alpha c_r) + f_e((s_o - c_o) + \alpha(l_r - c_r)) \\ \qquad - \frac{\beta f_e^2}{b(1-\delta)} (s_o - c_o + \alpha(l_r - c_r)) (s_o - s_n + \alpha(l_r - l_i)), \\ \qquad \text{if } s_n + \alpha l_i - \alpha l_r \le s_o \le \frac{b(1-\delta)}{\beta f_e} + s_n + \alpha l_i - \alpha l_r, \\ \Pi^{CCL}_{(5)}(p, s_o) = p - c - f_m(c_o + \alpha c_r), \\ \qquad \text{if } s_o > \frac{b(1-\delta)}{\beta f_e} + s_n + \alpha l_i - \alpha l_r: \end{cases}$$

1. *$\Pi^{CCL}(p, s_o)$ is continuous in s_o.*
2. *$\Pi^{CCL}_{(-3)}(p, s_o)$ is increasing in s_o.*

3. $\Pi^{CCL}_{(6)}(p, s_o)$ is concave in s_o.

Proposition 10 *The optimal price decision for the system, $s_o^*(p)$ for a given value of p under the centralized model, within the low TCO_i area of $p < \upsilon - \beta f_e(s_n + \alpha l_i)$ is as follows:*

$$
s_o^*(p) = \begin{cases}
s_n + \alpha l_i - \alpha l_r, \\
\qquad\qquad\qquad\qquad \textit{if } c_o < s_n + \alpha l_i - \frac{b(1-\delta)}{\beta f_e} - \alpha c_r, \\
\frac{b(1-\delta)}{2\beta f_e} + \frac{c_o + \alpha c_r + s_n + \alpha l_r}{2} - \alpha l_r, \\
\qquad \textit{if } s_n + \alpha l_i - \frac{b(1-\delta)}{\beta f_e} - \alpha c_r \leq c_o \leq s_n + \alpha l_i + \frac{b(1-\delta)}{\beta f_e} - \alpha c_r, \\
\left[s_n + \alpha l_i + \frac{b(1-\delta)}{\beta f_e} - \alpha l_r \;, \infty \right), \\
\qquad\qquad\qquad\qquad \textit{if } c_o > s_n + \alpha l_i + \frac{b(1-\delta)}{\beta f_e} - \alpha c_r.
\end{cases}
$$

Next, we continue with the analysis of the medium TCO_i area, i.e., $\upsilon - \beta f_e(s_n + \alpha l_i) < p < b\delta + \upsilon - \beta f_e(s_n + \alpha l_i)$:

Lemma 7 *The following statements are true regarding the profit function of the system, $(\Pi^{CCM}(p, s_o))$, within the medium TCO_i area of $\upsilon - \beta f_e(s_n + \alpha l_i) < p < b\delta + \upsilon - \beta f_e(s_n + \alpha l_i)$,*

$$
\Pi^{CCM}(p, s_o) =
$$

$$
\begin{cases}
\Pi^{CCM}_{(-2)}(p, s_o) = p - c - f_m(c_o + \alpha c_r) + f_e(s_o - c_o + \alpha(l_r - c_r)), \\
\qquad\qquad\qquad\qquad\qquad\qquad \textit{if } s_o < \frac{\upsilon - p}{\beta f_e} - \alpha l_r, \\
\Pi^{CCM}_{(0)}(p, s_o) = (p - c - f_m(c_o + \alpha c_r) + f_e(s_o - c_o + \alpha(l_r - c_r))) \\
\qquad\qquad\qquad\qquad \times \left(1 - \frac{p - \upsilon + \beta f_e(s_o + \alpha l_r)}{b}\right), \\
\qquad\qquad \textit{if } s_o < \frac{s_n + \alpha l_i}{\delta} - \frac{(1-\delta)(\upsilon - p)}{\delta \beta f_e} - \alpha l_r \textit{ and } s_o \geq \frac{\upsilon - p}{\beta f_e} - \alpha l_r, \\
\Pi^{CCM}_{(7)}(p, s_o) = (p - c - f_m(c_o + \alpha c_r)) \left(1 - \frac{p - \upsilon + \beta f_e(s_n + \alpha l_i)}{b\delta}\right) \\
\qquad\qquad + f_e(s_o - c_o + \alpha(l_r - c_r)) \left(1 - \frac{\beta f_e(s_o - s_n + \alpha(l_r - l_i))}{b(1-\delta)}\right), \\
\qquad \textit{if } s_o > \frac{s_n + \alpha l_i}{\delta} - \frac{(1-\delta)(\upsilon - p)}{\delta \beta f_e} - \alpha l_r \textit{ and } s_o \leq \frac{b(1-\delta)}{\beta f_e} + s_n + \alpha l_i - \alpha l_r, \\
\Pi^{CCM}_{(4)}(p, s_o) = (p - c - f_m(c_o + \alpha c_r)) \left(1 - \frac{p - \upsilon + \beta f_e(s_n + \alpha l_i)}{b\delta}\right), \\
\qquad\qquad\qquad\qquad\qquad \textit{if } s_o > \frac{b(1-\delta)}{\beta f_e} + s_n + \alpha l_i - \alpha l_r\textit{:}
\end{cases}
$$

1. $\Pi^{CCM}(p, s_o)$ *is continuous in* s_o.

2. $\Pi^{CCM}_{(-2)}(p, s_o)$ *is increasing in* s_o.

3. $\Pi^{CCM}_{(0)}(p, s_o)$ *is concave in* s_o.

4. $\Pi^{CCM}_{(7)}(p, s_o)$ *is concave in* s_o.

For the high TCO area, the solution is the same as the one presented for the monopoly model centralized model, which will be added for the full characterization of the solution.

4.2.2 Best Centralized Product Price, p, and Overall Equilibrium

Again, we start with the analysis for the low TCO_i area.

Lemma 8 *The following statements are true regarding the system's profit function given the spare parts retail price,* $\Pi^{CCL}(p, s_o^*(p))$, *within the highly-competitive independent workshop region of* $p < v - \beta f_e(s_n + \alpha l_i)$,

$$\Pi^{CCL}(p, s_o^*(p)) =$$

$$\begin{cases} \Pi^{CCL}_{(-2)}(p) = p - c - f_m(c_o + \alpha c_r) + f_e(s_n - c_o + \alpha(l_i - c_r), \\ \qquad\qquad \textit{if } c_o < s_n + \alpha l_i - \frac{b(1-\delta)}{\beta f_e} - \alpha c_r, \\ \Pi^{CCL}_{(6)}(p) = p - c - f_m(c_o + \alpha c_r) + \frac{f_e(s_n - c_o + \alpha(l_i - c_r))}{2} \\ \qquad\qquad + \frac{\beta f_e^2(s_n - c_o + \alpha(l_i - c_r))^2}{4b(1-\delta)} + \frac{b(1-\delta)}{4\beta}, \\ \qquad\qquad \textit{if } s_n + \alpha l_i - \frac{b(1-\delta)}{\beta f_e} - \alpha c_r \leq c_o \leq s_n + \alpha l_i + \frac{b(1-\delta)}{\beta f_e} - \alpha c_r, \\ \Pi^{CCL}_{(5)}(p) = p - c - f_m(c_o + \alpha l_r), \\ \qquad\qquad \textit{if } c_o > s_n + \alpha l_i + \frac{b(1-\delta)}{\beta f_e} - \alpha c_r : \end{cases}$$

1. $\Pi^{CCL}(p, s_o^*(p))$ *is continuous in* p.

2. $\Pi^{CCL}_{(-3)}(p, s_o^*(p))$ *is increasing in* p.

3. $\Pi^{CCL}_{(6)}(p, s_o^*(p))$ *is increasing in* p.

We did not characterize the full equilibrium, but combining the insights from the centralized version of the monopoly model with Lemma 8 strongly suggests that the optimal solution should occur within region ⓪, region ⑦, or one of their boundaries.

CHAPTER 5

NUMERICAL ANALYSIS OF THE DOWNSTREAM COMPETITION MODEL

In this chapter we conduct a numerical study of the downstream competition model and present our insights. As presented and analyzed in the previous chapter, the downstream competition model considers a manufacturer who is the monopolistic supplier of a sophisticated durable product. Her retailer provides warranty-covered after-sales services for all products sold, but faces downstream competition from an independent workshop for customer-paid after-sales services. The manufacturer sets the product retail price and the wholesale price of spare parts to the retailer, and the retailer sets the retail sales price of spare parts to the end customer.

In order to study how the downstream competition model equilibria behave and how they compare to the monopoly model, we conduct numerical experiments using the range of parameter values given in Tables 5.1 and 5.2. Overall, we test 187,500 instances, doing a full factorial experimental design. In the next sections, we first give our general observations regarding the competition model equilibria, followed by analysis of selected research questions in separate subsections.

Table 5.1: Parameter settings (those unchanged across instances)

α	c_o	l_r	c_r	ω_n	l_i	c_i	c
0.5	0.375	0.5	0.25	0	0.25	0	0.5

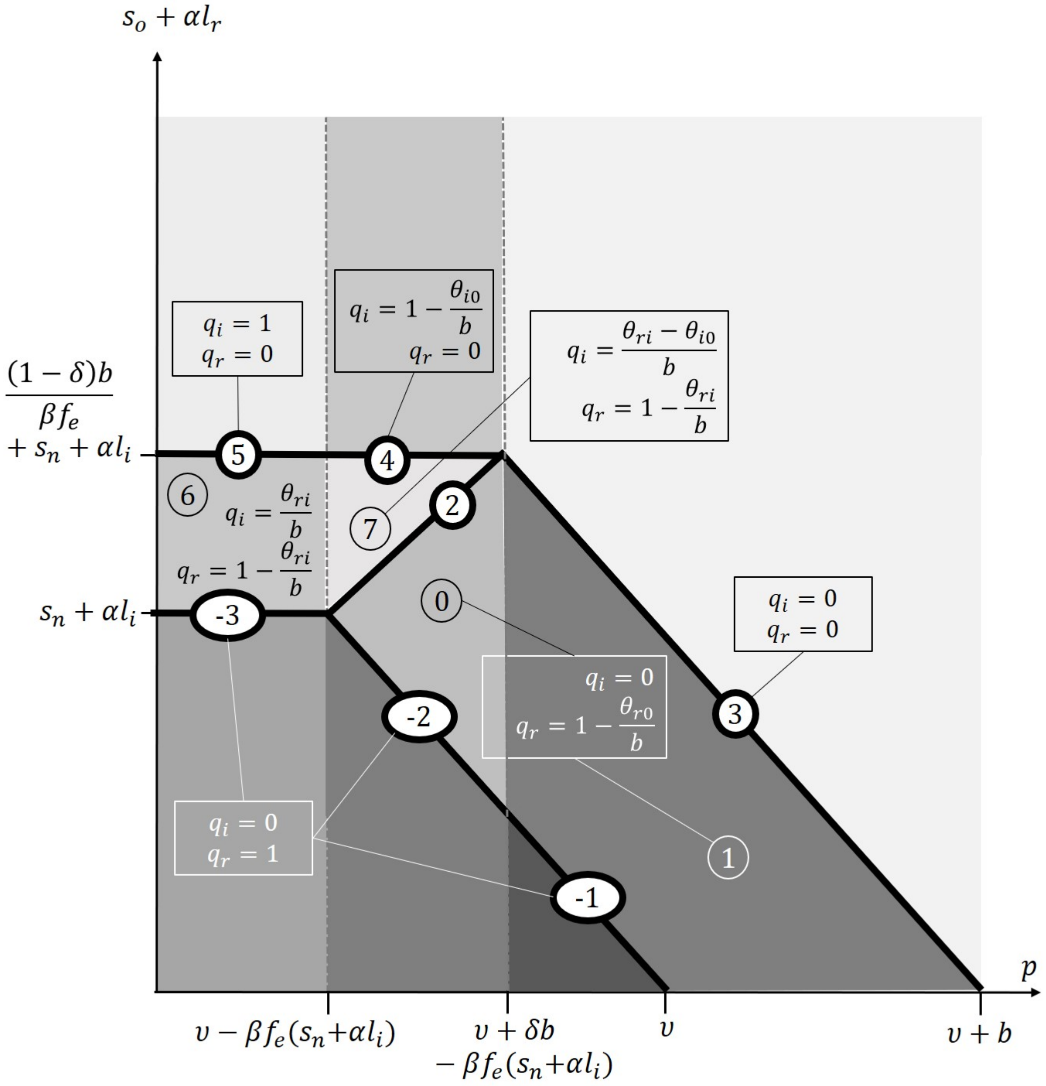


Figure 5.1: The downstream competition model's demand output and the retailer's best response where $\theta_{ri} = \frac{\beta f_e((s_o+\alpha l_r)-(s_n+\alpha l_i))}{1-\delta}$, $\theta_{r0} = \beta f_e(s_o + \alpha l_r) - (\upsilon - p)$, and $\theta_{i0} = \frac{\beta f_e(s_n+\alpha l_i)-(\upsilon-p)}{\delta}$

Table 5.2: Parameter settings (those changed across instances)

Factor	Levels
β	{0.05, 0.25, 0.50, 0.75, 0.95}
δ	{0.05, 0.25, 0.50, 0.75, 0.95}
s_n	{0.5, 1, 2, 3, 4, 5}
v	{1, 2, 3, 4, 5}
b	{1, 2, 3, 4, 5}
f_m/f	{0.05, 0.25, 0.50, 0.75, 0.95}
f	{1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

5.1 Competition Model Equilibria Observations

The graphical representation of the model's demand output in terms of the total after-sales price of the retailer per each repair ($s_o + \alpha l_r$) in relation to the product retail price (p) are given in Figure 5.1. Notice that the representation of regions in this figure is slightly different from that in Figure 4.2 within Chapter 4. From the analytical solution, we know that the retailer's best response, the spare part retail price (or, the way it's shown on the Figure, the total repair price per each after-sales service job, $s_o + \alpha l_r$), for a given ω_o, p, and a set of parameters is either realized on the boundaries indicated in bold lines, or at a concavity peak point within the regions they surround. The lower regions with full market coverage and retailer-monopolized after-sales ((-3), (-2), (-1)) take s_o values at the upper bound as compared to the region representations in the previous chapter. Here, the retailer picks the highest "low price" that he can apply for spare parts that still ensures full market coverage for himself. Reversely, the upper regions with no after-sales retail coverage by the retailer (⑤, ④, ③) take s_o values at the lower bound as compared to the region representations in the previous chapter. Here, the retailer is indifferent between this lower bound and the regions above, as all of them result in zero demand for original spare parts anyways. For sake of convenience, we report the lower boundary $s_o + \alpha l_r$ figures as the best response. Note that region ③ has no market coverage at all, whereas the market is partially and fully covered by the independent workshop in regions ④ and ⑤, respectively. In these two regions, the retailer's only income is from the warranty-period after-sales services that he performs and gets paid by the manufacturer. Marking of regions with partial aftermarket coverage by the retailer (⑥, ⑦, ⓪, ①) are unchanged. The retailer's objective function has convex properties in these regions and if one of those regions are seen in the best response, they indicate a peak point interior to that region, corresponding to partial market coverage of the retailer. The retailer shares the market with the independent retailer in regions ⑥ and ⑦, whereas monopolizing the market in regions ⓪ and ①. Finally, we have new boundary marked as region ② between regions ⓪ and ⑦, which is another potential best response of the retailer. This set of solutions is where the retailer dominates the market by himself with maximum possible partial coverage he can achieve (as boundary of region ⓪), as opposed to sharing the market with the independent workshop (although still with partial coverage overall, in the area marked with ⑦). We can interpret that in order to maximize his profits, the retailer maximizes his market coverage in region ⓪, and he maximizes his spare part retail price in region ②.

An important observation regarding the perceived total cost of ownership with after-sales services provided by the independent retailer, $TCO_i = p + \beta f_e(s_n + \alpha l_i)$, is that the $\beta f_e(s_n + \alpha l_i)$ component is exogenous and always positive. We also impose a nonnegative product price, $p \geq 0$. Therefore, any increase in consumer awareness of future costs (β), non-original spare part price (s_n), labor cost per repair in the independent workshop (l_i), total number of customer-paid repairs (f_e); or any decrease in the base valuation (v), similarity of the services by the independent workshop to those in the retailer (δ), customer heterogeneity (b) would push the model towards an equilibrium more to the regions on the right. We revisit this point in the aftermarket monopolization discussion.

Another remark we make is that regions with no retailer coverage but partial or full independent workshop coverage (④ and ⑤) are not represented in the equilibria across the instances we test.[1] Although we cannot generalize this finding, we can explain why it makes sense that this situation is not observed or is presumably rare. Given a reasonably low wholesale spare part price, ω_o, where the retailer can still set the spare part retail price s_o below the threshold $\frac{(1-\delta)b}{\beta f_e} + s_n + \alpha l_i$ and make a profit, he benefits from selling any number of after-sales services to customers. The low product

---

[1] In Table B.4, we report how the equilibria are distributed across regions for the tested instances.

price p and the independent retailer having the potential to take these customers show that TCO is sufficiently low to entice the customers to buy. Therefore, it would be enticing for the manufacturer to set her wholesale price low enough in order to capture some or all of this after-sales demand potential from the independent workshop, both for herself and the retailer. This explanation inherently assumes that the manufacturer still makes a profit from this low enough wholesale price, i.e., $\omega_o > c_o$. We have also tested some instances where we pushed c_o to higher values (not reported). In those instances, we observed that the manufacturer tends to increase p, choosing to operate in the high-TCO_i area. When c_o is too high, the equilibrium then just shifts to no market coverage (region ③). In conclusion, the manufacturer and the retailer do not allow the independent workshop to monopolize the after-sales market under the studied setup; but further analysis is required to generalize this point.

Next, we explore how the manufacturer's profit function, the retailer's profit function, and the consumer surplus behave in the downstream competition model equilibrium. Although we tested a wide range of numerical instances as reported in Table 5.2, we focus on displaying a consistent subset of them in the figures used within this main text to complement our discussion. In Figure 5.2, we report how the manufacturer's profit changes as the total number of repairs, f is changed. Figures B.1-B.2 showing how the manufacturer's profit changes as the spare parts price of the independent retailer, s_n, and customer's base valuation, υ, is changed are given in Appendix B. In all figures, we use $f = 5, s_n = 3, \upsilon = 3$ as the base case (subfigure (b)) and show the trend as each of these parameters are increased or decreased. Within each subfigure, we show the result for all tested instances of warranty coverage, f_m/f, customer heterogeneity, b, customer foresight, β, and independent workshop service similarity, δ, for the given $f, s_n,$ and υ values. Note that all following figures are presented in the same structure. For Figures 5.2 and B.1-B.2, the cells report the manufacturer's profit function for the corresponding parameter combination, where the background is highlighted in a black and white heatmap, darker colors representing higher values. The scale is consistent across the three figures for cross-comparison purposes.

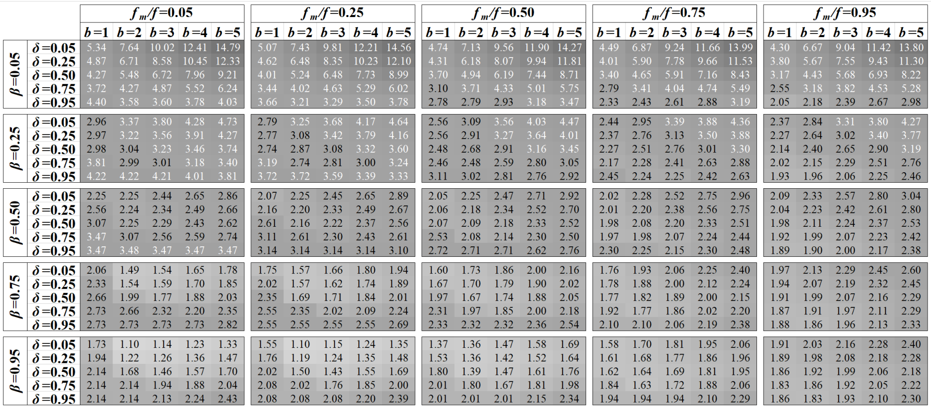
Similarly, Figures 5.3 and B.3-B.4 show the retailer's profit function; and Figures 5.4 and B.5-B.6 show the consumer surplus in the same manner. Because consumer surplus can become very negative, the low-end of the heatmap scale is adjusted to -15 and all values below this figure are highlighted in white (so that the changes in other values are easier to observe). Note that we make more generalized observations in this section and save more detailed discussion of some topics to later sections.

As it can be expected, manufacturer profit and consumer surplus behave exactly in the opposite manner of each other. As one increases, the other one decreases, and vice versa. Especially, low customer foresight (β) significantly hurts the customer and benefits the manufacturer.

Customer heterogeneity (b) has two different impact patterns on consumer surplus, depending on customers' awareness of future costs (β). When β is low, consumers significantly underestimate the future costs and tend to spend much more than they actually would. In this case, increased heterogeneity "tempts" more customers to buy, which is actually worse for the overall consumer surplus. Reversely, when β is high, consumers have a good sense of the product's TCO, making an informed buying decision. In this case, increased heterogeneity helps the product to reach to a wider customer base, increasing the total consumer surplus. For the manufacturer and the retailer, increased customer heterogeneity improves the profits regardless of the customers' awareness of future costs.

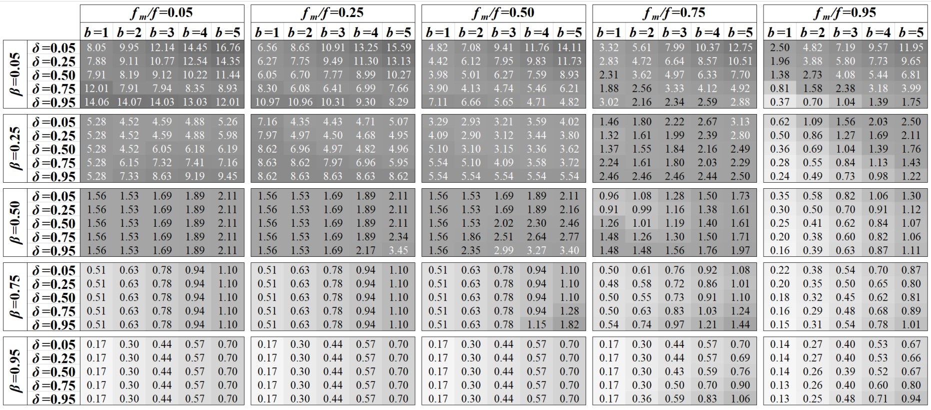
The total consumer surplus decreases as s_n increases. This is to be expected, since an alternative for the customer is getting more expensive. On the more detailed level, we see that the impact of s_n is intertwined with the impact of consumer foresight. For higher consumer foresight, changing s_n has no to little impact on consumer surplus. However, the lower the consumer awareness of future costs (β) is, the more effect s_n's change has on the consumer surplus. But consumer surplus is most hurt when low consumer awareness of future costs (β) is combined with low warranty coverage (f_m/f). However, we will later see in Section 5.2.2 that even this scenario with the downstream competition model is more preferable to the monopoly model, hurting the customer less.

The retailer achieves highest profits with combinations of high customer heterogeneity (b), low perception of independent workshop service quality (δ), and low consumer awareness of future costs (β).



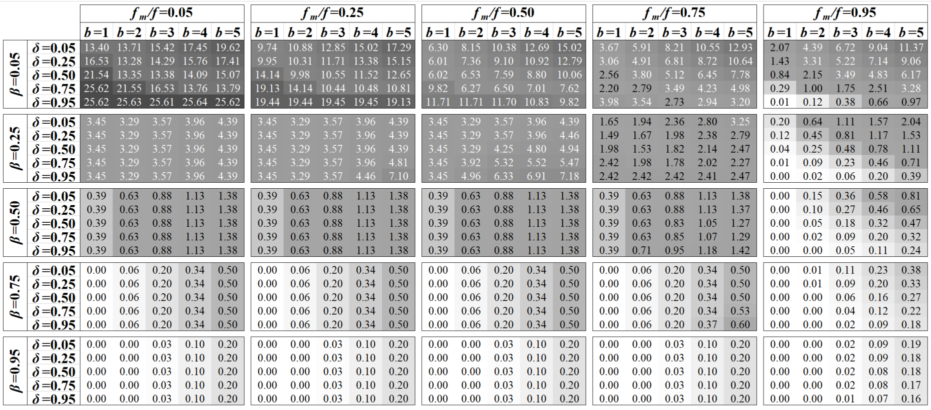
		f_m/f=0.05					f_m/f=0.25					f_m/f=0.50					f_m/f=0.75					f_m/f=0.95				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	5.34	7.64	10.02	12.41	14.79	5.07	7.43	9.81	12.21	14.56	4.74	7.13	9.56	11.90	14.27	4.49	6.87	9.24	11.66	13.99	4.30	6.67	9.04	11.42	13.80
	δ=0.25	4.87	6.71	8.58	10.45	12.33	4.62	6.48	8.35	10.23	12.10	4.31	6.18	8.07	9.94	11.81	4.01	5.90	7.78	9.66	11.53	3.80	5.67	7.55	9.43	11.30
	δ=0.50	4.27	5.48	6.72	7.96	9.21	4.01	5.24	6.48	7.73	8.99	3.70	4.94	6.19	7.44	8.71	3.40	4.65	5.91	7.16	8.43	3.17	4.43	5.68	6.93	8.22
	δ=0.75	3.72	4.27	4.87	5.52	6.24	3.44	4.02	4.63	5.29	6.02	3.10	3.71	4.33	5.01	5.75	2.79	3.41	4.04	4.74	5.49	2.55	3.18	3.82	4.53	5.28
	δ=0.95	4.40	3.58	3.60	3.78	4.03	3.66	3.21	3.29	3.50	3.78	2.78	2.79	2.93	3.18	3.47	2.33	2.43	2.61	2.88	3.19	2.05	2.18	2.39	2.67	2.98
β=0.25	δ=0.05	2.96	3.37	3.80	4.28	4.73	2.79	3.25	3.68	4.17	4.64	2.56	3.09	3.56	4.03	4.47	2.44	2.95	3.39	3.88	4.36	2.37	2.84	3.31	3.80	4.27
	δ=0.25	2.97	3.22	3.56	3.91	4.27	2.77	3.08	3.42	3.79	4.16	2.56	2.91	3.27	3.64	4.01	2.37	2.76	3.13	3.50	3.88	2.27	2.64	3.02	3.40	3.77
	δ=0.50	2.98	3.04	3.23	3.46	3.74	2.74	2.87	3.08	3.32	3.60	2.48	2.68	2.91	3.16	3.45	2.27	2.51	2.76	3.01	3.30	2.14	2.40	2.65	2.90	3.19
	δ=0.75	3.81	2.99	3.01	3.18	3.40	3.19	2.74	2.81	3.00	3.24	2.46	2.48	2.59	2.80	3.05	2.17	2.28	2.41	2.63	2.88	2.02	2.15	2.29	2.51	2.76
	δ=0.95	4.22	4.22	4.21	4.01	3.81	3.72	3.72	3.59	3.39	3.33	3.11	3.02	2.81	2.76	2.92	2.45	2.24	2.25	2.42	2.63	1.93	1.96	2.06	2.25	2.46
β=0.50	δ=0.05	2.25	2.25	2.44	2.65	2.86	2.07	2.25	2.45	2.65	2.89	2.05	2.25	2.47	2.71	2.92	2.02	2.28	2.52	2.75	2.96	2.09	2.33	2.57	2.80	3.04
	δ=0.25	2.56	2.24	2.34	2.49	2.66	2.16	2.20	2.33	2.49	2.67	2.06	2.18	2.34	2.52	2.70	2.01	2.20	2.38	2.56	2.75	2.04	2.23	2.42	2.61	2.80
	δ=0.50	3.07	2.25	2.29	2.43	2.62	2.61	2.16	2.22	2.37	2.56	2.07	2.09	2.18	2.33	2.52	1.98	2.08	2.20	2.33	2.51	1.98	2.11	2.24	2.37	2.53
	δ=0.75	3.47	3.07	2.56	2.59	2.74	3.11	2.61	2.30	2.43	2.61	2.53	2.08	2.14	2.30	2.50	1.97	1.98	2.07	2.24	2.44	1.92	1.99	2.07	2.23	2.42
	δ=0.95	3.47	3.48	3.47	3.47	3.47	3.14	3.14	3.14	3.14	3.10	2.72	2.71	2.71	2.62	2.76	2.30	2.25	2.15	2.30	2.48	1.89	1.90	2.00	2.17	2.38
β=0.75	δ=0.05	2.06	1.49	1.54	1.65	1.78	1.75	1.57	1.66	1.80	1.94	1.60	1.73	1.86	2.00	2.16	1.76	1.93	2.06	2.25	2.40	1.97	2.13	2.29	2.45	2.60
	δ=0.25	2.33	1.54	1.59	1.70	1.85	2.02	1.57	1.62	1.74	1.89	1.67	1.70	1.79	1.90	2.02	1.78	1.88	2.00	2.12	2.24	1.94	2.07	2.19	2.32	2.45
	δ=0.50	2.66	1.99	1.77	1.88	2.03	2.35	1.69	1.71	1.84	2.01	1.97	1.67	1.74	1.88	2.05	1.77	1.82	1.89	2.00	2.15	1.91	1.99	2.07	2.16	2.29
	δ=0.75	2.73	2.66	2.32	2.20	2.35	2.55	2.35	2.02	2.09	2.24	2.31	1.97	1.85	2.00	2.18	1.92	1.77	1.86	2.02	2.20	1.87	1.91	1.97	2.11	2.29
	δ=0.95	2.73	2.73	2.73	2.73	2.82	2.55	2.55	2.55	2.55	2.69	2.33	2.32	2.32	2.36	2.54	2.10	2.10	2.06	2.19	2.38	1.88	1.86	1.96	2.13	2.33
β=0.95	δ=0.05	1.73	1.10	1.14	1.23	1.33	1.55	1.10	1.15	1.24	1.35	1.37	1.36	1.47	1.58	1.69	1.58	1.70	1.81	1.95	2.06	1.91	2.03	2.16	2.28	2.40
	δ=0.25	1.94	1.22	1.26	1.36	1.47	1.76	1.19	1.24	1.35	1.48	1.53	1.36	1.42	1.52	1.64	1.61	1.68	1.77	1.86	1.96	1.89	1.98	2.08	2.18	2.28
	δ=0.50	2.14	1.68	1.46	1.57	1.70	2.02	1.50	1.43	1.55	1.69	1.80	1.39	1.47	1.61	1.76	1.62	1.64	1.69	1.81	1.95	1.86	1.92	1.99	2.06	2.18
	δ=0.75	2.14	2.14	1.94	1.88	2.04	2.08	2.02	1.76	1.85	2.00	2.01	1.80	1.67	1.81	1.98	1.84	1.63	1.72	1.88	2.06	1.83	1.86	1.92	2.05	2.22
	δ=0.95	2.14	2.14	2.13	2.24	2.43	2.08	2.08	2.08	2.20	2.39	2.01	2.01	2.01	2.15	2.34	1.94	1.94	1.94	2.10	2.29	1.86	1.83	1.93	2.10	2.30

(a) $f = 1$



		f_m/f=0.05					f_m/f=0.25					f_m/f=0.50					f_m/f=0.75					f_m/f=0.95				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	8.05	9.95	12.14	14.45	16.76	6.56	8.65	10.91	13.25	15.59	4.82	7.08	9.41	11.76	14.11	3.32	5.61	7.99	10.37	12.75	2.50	4.82	7.19	9.57	11.95
	δ=0.25	7.88	9.11	10.77	12.54	14.35	6.27	7.75	9.49	11.30	13.13	4.42	6.12	7.95	9.83	11.73	2.83	4.72	6.64	8.57	10.51	1.96	3.88	5.80	7.73	9.65
	δ=0.50	7.91	8.19	9.12	10.22	11.44	6.05	6.70	7.77	8.99	10.27	3.98	5.01	6.27	7.59	8.93	2.31	3.62	4.97	6.33	7.70	1.38	2.73	4.08	5.44	6.81
	δ=0.75	12.01	7.91	7.94	8.35	8.93	8.30	6.08	6.41	6.99	7.66	3.90	4.13	4.74	5.46	6.21	1.88	2.56	3.33	4.12	4.92	0.81	1.58	2.38	3.18	3.99
	δ=0.95	14.06	14.07	14.03	13.03	12.01	10.97	10.96	10.31	9.30	8.29	7.11	6.66	5.65	4.71	4.82	3.02	2.16	2.34	2.59	2.88	0.37	0.70	1.04	1.39	1.75
β=0.25	δ=0.05	5.28	4.52	4.59	4.88	5.26	7.16	4.35	4.43	4.71	5.07	3.29	2.93	3.21	3.59	4.02	1.46	1.80	2.22	2.67	3.13	0.62	1.09	1.56	2.03	2.50
	δ=0.25	5.28	4.52	4.59	4.88	5.98	7.97	4.97	4.50	4.68	4.95	4.09	2.90	3.12	3.44	3.80	1.32	1.61	1.99	2.39	2.80	0.50	0.86	1.27	1.69	2.11
	δ=0.50	5.28	4.52	6.05	6.18	6.19	8.62	6.96	4.97	4.82	4.96	5.10	3.10	3.15	3.36	3.62	1.37	1.55	1.84	2.16	2.49	0.36	0.69	1.04	1.39	1.76
	δ=0.75	5.28	6.15	7.32	7.41	7.16	8.63	8.62	7.97	6.96	5.95	5.54	5.10	4.09	3.58	3.72	2.24	1.61	1.80	2.03	2.29	0.28	0.55	0.84	1.13	1.43
	δ=0.95	5.28	7.33	8.63	9.19	9.45	8.62	8.63	8.63	8.63	8.62	5.54	5.54	5.54	5.54	5.54	2.46	2.46	2.46	2.44	2.50	0.24	0.49	0.73	0.98	1.22
β=0.50	δ=0.05	1.56	1.53	1.69	1.89	2.11	1.56	1.53	1.69	1.89	2.11	1.56	1.53	1.69	1.89	2.11	0.96	1.08	1.28	1.50	1.73	0.35	0.58	0.82	1.06	1.30
	δ=0.25	1.56	1.53	1.69	1.89	2.11	1.56	1.53	1.69	1.89	2.11	1.56	1.53	1.69	1.89	2.16	0.91	0.99	1.16	1.38	1.61	0.30	0.50	0.70	0.91	1.12
	δ=0.50	1.56	1.53	1.69	1.89	2.11	1.56	1.53	1.69	1.89	2.11	1.56	1.53	2.02	2.30	2.46	1.26	1.01	1.19	1.40	1.61	0.25	0.41	0.62	0.84	1.07
	δ=0.75	1.56	1.53	1.69	1.89	2.11	1.56	1.53	1.69	1.89	2.34	1.56	1.86	2.51	2.64	2.77	1.48	1.26	1.30	1.50	1.71	0.20	0.38	0.60	0.82	1.06
	δ=0.95	1.56	1.53	1.69	1.89	2.11	1.56	1.53	1.69	2.17	3.45	1.56	2.35	2.99	3.27	3.40	1.48	1.48	1.56	1.76	1.97	0.16	0.39	0.63	0.87	1.11
β=0.75	δ=0.05	0.51	0.63	0.78	0.94	1.10	0.51	0.63	0.78	0.94	1.10	0.51	0.63	0.78	0.94	1.10	0.50	0.61	0.76	0.92	1.08	0.22	0.38	0.54	0.70	0.87
	δ=0.25	0.51	0.63	0.78	0.94	1.10	0.51	0.63	0.78	0.94	1.10	0.51	0.63	0.78	0.94	1.10	0.48	0.58	0.72	0.86	1.01	0.20	0.35	0.50	0.65	0.80
	δ=0.50	0.51	0.63	0.78	0.94	1.10	0.51	0.63	0.78	0.94	1.10	0.51	0.63	0.78	0.94	1.10	0.50	0.55	0.73	0.91	1.10	0.18	0.32	0.45	0.62	0.81
	δ=0.75	0.51	0.63	0.78	0.94	1.10	0.51	0.63	0.78	0.94	1.10	0.51	0.63	0.78	0.94	1.28	0.50	0.63	0.83	1.03	1.24	0.16	0.29	0.48	0.68	0.89
	δ=0.95	0.51	0.63	0.78	0.94	1.10	0.51	0.63	0.78	0.94	1.10	0.51	0.63	0.78	1.15	1.82	0.54	0.74	0.97	1.21	1.44	0.15	0.31	0.54	0.78	1.01
β=0.95	δ=0.05	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.44	0.57	0.70	0.14	0.27	0.40	0.53	0.67
	δ=0.25	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.44	0.57	0.69	0.14	0.27	0.40	0.53	0.66
	δ=0.50	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.43	0.59	0.76	0.14	0.26	0.39	0.52	0.67
	δ=0.75	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.50	0.70	0.90	0.13	0.26	0.40	0.60	0.80
	δ=0.95	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.44	0.57	0.70	0.17	0.36	0.59	0.83	1.06	0.13	0.25	0.48	0.71	0.94

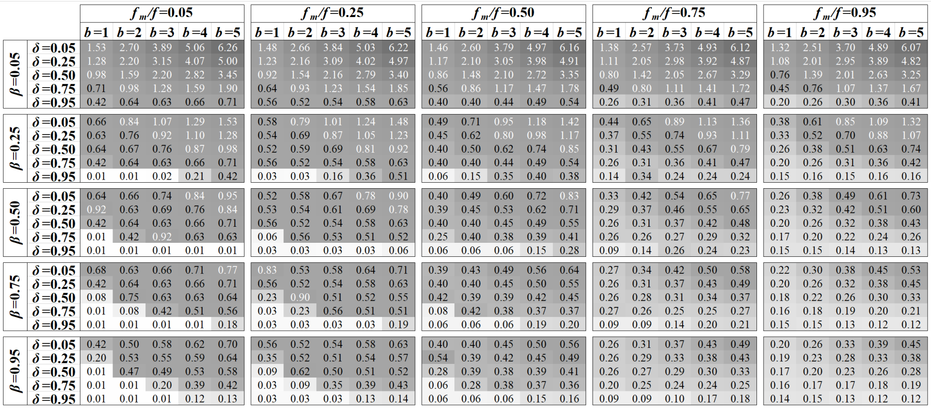
(b) $f = 5$



		f_m/f=0.05					f_m/f=0.25					f_m/f=0.50					f_m/f=0.75					f_m/f=0.95				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	13.40	13.71	15.42	17.45	19.62	9.74	10.88	12.85	15.02	17.29	6.30	8.15	10.38	12.69	15.02	3.67	5.91	8.21	10.55	12.93	2.07	4.39	6.72	9.04	11.37
	δ=0.25	16.53	13.28	14.29	15.76	17.41	9.95	10.31	11.71	13.38	15.15	6.01	7.36	9.10	10.92	12.79	3.06	4.91	6.81	8.72	10.64	1.43	3.31	5.22	7.14	9.06
	δ=0.50	21.54	13.35	13.38	14.09	15.07	14.14	9.98	10.55	11.52	12.65	6.02	6.53	7.59	8.80	10.06	2.56	3.80	5.12	6.45	7.78	0.84	2.15	3.49	4.83	6.17
	δ=0.75	25.62	21.55	16.53	13.76	13.79	19.13	14.14	10.44	10.48	10.81	9.82	6.27	6.50	7.01	7.62	2.20	2.79	3.49	4.23	4.98	0.29	1.00	1.75	2.51	3.28
	δ=0.95	25.62	25.63	25.61	25.64	25.62	19.44	19.44	19.45	19.45	19.13	11.71	11.71	11.70	10.83	9.82	3.98	3.54	2.73	2.94	3.20	0.01	0.12	0.38	0.66	0.97
β=0.25	δ=0.05	3.45	3.29	3.57	3.96	4.39	3.45	3.29	3.57	3.96	4.39	3.45	3.29	3.57	3.96	4.39	1.65	1.94	2.36	2.80	3.25	0.20	0.64	1.11	1.57	2.04
	δ=0.25	3.45	3.29	3.57	3.96	4.39	3.45	3.29	3.57	3.96	4.39	3.45	3.29	3.57	3.96	4.46	1.49	1.67	1.98	2.38	2.79	0.12	0.45	0.81	1.17	1.53
	δ=0.50	3.45	3.29	3.57	3.96	4.39	3.45	3.29	3.57	3.96	4.39	3.45	3.29	4.25	4.80	4.94	1.98	1.53	1.82	2.14	2.47	0.04	0.25	0.48	0.78	1.11
	δ=0.75	3.45	3.29	3.57	3.96	4.39	3.45	3.29	3.57	3.96	4.81	3.45	3.92	5.32	5.52	5.47	2.42	1.98	1.78	2.02	2.27	0.01	0.09	0.23	0.46	0.71
	δ=0.95	3.45	3.29	3.57	3.96	4.39	3.45	3.29	3.57	4.46	7.10	3.45	4.96	6.33	6.91	7.18	2.42	2.42	2.42	2.41	2.47	0.00	0.02	0.06	0.20	0.39
β=0.50	δ=0.05	0.39	0.63	0.88	1.13	1.38	0.39	0.63	0.88	1.13	1.38	0.39	0.63	0.88	1.13	1.38	0.39	0.63	0.88	1.13	1.38	0.00	0.15	0.36	0.58	0.81
	δ=0.25	0.39	0.63	0.88	1.13	1.38	0.39	0.63	0.88	1.13	1.38	0.39	0.63	0.88	1.13	1.38	0.39	0.63	0.88	1.13	1.37	0.00	0.10	0.27	0.46	0.65
	δ=0.50	0.39	0.63	0.88	1.13	1.38	0.39	0.63	0.88	1.13	1.38	0.39	0.63	0.88	1.13	1.38	0.39	0.63	0.83	1.05	1.27	0.00	0.05	0.18	0.32	0.47
	δ=0.75	0.39	0.63	0.88	1.13	1.38	0.39	0.63	0.88	1.13	1.38	0.39	0.63	0.88	1.13	1.38	0.39	0.63	0.85	1.07	1.29	0.00	0.02	0.09	0.20	0.32
	δ=0.95	0.39	0.63	0.88	1.13	1.38	0.39	0.63	0.88	1.13	1.38	0.39	0.63	0.88	1.13	1.38	0.39	0.71	0.95	1.18	1.42	0.00	0.00	0.05	0.11	0.24
β=0.75	δ=0.05	0.00	0.06	0.20	0.34	0.50	0.00	0.06	0.20	0.34	0.50	0.00	0.06	0.20	0.34	0.50	0.00	0.06	0.20	0.34	0.50	0.00	0.01	0.11	0.23	0.38
	δ=0.25	0.00	0.06	0.20	0.34	0.50	0.00	0.06	0.20	0.34	0.50	0.00	0.06	0.20	0.34	0.50	0.00	0.06	0.20	0.34	0.50	0.00	0.01	0.09	0.20	0.33
	δ=0.50	0.00	0.06	0.20	0.34	0.50	0.00	0.06	0.20	0.34	0.50	0.00	0.06	0.20	0.34	0.50	0.00	0.06	0.20	0.34	0.50	0.00	0.00	0.06	0.16	0.27
	δ=0.75	0.00	0.06	0.20	0.34	0.50	0.00	0.06	0.20	0.34	0.50	0.00	0.06	0.20	0.34	0.50	0.00	0.06	0.20	0.34	0.53	0.00	0.00	0.04	0.12	0.22
	δ=0.95	0.00	0.06	0.20	0.34	0.50	0.00	0.06	0.20	0.34	0.50	0.00	0.06	0.20	0.34	0.50	0.00	0.06	0.20	0.37	0.60	0.00	0.00	0.02	0.09	0.18
β=0.95	δ=0.05	0.00	0.00	0.03	0.10	0.20	0.00	0.00	0.03	0.10	0.20	0.00	0.00	0.03	0.10	0.20	0.00	0.00	0.03	0.10	0.20	0.00	0.00	0.02	0.09	0.19
	δ=0.25	0.00	0.00	0.03	0.10	0.20	0.00	0.00	0.03	0.10	0.20	0.00	0.00	0.03	0.10	0.20	0.00	0.00	0.03	0.10	0.20	0.00	0.00	0.02	0.09	0.18
	δ=0.50	0.00	0.00	0.03	0.10	0.20	0.00	0.00	0.03	0.10	0.20	0.00	0.00	0.03	0.10	0.20	0.00	0.00	0.03	0.10	0.20	0.00	0.00	0.02	0.08	0.18
	δ=0.75	0.00	0.00	0.03	0.10	0.20	0.00	0.00	0.03	0.10	0.20	0.00	0.00	0.03	0.10	0.20	0.00	0.00	0.03	0.10	0.20	0.00	0.00	0.02	0.08	0.17
	δ=0.95	0.00	0.00	0.03	0.10	0.20	0.00	0.00	0.03	0.10	0.20	0.00	0.00	0.03	0.10	0.20	0.00	0.00	0.03	0.10	0.20	0.00	0.00	0.01	0.07	0.16

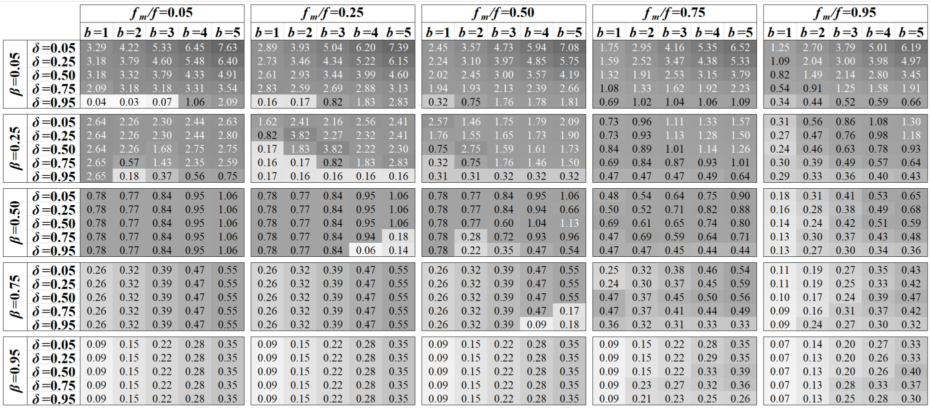
(c) $f = 10$

Figure 5.2: Impact of f on the manufacturer's profit function, where the parameters are set to $s_n = 3, \upsilon = 3$, and values from 0 to 45.68 are highlighted as a heatmap from white to black background



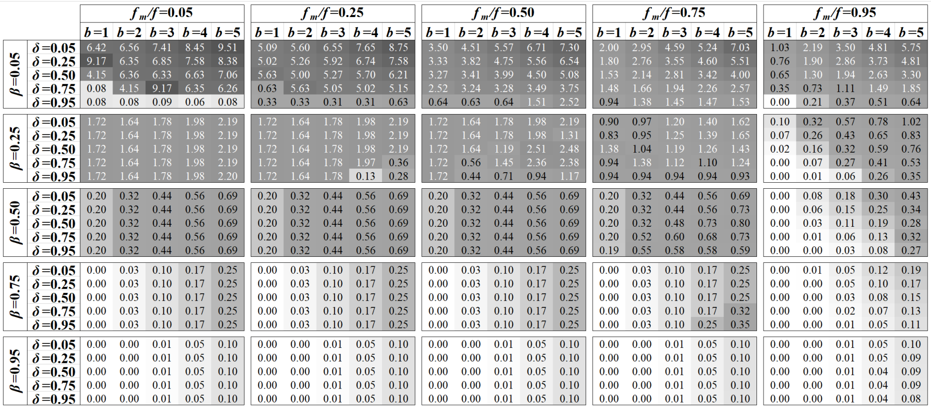
		f_m/f=0.05					f_m/f=0.25					f_m/f=0.50					f_m/f=0.75					f_m/f=0.95				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	1.53	2.70	3.89	5.06	6.26	1.48	2.66	3.84	5.03	6.22	1.46	2.60	3.79	4.97	6.16	1.38	2.57	3.73	4.93	6.12	1.32	2.51	3.70	4.89	6.07
	δ=0.25	1.28	2.20	3.15	4.07	5.00	1.23	2.16	3.09	4.02	4.97	1.17	2.10	3.05	3.98	4.91	1.11	2.05	2.98	3.92	4.87	1.08	2.01	2.95	3.89	4.82
	δ=0.50	0.98	1.59	2.20	2.82	3.45	0.92	1.54	2.16	2.79	3.40	0.86	1.48	2.10	2.72	3.35	0.80	1.42	2.05	2.67	3.29	0.76	1.39	2.01	2.63	3.25
	δ=0.75	0.71	0.98	1.28	1.59	1.90	0.64	0.93	1.23	1.54	1.85	0.56	0.86	1.17	1.47	1.78	0.49	0.80	1.11	1.41	1.72	0.45	0.76	1.07	1.37	1.67
	δ=0.95	0.42	0.64	0.63	0.66	0.71	0.56	0.52	0.54	0.58	0.63	0.40	0.40	0.44	0.49	0.54	0.26	0.31	0.36	0.41	0.47	0.20	0.26	0.30	0.36	0.41
β=0.25	δ=0.05	0.66	0.84	1.07	1.29	1.53	0.58	0.79	1.01	1.24	1.48	0.49	0.71	0.95	1.18	1.42	0.44	0.65	0.89	1.13	1.36	0.38	0.61	0.85	1.09	1.32
	δ=0.25	0.63	0.76	0.92	1.10	1.28	0.54	0.69	0.87	1.05	1.23	0.45	0.62	0.80	0.98	1.17	0.37	0.55	0.74	0.93	1.11	0.33	0.52	0.70	0.88	1.07
	δ=0.50	0.64	0.67	0.76	0.87	0.98	0.52	0.59	0.69	0.81	0.92	0.40	0.50	0.62	0.74	0.85	0.31	0.43	0.55	0.67	0.79	0.26	0.38	0.51	0.63	0.74
	δ=0.75	0.42	0.64	0.63	0.66	0.71	0.56	0.52	0.54	0.58	0.63	0.40	0.40	0.44	0.49	0.54	0.26	0.31	0.36	0.41	0.47	0.20	0.26	0.31	0.36	0.42
	δ=0.95	0.01	0.01	0.02	0.21	0.42	0.03	0.03	0.16	0.36	0.51	0.06	0.15	0.35	0.40	0.38	0.14	0.34	0.24	0.24	0.24	0.15	0.16	0.15	0.16	0.16
β=0.50	δ=0.05	0.64	0.66	0.74	0.84	0.95	0.52	0.58	0.67	0.78	0.90	0.40	0.49	0.60	0.72	0.83	0.33	0.42	0.54	0.65	0.77	0.26	0.38	0.49	0.61	0.73
	δ=0.25	0.92	0.63	0.69	0.76	0.84	0.53	0.54	0.61	0.69	0.78	0.39	0.45	0.53	0.62	0.71	0.29	0.37	0.46	0.55	0.65	0.23	0.32	0.42	0.51	0.60
	δ=0.50	0.42	0.64	0.63	0.66	0.71	0.56	0.52	0.54	0.58	0.63	0.40	0.40	0.45	0.49	0.55	0.26	0.31	0.37	0.42	0.48	0.20	0.26	0.32	0.38	0.43
	δ=0.75	0.01	0.42	0.92	0.63	0.63	0.06	0.56	0.53	0.51	0.52	0.25	0.40	0.38	0.39	0.41	0.26	0.26	0.27	0.29	0.32	0.17	0.20	0.22	0.24	0.26
	δ=0.95	0.01	0.01	0.01	0.01	0.01	0.03	0.03	0.03	0.03	0.06	0.06	0.06	0.06	0.15	0.28	0.09	0.14	0.26	0.24	0.23	0.15	0.15	0.14	0.13	0.13
β=0.75	δ=0.05	0.68	0.63	0.66	0.71	0.77	0.83	0.53	0.58	0.64	0.71	0.39	0.43	0.49	0.56	0.64	0.27	0.34	0.42	0.50	0.58	0.22	0.30	0.38	0.45	0.53
	δ=0.25	0.42	0.64	0.63	0.66	0.71	0.56	0.52	0.54	0.58	0.63	0.40	0.40	0.45	0.50	0.55	0.26	0.31	0.37	0.43	0.49	0.20	0.26	0.32	0.38	0.45
	δ=0.50	0.08	0.75	0.63	0.63	0.64	0.23	0.90	0.51	0.52	0.55	0.42	0.39	0.39	0.42	0.45	0.26	0.28	0.31	0.34	0.37	0.18	0.22	0.26	0.30	0.33
	δ=0.75	0.01	0.08	0.42	0.51	0.56	0.03	0.23	0.56	0.51	0.51	0.08	0.42	0.38	0.37	0.37	0.27	0.26	0.25	0.25	0.27	0.16	0.18	0.19	0.20	0.21
	δ=0.95	0.01	0.01	0.01	0.01	0.18	0.03	0.03	0.03	0.03	0.19	0.06	0.06	0.06	0.19	0.20	0.09	0.09	0.14	0.20	0.21	0.15	0.15	0.13	0.12	0.12
β=0.95	δ=0.05	0.42	0.50	0.58	0.62	0.70	0.56	0.52	0.54	0.58	0.63	0.40	0.40	0.45	0.50	0.56	0.26	0.31	0.37	0.43	0.49	0.20	0.26	0.33	0.39	0.45
	δ=0.25	0.20	0.53	0.55	0.59	0.64	0.35	0.52	0.51	0.54	0.57	0.54	0.39	0.42	0.45	0.49	0.26	0.29	0.33	0.38	0.43	0.19	0.23	0.28	0.33	0.38
	δ=0.50	0.01	0.47	0.49	0.53	0.58	0.09	0.62	0.50	0.51	0.52	0.28	0.39	0.38	0.39	0.41	0.26	0.27	0.29	0.30	0.33	0.17	0.20	0.23	0.26	0.28
	δ=0.75	0.01	0.01	0.20	0.39	0.42	0.03	0.09	0.35	0.39	0.43	0.06	0.28	0.38	0.37	0.36	0.20	0.25	0.24	0.24	0.25	0.16	0.17	0.17	0.18	0.19
	δ=0.95	0.01	0.01	0.01	0.12	0.13	0.03	0.03	0.03	0.13	0.14	0.06	0.06	0.06	0.15	0.16	0.09	0.09	0.10	0.17	0.18	0.14	0.15	0.13	0.12	0.12

(a) $f = 1$



		f_m/f=0.05					f_m/f=0.25					f_m/f=0.50					f_m/f=0.75					f_m/f=0.95				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	3.29	4.22	5.33	6.45	7.63	2.89	3.93	5.04	6.20	7.39	2.45	3.57	4.73	5.94	7.08	1.75	2.95	4.16	5.35	6.52	1.25	2.70	3.79	5.01	6.19
	δ=0.25	3.18	3.79	4.60	5.48	6.40	2.73	3.46	4.34	5.22	6.15	2.24	3.10	3.97	4.85	5.75	1.59	2.52	3.47	4.38	5.33	1.09	2.04	3.00	3.98	4.97
	δ=0.50	3.18	3.32	3.79	4.33	4.91	2.61	2.93	3.44	3.99	4.60	2.02	2.45	3.00	3.57	4.19	1.32	1.91	2.53	3.15	3.79	0.82	1.49	2.14	2.80	3.45
	δ=0.75	2.09	3.18	3.18	3.31	3.54	2.83	2.59	2.69	2.88	3.13	1.94	1.93	2.13	2.39	2.66	1.08	1.33	1.62	1.92	2.23	0.54	0.91	1.25	1.58	1.91
	δ=0.95	0.04	0.03	0.07	1.06	2.09	0.16	0.17	0.82	1.83	2.83	0.32	0.75	1.76	1.78	1.81	0.69	1.02	1.04	1.06	1.09	0.34	0.44	0.52	0.59	0.66
β=0.25	δ=0.05	2.64	2.26	2.30	2.44	2.63	1.62	2.41	2.16	2.56	2.41	2.57	1.46	1.75	1.79	2.09	0.73	0.96	1.11	1.33	1.57	0.31	0.56	0.86	1.08	1.30
	δ=0.25	2.64	2.26	2.30	2.44	2.80	0.82	3.82	2.27	2.32	2.41	1.76	1.55	1.65	1.73	1.90	0.73	0.93	1.13	1.28	1.50	0.27	0.47	0.76	0.98	1.18
	δ=0.50	2.64	2.26	1.68	2.75	2.75	0.17	1.83	3.82	2.22	2.30	0.75	2.75	1.59	1.61	1.73	0.84	0.89	1.01	1.14	1.26	0.24	0.46	0.63	0.78	0.93
	δ=0.75	2.65	0.57	1.43	2.35	2.59	0.16	0.17	0.82	1.83	2.83	0.32	0.75	1.76	1.46	1.50	0.69	0.84	0.87	0.93	1.01	0.30	0.39	0.49	0.57	0.64
	δ=0.95	2.65	0.18	0.37	0.56	0.75	0.17	0.16	0.16	0.16	0.16	0.31	0.31	0.32	0.32	0.32	0.47	0.47	0.47	0.49	0.64	0.29	0.33	0.36	0.40	0.43
β=0.50	δ=0.05	0.78	0.77	0.84	0.95	1.06	0.78	0.77	0.84	0.95	1.06	0.78	0.77	0.84	0.95	1.06	0.48	0.54	0.64	0.75	0.90	0.18	0.31	0.41	0.53	0.65
	δ=0.25	0.78	0.77	0.84	0.95	1.06	0.78	0.77	0.84	0.95	1.06	0.78	0.77	0.84	0.94	0.66	0.50	0.52	0.71	0.82	0.88	0.16	0.28	0.38	0.49	0.68
	δ=0.50	0.78	0.77	0.84	0.95	1.06	0.78	0.77	0.84	0.95	1.06	0.78	0.77	0.60	1.04	1.13	0.69	0.61	0.65	0.74	0.80	0.14	0.24	0.42	0.51	0.59
	δ=0.75	0.78	0.77	0.84	0.95	1.06	0.78	0.77	0.84	0.94	0.18	0.78	0.28	0.72	0.93	0.96	0.47	0.69	0.59	0.64	0.71	0.13	0.30	0.37	0.43	0.48
	δ=0.95	0.78	0.77	0.84	0.95	1.06	0.78	0.77	0.84	0.06	0.14	0.78	0.22	0.35	0.47	0.54	0.47	0.47	0.45	0.44	0.44	0.13	0.27	0.30	0.34	0.36
β=0.75	δ=0.05	0.26	0.32	0.39	0.47	0.55	0.26	0.32	0.39	0.47	0.55	0.26	0.32	0.39	0.47	0.55	0.25	0.32	0.38	0.46	0.54	0.11	0.19	0.27	0.35	0.43
	δ=0.25	0.26	0.32	0.39	0.47	0.55	0.26	0.32	0.39	0.47	0.55	0.26	0.32	0.39	0.47	0.55	0.24	0.30	0.37	0.45	0.59	0.11	0.19	0.25	0.33	0.42
	δ=0.50	0.26	0.32	0.39	0.47	0.55	0.26	0.32	0.39	0.47	0.55	0.26	0.32	0.39	0.47	0.55	0.47	0.37	0.45	0.50	0.56	0.10	0.17	0.24	0.39	0.47
	δ=0.75	0.26	0.32	0.39	0.47	0.55	0.26	0.32	0.39	0.47	0.55	0.26	0.32	0.39	0.47	0.17	0.47	0.37	0.41	0.44	0.49	0.09	0.16	0.31	0.37	0.42
	δ=0.95	0.26	0.32	0.39	0.47	0.55	0.26	0.32	0.39	0.47	0.55	0.26	0.32	0.39	0.09	0.18	0.36	0.32	0.31	0.33	0.33	0.09	0.24	0.27	0.30	0.32
β=0.95	δ=0.05	0.09	0.15	0.22	0.28	0.35	0.09	0.15	0.22	0.28	0.35	0.09	0.15	0.22	0.28	0.35	0.09	0.15	0.22	0.28	0.35	0.07	0.14	0.20	0.27	0.33
	δ=0.25	0.09	0.15	0.22	0.28	0.35	0.09	0.15	0.22	0.28	0.35	0.09	0.15	0.22	0.28	0.35	0.09	0.15	0.22	0.29	0.35	0.07	0.13	0.20	0.26	0.33
	δ=0.50	0.09	0.15	0.22	0.28	0.35	0.09	0.15	0.22	0.28	0.35	0.09	0.15	0.22	0.28	0.35	0.09	0.15	0.22	0.33	0.39	0.07	0.13	0.20	0.26	0.40
	δ=0.75	0.09	0.15	0.22	0.28	0.35	0.09	0.15	0.22	0.28	0.35	0.09	0.15	0.22	0.28	0.35	0.09	0.23	0.27	0.32	0.36	0.07	0.13	0.28	0.33	0.37
	δ=0.95	0.09	0.15	0.22	0.28	0.35	0.09	0.15	0.22	0.28	0.35	0.09	0.15	0.22	0.28	0.35	0.09	0.21	0.23	0.25	0.26	0.07	0.13	0.25	0.28	0.30

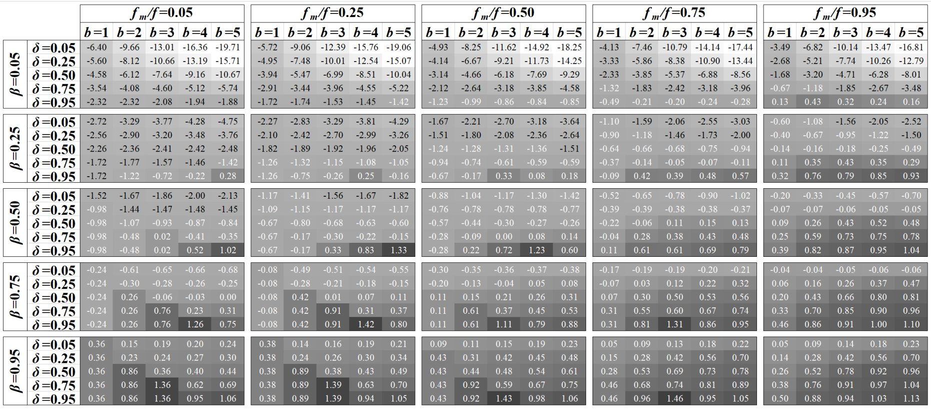
(b) $f = 5$



		f_m/f=0.05					f_m/f=0.25					f_m/f=0.50					f_m/f=0.75					f_m/f=0.95				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	6.42	6.56	7.41	8.45	9.51	5.09	5.60	6.55	7.65	8.75	3.50	4.51	5.57	6.71	7.30	2.00	2.95	4.59	5.24	7.03	1.03	2.19	3.50	4.81	5.75
	δ=0.25	9.17	6.35	6.85	7.58	8.38	5.02	5.26	5.92	6.74	7.58	3.33	3.82	4.75	5.56	6.54	1.80	2.76	3.55	4.60	5.51	0.76	1.90	2.86	3.73	4.81
	δ=0.50	4.15	6.36	6.33	6.63	7.06	5.63	5.00	5.27	5.70	6.21	3.27	3.41	3.99	4.50	5.08	1.53	2.14	2.81	3.42	4.00	0.65	1.30	1.94	2.63	3.30
	δ=0.75	0.08	4.15	9.17	6.35	6.26	0.63	5.63	5.05	5.02	5.15	2.52	3.24	3.28	3.49	3.75	1.48	1.66	1.94	2.26	2.57	0.35	0.73	1.11	1.49	1.85
	δ=0.95	0.08	0.08	0.09	0.06	0.08	0.33	0.33	0.31	0.31	0.63	0.64	0.63	0.64	1.51	2.52	0.94	1.38	1.45	1.47	1.53	0.00	0.21	0.37	0.51	0.64
β=0.25	δ=0.05	1.72	1.64	1.78	1.98	2.19	1.72	1.64	1.78	1.98	2.19	1.72	1.64	1.78	1.98	2.19	0.90	0.97	1.20	1.40	1.62	0.10	0.32	0.57	0.78	1.02
	δ=0.25	1.72	1.64	1.78	1.98	2.19	1.72	1.64	1.78	1.98	2.19	1.72	1.64	1.78	1.98	1.31	0.83	0.95	1.25	1.39	1.65	0.07	0.26	0.43	0.65	0.83
	δ=0.50	1.72	1.64	1.78	1.98	2.19	1.72	1.64	1.78	1.98	2.19	1.72	1.64	1.19	2.51	2.48	1.38	1.04	1.19	1.26	1.43	0.02	0.16	0.32	0.59	0.76
	δ=0.75	1.72	1.64	1.78	1.98	2.19	1.72	1.64	1.78	1.97	0.36	1.72	0.56	1.45	2.36	2.38	0.94	1.38	1.12	1.10	1.24	0.00	0.07	0.27	0.41	0.53
	δ=0.95	1.72	1.64	1.78	1.98	2.20	1.72	1.64	1.78	0.13	0.28	1.72	0.44	0.71	0.94	1.17	0.94	0.94	0.94	0.94	0.93	0.00	0.01	0.06	0.26	0.35
β=0.50	δ=0.05	0.20	0.32	0.44	0.56	0.69	0.20	0.32	0.44	0.56	0.69	0.20	0.32	0.44	0.56	0.69	0.20	0.32	0.44	0.56	0.69	0.00	0.08	0.18	0.30	0.43
	δ=0.25	0.20	0.32	0.44	0.56	0.69	0.20	0.32	0.44	0.56	0.69	0.20	0.32	0.44	0.56	0.69	0.20	0.32	0.44	0.56	0.73	0.00	0.06	0.15	0.25	0.34
	δ=0.50	0.20	0.32	0.44	0.56	0.69	0.20	0.32	0.44	0.56	0.69	0.20	0.32	0.44	0.56	0.69	0.20	0.32	0.48	0.73	0.80	0.00	0.03	0.11	0.19	0.28
	δ=0.75	0.20	0.32	0.44	0.56	0.69	0.20	0.32	0.44	0.56	0.69	0.20	0.32	0.44	0.56	0.69	0.20	0.52	0.60	0.68	0.73	0.00	0.01	0.06	0.13	0.32
	δ=0.95	0.20	0.32	0.44	0.56	0.69	0.20	0.32	0.44	0.56	0.69	0.20	0.32	0.44	0.56	0.69	0.19	0.55	0.58	0.58	0.59	0.00	0.00	0.03	0.08	0.27
β=0.75	δ=0.05	0.00	0.03	0.10	0.17	0.25	0.00	0.03	0.10	0.17	0.25	0.00	0.03	0.10	0.17	0.25	0.00	0.03	0.10	0.17	0.25	0.00	0.01	0.05	0.12	0.19
	δ=0.25	0.00	0.03	0.10	0.17	0.25	0.00	0.03	0.10	0.17	0.25	0.00	0.03	0.10	0.17	0.25	0.00	0.03	0.10	0.17	0.25	0.00	0.00	0.05	0.10	0.17
	δ=0.50	0.00	0.03	0.10	0.17	0.25	0.00	0.03	0.10	0.17	0.25	0.00	0.03	0.10	0.17	0.25	0.00	0.03	0.10	0.17	0.25	0.00	0.00	0.03	0.08	0.15
	δ=0.75	0.00	0.03	0.10	0.17	0.25	0.00	0.03	0.10	0.17	0.25	0.00	0.03	0.10	0.17	0.25	0.00	0.03	0.10	0.17	0.32	0.00	0.00	0.02	0.07	0.13
	δ=0.95	0.00	0.03	0.10	0.17	0.25	0.00	0.03	0.10	0.17	0.25	0.00	0.03	0.10	0.17	0.25	0.00	0.03	0.10	0.25	0.35	0.00	0.00	0.01	0.05	0.11
β=0.95	δ=0.05	0.00	0.00	0.01	0.05	0.10	0.00	0.00	0.01	0.05	0.10	0.00	0.00	0.01	0.05	0.10	0.00	0.00	0.01	0.05	0.10	0.00	0.00	0.01	0.05	0.10
	δ=0.25	0.00	0.00	0.01	0.05	0.10	0.00	0.00	0.01	0.05	0.10	0.00	0.00	0.01	0.05	0.10	0.00	0.00	0.01	0.05	0.10	0.00	0.00	0.01	0.05	0.09
	δ=0.50	0.00	0.00	0.01	0.05	0.10	0.00	0.00	0.01	0.05	0.10	0.00	0.00	0.01	0.05	0.10	0.00	0.00	0.01	0.05	0.10	0.00	0.00	0.01	0.04	0.09
	δ=0.75	0.00	0.00	0.01	0.05	0.10	0.00	0.00	0.01	0.05	0.10	0.00	0.00	0.01	0.05	0.10	0.00	0.00	0.01	0.05	0.10	0.00	0.00	0.01	0.04	0.09
	δ=0.95	0.00	0.00	0.01	0.05	0.10	0.00	0.00	0.01	0.05	0.10	0.00	0.00	0.01	0.05	0.10	0.00	0.00	0.01	0.05	0.10	0.00	0.00	0.01	0.04	0.08

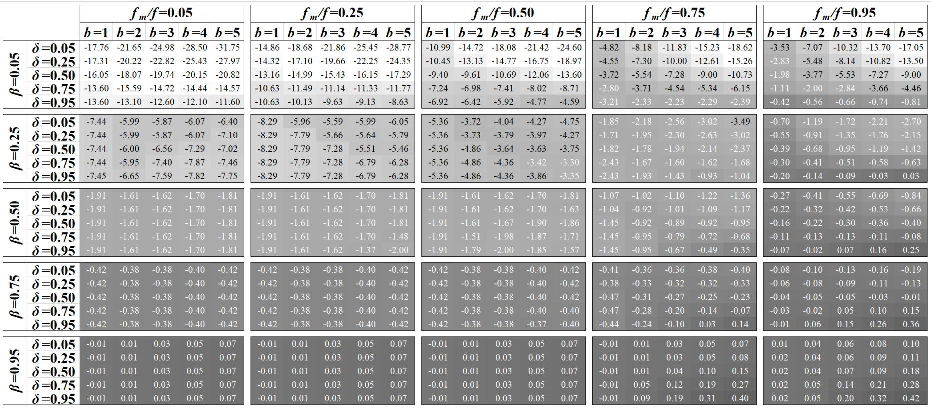
(c) $f = 10$

Figure 5.3: Impact of f on the retailer's profit function, where the parameters are set to $s_n = 3, \upsilon = 3$, and values from 0 to 19.19 are highlighted as a heatmap from white to black background



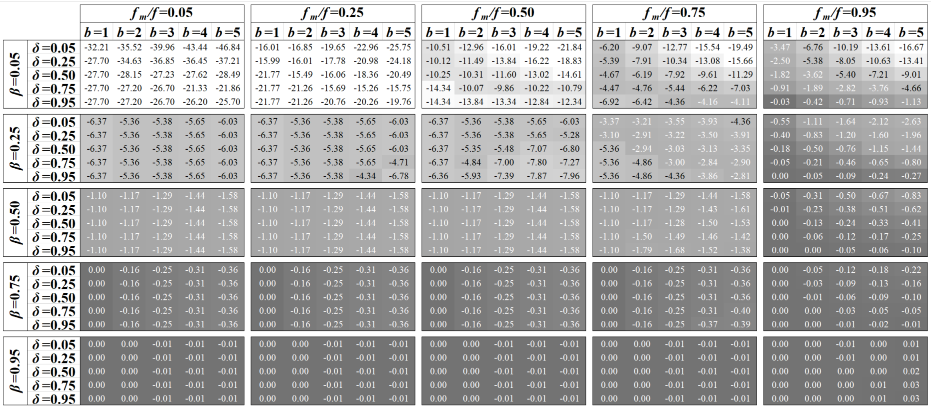
		f_m/f=0.05					f_m/f=0.25					f_m/f=0.50					f_m/f=0.75					f_m/f=0.95				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	-6.40	-9.66	-13.01	-16.36	-19.71	-5.72	-9.06	-12.39	-15.76	-19.06	-4.93	-8.25	-11.62	-14.92	-18.25	-4.13	-7.46	-10.79	-14.14	-17.44	-3.49	-6.82	-10.14	-13.47	-16.81
	δ=0.25	-5.60	-8.12	-10.66	-13.19	-15.71	-4.95	-7.48	-10.01	-12.54	-15.07	-4.14	-6.67	-9.21	-11.73	-14.25	-3.33	-5.86	-8.38	-10.90	-13.44	-2.68	-5.21	-7.74	-10.26	-12.79
	δ=0.50	-4.58	-6.12	-7.64	-9.16	-10.67	-3.94	-5.47	-6.99	-8.51	-10.04	-3.14	-4.66	-6.18	-7.69	-9.29	-2.33	-3.85	-5.37	-6.88	-8.56	-1.68	-3.20	-4.71	-6.28	-8.01
	δ=0.75	-3.54	-4.08	-4.60	-5.12	-5.74	-2.91	-3.44	-3.96	-4.55	-5.22	-2.12	-2.64	-3.18	-3.85	-4.58	-1.32	-1.83	-2.42	-3.18	-3.96	-0.67	-1.18	-1.85	-2.67	-3.48
	δ=0.95	-2.32	-2.32	-2.08	-1.94	-1.88	-1.72	-1.74	-1.53	-1.45	-1.42	-1.23	-0.99	-0.86	-0.84	-0.85	-0.49	-0.21	-0.20	-0.24	-0.28	0.13	0.43	0.32	0.24	0.16
β=0.25	δ=0.05	-2.72	-3.29	-3.77	-4.28	-4.75	-2.27	-2.83	-3.29	-3.81	-4.29	-1.67	-2.21	-2.70	-3.18	-3.64	-1.10	-1.59	-2.06	-2.55	-3.03	-0.60	-1.08	-1.56	-2.05	-2.52
	δ=0.25	-2.56	-2.90	-3.20	-3.48	-3.76	-2.10	-2.42	-2.70	-2.99	-3.26	-1.51	-1.80	-2.08	-2.36	-2.64	-0.90	-1.18	-1.46	-1.73	-2.00	-0.40	-0.67	-0.95	-1.22	-1.50
	δ=0.50	-2.26	-2.36	-2.41	-2.42	-2.48	-1.82	-1.89	-1.92	-1.96	-2.05	-1.24	-1.28	-1.31	-1.36	-1.51	-0.64	-0.66	-0.68	-0.75	-0.94	-0.14	-0.16	-0.18	-0.25	-0.49
	δ=0.75	-1.72	-1.77	-1.57	-1.46	-1.42	-1.26	-1.32	-1.15	-1.08	-1.05	-0.94	-0.74	-0.61	-0.59	-0.59	-0.37	-0.14	-0.05	-0.07	-0.11	0.11	0.35	0.43	0.35	0.29
	δ=0.95	-1.72	-1.22	-0.72	-0.22	0.28	-1.26	-0.75	-0.26	0.25	-0.16	-0.67	-0.17	0.33	0.08	0.18	-0.09	0.42	0.39	0.48	0.57	0.32	0.76	0.79	0.85	0.93
β=0.50	δ=0.05	-1.52	-1.67	-1.86	-2.00	-2.13	-1.17	-1.41	-1.56	-1.67	-1.82	-0.88	-1.04	-1.17	-1.30	-1.42	-0.52	-0.65	-0.78	-0.90	-1.02	-0.20	-0.33	-0.45	-0.57	-0.70
	δ=0.25	-0.98	-1.44	-1.47	-1.48	-1.45	-1.09	-1.15	-1.17	-1.17	-1.17	-0.76	-0.78	-0.78	-0.78	-0.77	-0.39	-0.39	-0.38	-0.38	-0.37	-0.07	-0.07	-0.06	-0.05	-0.05
	δ=0.50	-0.98	-1.07	-0.93	-0.87	-0.84	-0.67	-0.80	-0.68	-0.63	-0.60	-0.57	-0.44	-0.30	-0.27	-0.26	-0.22	-0.06	0.11	0.15	0.13	0.09	0.26	0.43	0.52	0.48
	δ=0.75	-0.98	-0.48	0.02	-0.41	-0.35	-0.67	-0.17	-0.30	-0.22	-0.15	-0.28	-0.09	0.00	0.08	0.14	-0.04	0.28	0.38	0.43	0.48	0.25	0.59	0.73	0.75	0.78
	δ=0.95	-0.98	-0.48	0.02	0.52	1.02	-0.67	-0.17	0.33	0.83	1.33	-0.28	0.22	0.72	1.23	0.60	0.11	0.61	0.61	0.69	0.79	0.39	0.82	0.87	0.95	1.04
β=0.75	δ=0.05	-0.24	-0.61	-0.65	-0.66	-0.68	-0.08	-0.49	-0.51	-0.54	-0.55	-0.30	-0.35	-0.36	-0.37	-0.38	-0.17	-0.19	-0.19	-0.20	-0.21	-0.04	-0.04	-0.05	-0.06	-0.06
	δ=0.25	-0.24	-0.30	-0.28	-0.26	-0.25	-0.08	-0.28	-0.21	-0.18	-0.15	-0.20	-0.13	-0.04	0.05	0.08	-0.07	0.03	0.12	0.22	0.32	0.06	0.16	0.26	0.37	0.47
	δ=0.50	-0.24	0.26	-0.06	-0.03	0.00	-0.08	0.42	0.01	0.07	0.11	0.11	0.15	0.21	0.26	0.31	0.07	0.30	0.50	0.53	0.56	0.20	0.43	0.66	0.80	0.81
	δ=0.75	-0.24	0.26	0.76	0.23	0.31	-0.08	0.42	0.91	0.31	0.37	0.11	0.61	0.37	0.45	0.53	0.31	0.55	0.60	0.67	0.74	0.33	0.70	0.85	0.90	0.96
	δ=0.95	-0.24	0.26	0.76	1.26	0.75	-0.08	0.42	0.91	1.42	0.80	0.11	0.61	1.11	0.79	0.88	0.31	0.81	1.31	0.86	0.95	0.46	0.86	0.91	1.00	1.10
β=0.95	δ=0.05	0.36	0.15	0.19	0.20	0.24	0.38	0.14	0.16	0.19	0.21	0.09	0.11	0.15	0.19	0.23	0.05	0.09	0.13	0.18	0.22	0.05	0.09	0.14	0.18	0.23
	δ=0.25	0.36	0.23	0.24	0.27	0.30	0.38	0.24	0.26	0.30	0.34	0.43	0.31	0.42	0.45	0.48	0.15	0.28	0.42	0.56	0.70	0.14	0.28	0.42	0.56	0.70
	δ=0.50	0.36	0.86	0.36	0.40	0.44	0.38	0.89	0.38	0.43	0.49	0.43	0.44	0.48	0.54	0.61	0.28	0.53	0.69	0.73	0.78	0.26	0.52	0.78	0.92	0.96
	δ=0.75	0.36	0.86	1.36	0.62	0.69	0.38	0.89	1.39	0.63	0.70	0.43	0.92	0.59	0.67	0.75	0.46	0.68	0.74	0.81	0.89	0.38	0.76	0.91	0.97	1.04
	δ=0.95	0.36	0.86	1.36	0.95	1.06	0.38	0.89	1.39	0.94	1.05	0.43	0.92	1.43	0.98	1.06	0.46	0.96	1.46	0.95	1.05	0.50	0.88	0.94	1.03	1.13

(a) $f = 1$



		f_m/f=0.05					f_m/f=0.25					f_m/f=0.50					f_m/f=0.75					f_m/f=0.95				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	-17.76	-21.65	-24.98	-28.50	-31.75	-14.86	-18.68	-21.86	-25.45	-28.77	-10.99	-14.72	-18.08	-21.42	-24.60	-4.82	-8.18	-11.83	-15.23	-18.62	-3.53	-7.07	-10.32	-13.70	-17.05
	δ=0.25	-17.31	-20.22	-22.82	-25.43	-27.97	-14.32	-17.10	-19.66	-22.25	-24.35	-10.45	-13.13	-14.77	-16.75	-18.97	-4.55	-7.30	-10.00	-12.61	-15.26	-2.83	-5.48	-8.14	-10.82	-13.50
	δ=0.50	-16.05	-18.07	-19.74	-20.15	-20.82	-13.16	-14.99	-15.43	-16.15	-17.29	-9.40	-9.61	-10.69	-12.06	-13.60	-3.72	-5.54	-7.28	-9.00	-10.73	-1.98	-3.77	-5.53	-7.27	-9.00
	δ=0.75	-13.60	-15.59	-14.72	-14.44	-14.57	-10.63	-11.49	-11.14	-11.33	-11.77	-7.24	-6.98	-7.41	-8.02	-8.71	-2.80	-3.71	-4.54	-5.34	-6.15	-1.11	-2.00	-2.84	-3.66	-4.46
	δ=0.95	-13.60	-13.10	-12.60	-12.10	-11.60	-10.63	-10.13	-9.63	-9.13	-8.63	-6.92	-6.42	-5.92	-4.77	-4.59	-3.21	-2.33	-2.23	-2.29	-2.39	-0.42	-0.56	-0.66	-0.74	-0.81
β=0.25	δ=0.05	-7.44	-5.99	-5.87	-6.07	-6.40	-8.29	-5.96	-5.59	-5.99	-6.05	-5.36	-3.72	-4.04	-4.27	-4.75	-1.85	-2.18	-2.56	-3.02	-3.49	-0.70	-1.19	-1.72	-2.21	-2.70
	δ=0.25	-7.44	-5.99	-5.87	-6.07	-7.10	-8.29	-7.79	-5.66	-5.64	-5.79	-5.36	-3.73	-3.79	-3.97	-4.27	-1.71	-1.95	-2.30	-2.63	-3.02	-0.55	-0.91	-1.35	-1.76	-2.15
	δ=0.50	-7.44	-6.00	-6.56	-7.29	-7.02	-8.29	-7.79	-7.28	-5.51	-5.46	-5.36	-4.86	-3.64	-3.63	-3.75	-1.82	-1.78	-1.94	-2.14	-2.37	-0.39	-0.68	-0.95	-1.19	-1.42
	δ=0.75	-7.44	-5.95	-7.40	-7.87	-7.46	-8.29	-7.79	-7.28	-6.79	-6.28	-5.36	-4.86	-4.36	-3.42	-3.30	-2.43	-1.67	-1.60	-1.62	-1.68	-0.30	-0.41	-0.51	-0.58	-0.63
	δ=0.95	-7.45	-6.65	-7.59	-7.82	-7.75	-8.29	-7.79	-7.28	-6.79	-6.28	-5.36	-4.86	-4.36	-3.86	-3.35	-2.43	-1.93	-1.43	-0.93	-1.04	-0.20	-0.14	-0.09	-0.03	0.03
β=0.50	δ=0.05	-1.91	-1.61	-1.62	-1.70	-1.81	-1.91	-1.61	-1.62	-1.70	-1.81	-1.91	-1.61	-1.62	-1.70	-1.81	-1.07	-1.02	-1.10	-1.22	-1.36	-0.27	-0.41	-0.55	-0.69	-0.84
	δ=0.25	-1.91	-1.61	-1.62	-1.70	-1.81	-1.91	-1.61	-1.62	-1.70	-1.81	-1.91	-1.61	-1.62	-1.70	-1.63	-1.04	-0.92	-1.01	-1.09	-1.17	-0.22	-0.32	-0.42	-0.53	-0.66
	δ=0.50	-1.91	-1.61	-1.62	-1.70	-1.81	-1.91	-1.61	-1.62	-1.70	-1.81	-1.91	-1.61	-1.67	-1.90	-1.86	-1.45	-0.92	-0.89	-0.92	-0.95	-0.16	-0.22	-0.30	-0.36	-0.40
	δ=0.75	-1.91	-1.61	-1.62	-1.70	-1.81	-1.91	-1.61	-1.62	-1.70	-1.48	-1.91	-1.51	-1.98	-1.87	-1.71	-1.45	-0.95	-0.79	-0.72	-0.68	-0.11	-0.13	-0.13	-0.11	-0.08
	δ=0.95	-1.91	-1.61	-1.62	-1.70	-1.81	-1.91	-1.61	-1.62	-1.37	-2.00	-1.91	-1.79	-2.00	-1.85	-1.57	-1.45	-0.95	-0.67	-0.49	-0.35	-0.07	-0.02	0.07	0.16	0.25
β=0.75	δ=0.05	-0.42	-0.38	-0.38	-0.40	-0.42	-0.42	-0.38	-0.38	-0.40	-0.42	-0.42	-0.38	-0.38	-0.40	-0.42	-0.41	-0.36	-0.36	-0.38	-0.40	-0.08	-0.10	-0.13	-0.16	-0.19
	δ=0.25	-0.42	-0.38	-0.38	-0.40	-0.42	-0.42	-0.38	-0.38	-0.40	-0.42	-0.42	-0.38	-0.38	-0.40	-0.42	-0.38	-0.33	-0.32	-0.32	-0.33	-0.06	-0.08	-0.09	-0.11	-0.13
	δ=0.50	-0.42	-0.38	-0.38	-0.40	-0.42	-0.42	-0.38	-0.38	-0.40	-0.42	-0.42	-0.38	-0.38	-0.40	-0.42	-0.47	-0.31	-0.27	-0.25	-0.23	-0.04	-0.05	-0.05	-0.03	-0.01
	δ=0.75	-0.42	-0.38	-0.38	-0.40	-0.42	-0.42	-0.38	-0.38	-0.40	-0.42	-0.42	-0.38	-0.38	-0.40	-0.40	-0.47	-0.28	-0.20	-0.14	-0.07	-0.03	-0.02	0.05	0.10	0.15
	δ=0.95	-0.42	-0.38	-0.38	-0.40	-0.42	-0.42	-0.38	-0.38	-0.40	-0.42	-0.42	-0.38	-0.38	-0.37	-0.40	-0.44	-0.24	-0.10	0.03	0.14	-0.01	0.06	0.15	0.26	0.36
β=0.95	δ=0.05	-0.01	0.01	0.03	0.05	0.07	-0.01	0.01	0.03	0.05	0.07	-0.01	0.01	0.03	0.05	0.07	-0.01	0.01	0.03	0.05	0.07	0.01	0.04	0.06	0.08	0.10
	δ=0.25	-0.01	0.01	0.03	0.05	0.07	-0.01	0.01	0.03	0.05	0.07	-0.01	0.01	0.03	0.05	0.07	-0.01	0.01	0.03	0.05	0.08	0.02	0.04	0.06	0.09	0.11
	δ=0.50	-0.01	0.01	0.03	0.05	0.07	-0.01	0.01	0.03	0.05	0.07	-0.01	0.01	0.03	0.05	0.07	-0.01	0.01	0.04	0.10	0.15	0.02	0.04	0.07	0.09	0.18
	δ=0.75	-0.01	0.01	0.03	0.05	0.07	-0.01	0.01	0.03	0.05	0.07	-0.01	0.01	0.03	0.05	0.07	-0.01	0.05	0.12	0.19	0.27	0.02	0.05	0.14	0.21	0.28
	δ=0.95	-0.01	0.01	0.03	0.05	0.07	-0.01	0.01	0.03	0.05	0.07	-0.01	0.01	0.03	0.05	0.07	-0.01	0.09	0.19	0.31	0.40	0.02	0.05	0.20	0.32	0.42

(b) $f = 5$



		f_m/f=0.05					f_m/f=0.25					f_m/f=0.50					f_m/f=0.75					f_m/f=0.95				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	-32.21	-35.52	-39.96	-43.44	-46.84	-16.01	-16.85	-19.65	-22.96	-25.75	-10.51	-12.96	-16.01	-19.22	-21.84	-6.20	-9.07	-12.77	-15.54	-19.49	-3.47	-6.76	-10.19	-13.61	-16.67
	δ=0.25	-27.70	-34.63	-36.85	-36.45	-37.21	-15.99	-16.01	-17.78	-20.98	-24.18	-10.12	-11.49	-13.84	-16.22	-18.83	-5.39	-7.91	-10.34	-13.08	-15.66	-2.50	-5.38	-8.05	-10.63	-13.41
	δ=0.50	-27.70	-28.15	-27.23	-27.62	-28.49	-21.77	-15.49	-16.06	-18.36	-20.49	-10.25	-10.31	-11.60	-13.02	-14.61	-4.67	-6.19	-7.92	-9.61	-11.29	-1.82	-3.62	-5.40	-7.21	-9.01
	δ=0.75	-27.70	-27.20	-26.70	-21.33	-21.86	-21.77	-21.26	-15.69	-15.26	-15.75	-14.34	-10.07	-9.86	-10.22	-10.79	-4.47	-4.76	-5.44	-6.22	-7.03	-0.91	-1.89	-2.82	-3.76	-4.66
	δ=0.95	-27.70	-27.20	-26.70	-26.20	-25.70	-21.77	-21.26	-20.76	-20.26	-19.76	-14.34	-13.84	-13.34	-12.84	-12.34	-6.92	-6.42	-4.36	-4.16	-4.11	-0.03	-0.42	-0.71	-0.93	-1.13
β=0.25	δ=0.05	-6.37	-5.36	-5.38	-5.65	-6.03	-6.37	-5.36	-5.38	-5.65	-6.03	-6.37	-5.36	-5.38	-5.65	-6.03	-3.37	-3.21	-3.55	-3.93	-4.36	-0.55	-1.11	-1.64	-2.12	-2.63
	δ=0.25	-6.37	-5.36	-5.38	-5.65	-6.03	-6.37	-5.36	-5.38	-5.65	-6.03	-6.37	-5.36	-5.38	-5.65	-5.28	-3.10	-2.91	-3.22	-3.50	-3.91	-0.40	-0.83	-1.20	-1.60	-1.96
	δ=0.50	-6.37	-5.36	-5.38	-5.65	-6.03	-6.37	-5.36	-5.38	-5.65	-6.03	-6.37	-5.35	-5.48	-7.07	-6.80	-5.36	-2.94	-3.03	-3.13	-3.35	-0.18	-0.50	-0.76	-1.15	-1.44
	δ=0.75	-6.37	-5.36	-5.38	-5.65	-6.03	-6.37	-5.36	-5.38	-5.65	-4.71	-6.37	-4.84	-7.00	-7.80	-7.27	-5.36	-4.86	-3.00	-2.84	-2.90	-0.05	-0.21	-0.46	-0.65	-0.80
	δ=0.95	-6.37	-5.36	-5.38	-5.65	-6.03	-6.37	-5.36	-5.38	-4.34	-6.78	-6.36	-5.93	-7.39	-7.87	-7.96	-5.36	-4.86	-4.36	-3.86	-2.81	0.00	-0.05	-0.09	-0.24	-0.27
β=0.50	δ=0.05	-1.10	-1.17	-1.29	-1.44	-1.58	-1.10	-1.17	-1.29	-1.44	-1.58	-1.10	-1.17	-1.29	-1.44	-1.58	-1.10	-1.17	-1.29	-1.44	-1.58	-0.05	-0.31	-0.50	-0.67	-0.83
	δ=0.25	-1.10	-1.17	-1.29	-1.44	-1.58	-1.10	-1.17	-1.29	-1.44	-1.58	-1.10	-1.17	-1.29	-1.44	-1.58	-1.10	-1.17	-1.29	-1.43	-1.61	-0.01	-0.23	-0.38	-0.51	-0.62
	δ=0.50	-1.10	-1.17	-1.29	-1.44	-1.58	-1.10	-1.17	-1.29	-1.44	-1.58	-1.10	-1.17	-1.29	-1.44	-1.58	-1.10	-1.17	-1.28	-1.50	-1.53	0.00	-0.13	-0.24	-0.33	-0.41
	δ=0.75	-1.10	-1.17	-1.29	-1.44	-1.58	-1.10	-1.17	-1.29	-1.44	-1.58	-1.10	-1.17	-1.29	-1.44	-1.58	-1.10	-1.50	-1.49	-1.46	-1.42	0.00	-0.06	-0.12	-0.17	-0.25
	δ=0.95	-1.10	-1.17	-1.29	-1.44	-1.58	-1.10	-1.17	-1.29	-1.44	-1.58	-1.10	-1.17	-1.29	-1.44	-1.58	-1.10	-1.79	-1.68	-1.52	-1.38	0.00	0.00	-0.05	-0.06	-0.10
β=0.75	δ=0.05	0.00	-0.16	-0.25	-0.31	-0.36	0.00	-0.16	-0.25	-0.31	-0.36	0.00	-0.16	-0.25	-0.31	-0.36	0.00	-0.16	-0.25	-0.31	-0.36	0.00	-0.05	-0.12	-0.18	-0.22
	δ=0.25	0.00	-0.16	-0.25	-0.31	-0.36	0.00	-0.16	-0.25	-0.31	-0.36	0.00	-0.16	-0.25	-0.31	-0.36	0.00	-0.16	-0.25	-0.31	-0.36	0.00	-0.03	-0.09	-0.13	-0.16
	δ=0.50	0.00	-0.16	-0.25	-0.31	-0.36	0.00	-0.16	-0.25	-0.31	-0.36	0.00	-0.16	-0.25	-0.31	-0.36	0.00	-0.16	-0.25	-0.31	-0.36	0.00	-0.01	-0.06	-0.09	-0.10
	δ=0.75	0.00	-0.16	-0.25	-0.31	-0.36	0.00	-0.16	-0.25	-0.31	-0.36	0.00	-0.16	-0.25	-0.31	-0.36	0.00	-0.16	-0.25	-0.31	-0.40	0.00	0.00	-0.03	-0.05	-0.05
	δ=0.95	0.00	-0.16	-0.25	-0.31	-0.36	0.00	-0.16	-0.25	-0.31	-0.36	0.00	-0.16	-0.25	-0.31	-0.36	0.00	-0.16	-0.25	-0.37	-0.39	0.00	0.00	-0.01	-0.02	-0.01
β=0.95	δ=0.05	0.00	0.00	-0.01	-0.01	-0.01	0.00	0.00	-0.01	-0.01	-0.01	0.00	0.00	-0.01	-0.01	-0.01	0.00	0.00	-0.01	-0.01	-0.01	0.00	0.00	-0.01	0.00	0.01
	δ=0.25	0.00	0.00	-0.01	-0.01	-0.01	0.00	0.00	-0.01	-0.01	-0.01	0.00	0.00	-0.01	-0.01	-0.01	0.00	0.00	-0.01	-0.01	-0.01	0.00	0.00	-0.01	0.00	0.01
	δ=0.50	0.00	0.00	-0.01	-0.01	-0.01	0.00	0.00	-0.01	-0.01	-0.01	0.00	0.00	-0.01	-0.01	-0.01	0.00	0.00	-0.01	-0.01	-0.01	0.00	0.00	0.00	0.00	0.02
	δ=0.75	0.00	0.00	-0.01	-0.01	-0.01	0.00	0.00	-0.01	-0.01	-0.01	0.00	0.00	-0.01	-0.01	-0.01	0.00	0.00	-0.01	-0.01	-0.01	0.00	0.00	0.00	0.01	0.03
	δ=0.95	0.00	0.00	-0.01	-0.01	-0.01	0.00	0.00	-0.01	-0.01	-0.01	0.00	0.00	-0.01	-0.01	-0.01	0.00	0.00	-0.01	-0.01	-0.01	0.00	0.00	0.00	0.01	0.03

(c) $f = 10$

Figure 5.4: Impact of f on consumer surplus, where the parameters are set to $s_n = 3, \upsilon = 3$, values from -15.00 to 2.48 are highlighted as a heatmap from white to black background, and values below -15.00 are shown with a white background

5.1.1 Downstream Competition Model Equilibria Comparison with Monopoly Model Equilibria

Similar behavior: Manufacturer profit and consumer surplus behave in the opposite directions. Low customer foresight results in high spare parts prices, high manufacturer profit and low consumer surplus.

High TCO_i area comparison with monopoly model: In the previous chapter, we had explained that the high TCO_i area has the same analytical results as the monopoly model. However, this does not mean that we will observe the same equilibrium decisions as in the monopoly model in these cases. The important factor here is that the monopoly model had only $p \geq 0$ boundary on the product price. The downstream competition model also has the limitation that this solution applies only when $p \geq v+\delta b-\beta f_e(s_n+\alpha l_i)$. When $v+\delta b-\beta f_e(s_n+\alpha l_i) \leq 0$, it is possible for the two models to achieve the same equilibria where $p^* = 0$. However, when this condition is not satisfied, the manufacturer needs $p > 0$, which then cascades down to the other two decision variables, ω_o and s_o, still resulting in the same analytical solution as the monopoly model, but practically shifting the equilibrium to a different point.[2] Thus, we observe the manufacturer intentionally setting a positive product price (along with adjusted spare part wholesale and retail prices) to keep the independent workshop out of the market.

No coverage comparison with monopoly model: Another interesting remark is that the downstream competition model actually produces a higher number of no coverage instances than the monopoly model. In other words, it is more likely that no one buys in the downstream competition model as compared to the monopoly model. Across the tested instances, all no coverage equilibria of the monopoly model (7,650 instances) also have no coverage in the downstream competition model. However, the monopoly model has partial coverage in the remaining 1,050 instances where the downstream model has no coverage. This is due to the manufacturer and the retailer not being able to increase the product and spare parts prices freely, due to the availability of the non-original alternative. In all such instances, we observe that

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[2] Checking the numerical results, we see that there are 4,376 such instances with $p > 0$ and hence different equilibria from the monopoly model (out of 55,249 instances with equilibria in the high TCO_i region) and the rest have identical equilibria to the monopoly model.

the equilibrium spare parts wholesale and retail prices of the downstream competition model are lower than those of the monopoly model. Reversely, the product price is positive, pushing the equilibrium to the monopoly model area. Because the positive product price reduces the customers' utility, the remaining positive component of the utility function isn't sufficiently high to allow a high-enough spare part retail price that would allow the retailer to make a profit.

Comparison of market coverages: When we compare market coverage across the tested instances, we see that 58,946 (31%) of them produce identical total market coverage to the monopoly model (52,883, 28%, of which are in the high TCO_i area); 83,993 (45%) have greater coverage, serving on average 30% more of the total market; the remaining 44,561 (24%) have less coverage, serving on average 12% less of the total market. Thus, existence of a competitor may end up decreasing the total volume of customers served.

5.2 Research Questions

We study the following research questions:

- Under what conditions does the manufacturer (and the retailer) monopolize the after-sales market? Alternatively, does the independent workshop ever monopolize the after-sales market?
- Does the manufacturer benefit from the existence of an independent workshop at all? Under what conditions? How about the consumer surplus? Is there any case where having competition in the after-sales market worsens / does not improve the consumer surplus? When do the interests of the consumer and the manufacturer align and when do they differ?
- What is the best warranty coverage decision from the manufacturer's perspective vs. the consumer's perspective?
- How does the manufacturer price the product and spare parts? As a result, what is the manufacturer's main source of profits: sales or after-sales? Does she accept losses on either side?

- How strong is double marginalization? How are the chain profits and manufacturer profits affected by the existence of the independent workshop?

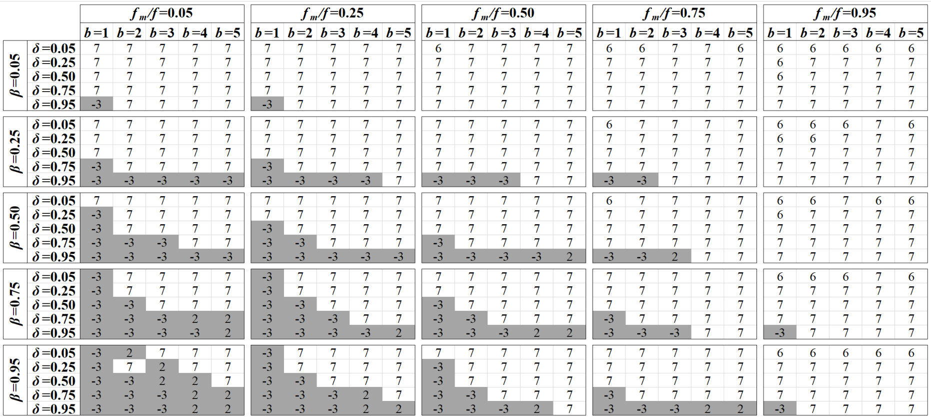
5.2.1 Aftermarket Monopolization

In this section, we study under what conditions the after-sales market is monopolized. As we have already stated before, our experimental design setting didn't produce any equilibria where the independent workshop monopolizes the after-sales market. Therefore, we focus on the manufacturer and hence the retailer's aftermarket monopolization. An important point is that the manufacturer drives the aftermarket monopolization by setting both the product price (p) and the spare part wholesale price (ω_o). By setting p, the manufacturer sets which TCO_i region the market will operate in. By setting ω_o with the knowledge that the retailer will maximize his profit, she then also determines exactly what market coverage region they will end up with. Therefore, we simply refer to the situation as *the manufacturer monopolizing the aftermarket*.

From Table B.4, we had already seen that the manufacturer ends up monopolizing the after-sales market for 55% of the tested instances (regions (-3), (-2), (-1), ⓪, ①, and ②), thereof 25% corresponding to the monopoly model equilibria (regions (-1) and ①). The manufacturer targets joint coverage of the after-sales market with the independent workshop for 40% of the tested instances where both original and non-original parts are sold (regions ⑥ and ⑦).

In Figures 5.5-5.7 we report how the manufacturer's aftermarket monopolization changes as the total number of repairs, f, spare parts price of the independent retailer, s_n, and customer's base valuation, v are changed. In parallel with the previous figures, we use $f = 5, s_n = 3, v = 3$ as the base case (subfigure (b)) and show the trend as each of these parameters are increased or decreased. For these three figures, the cells report the equilibrium region of the corresponding parameter combination, where the regions the manufacturer monopolizes the aftermarket are highlighted in grey and the parameter combinations that result in no market coverage (region ③) are highlighted in black.

From the figures, we make the following observations concerning the manufacturer's



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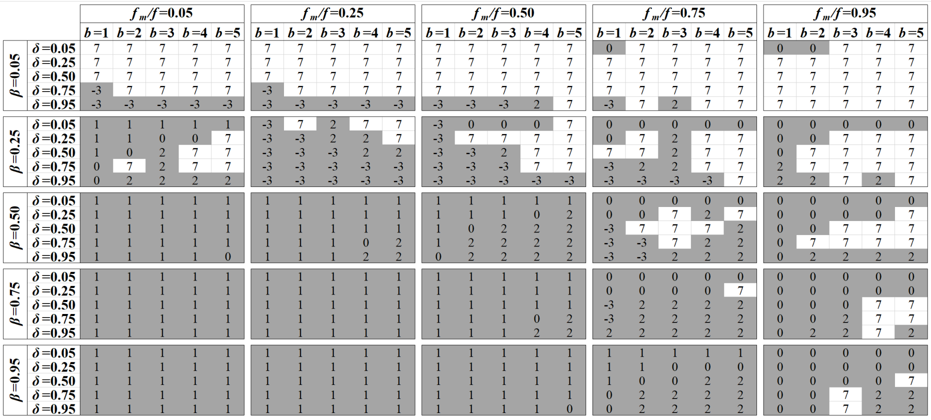
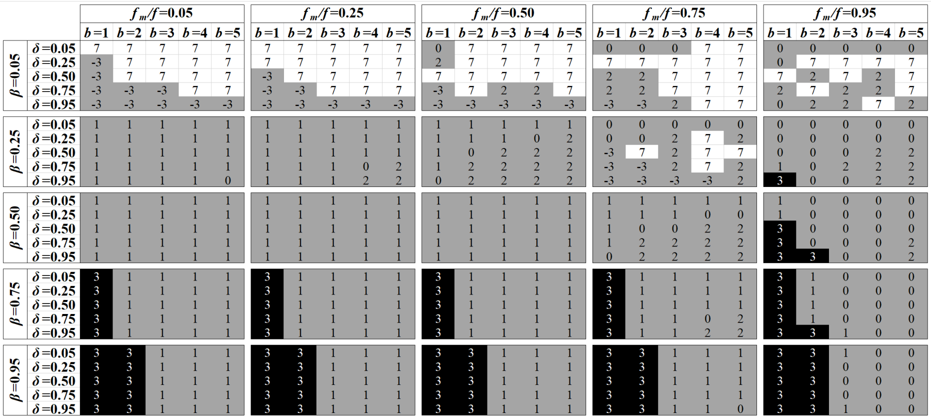
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	7	7	7	7	7	7	7	7	7	7	6	7	7	7	7	6	6	7	7	6	6	6	6	6	6
	δ=0.25	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	6	7	7	7	7
	δ=0.50	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	6	7	7	7	7
	δ=0.75	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
	δ=0.95	-3	7	7	7	7	-3	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
β=0.25	δ=0.05	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	6	7	7	7	7	6	6	6	7	6
	δ=0.25	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	6	6	7	7	7
	δ=0.50	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
	δ=0.75	-3	7	7	7	7	-3	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
	δ=0.95	-3	-3	-3	-3	-3	-3	-3	-3	-3	7	-3	-3	-3	7	7	-3	-3	7	7	7	7	7	7	7	7
β=0.50	δ=0.05	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	6	7	7	7	7	6	6	7	6	6
	δ=0.25	-3	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	6	7	7	7	7
	δ=0.50	-3	7	7	7	7	-3	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
	δ=0.75	-3	-3	-3	7	7	-3	-3	7	7	7	-3	7	7	7	7	7	7	7	7	7	7	7	7	7	7
	δ=0.95	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	2	-3	-3	2	7	7	7	7	7	7	7
β=0.75	δ=0.05	-3	7	7	7	7	-3	7	7	7	7	7	7	7	7	7	7	7	7	7	7	6	6	6	7	6
	δ=0.25	-3	7	7	7	7	-3	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
	δ=0.50	-3	-3	7	7	7	-3	-3	7	7	7	-3	7	7	7	7	7	7	7	7	7	7	7	7	7	7
	δ=0.75	-3	-3	-3	2	2	-3	-3	-3	7	7	-3	-3	7	7	7	-3	7	7	7	7	7	7	7	7	7
	δ=0.95	-3	-3	-3	-3	2	-3	-3	-3	-3	2	-3	-3	-3	2	2	-3	-3	-3	7	7	-3	7	7	7	7
β=0.95	δ=0.05	-3	2	7	7	7	-3	7	7	7	7	7	7	7	7	7	7	7	7	7	7	6	6	6	6	6
	δ=0.25	-3	7	2	7	7	-3	7	7	7	7	-3	7	7	7	7	7	7	7	7	7	7	7	7	7	7
	δ=0.50	-3	-3	2	2	7	-3	-3	7	7	7	-3	7	7	7	7	7	7	7	7	7	7	7	7	7	7
	δ=0.75	-3	-3	-3	2	2	-3	-3	-3	2	7	-3	-3	7	7	7	-3	7	7	7	7	7	7	7	7	7
	δ=0.95	-3	-3	-3	2	2	-3	-3	-3	2	2	-3	-3	-3	2	7	-3	-3	-3	2	2	-3	7	7	7	7

(a) $f = 1$



		f_m/f=0.05					f_m/f=0.25					f_m/f=0.50					f_m/f=0.75					f_m/f=0.95				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	0	7	7	7	7	0	0	7	7	7
	δ=0.25	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
	δ=0.50	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
	δ=0.75	-3	7	7	7	7	-3	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
	δ=0.95	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	2	7	-3	7	2	7	7	7	7	7	7	7
β=0.25	δ=0.05	1	1	1	1	1	-3	7	2	7	7	-3	0	0	0	7	0	0	0	0	0	0	0	0	0	0
	δ=0.25	1	1	0	0	7	-3	-3	2	2	7	-3	7	7	7	7	0	7	2	7	7	0	0	7	7	7
	δ=0.50	1	0	2	7	7	-3	-3	-3	2	2	-3	-3	2	7	7	7	7	2	7	7	0	7	7	7	7
	δ=0.75	0	7	2	7	7	-3	-3	-3	-3	-3	-3	-3	-3	7	7	-3	2	2	7	7	2	7	7	7	7
	δ=0.95	0	2	2	2	2	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	7	2	2	7	2	7
β=0.50	δ=0.05	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
	δ=0.25	1	1	1	1	1	1	1	1	1	1	1	1	1	0	2	0	0	7	2	7	0	0	0	0	7
	δ=0.50	1	1	1	1	1	1	1	1	1	1	1	0	2	2	2	-3	7	7	7	2	0	0	7	7	7
	δ=0.75	1	1	1	1	1	1	1	1	0	2	1	2	2	2	2	-3	-3	7	2	2	0	7	7	7	7
	δ=0.95	1	1	1	1	0	1	1	1	2	2	0	2	2	2	2	-3	-3	2	2	2	0	2	2	2	2
β=0.75	δ=0.05	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
	δ=0.25	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	7	0	0	0	0	0
	δ=0.50	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-3	2	2	2	2	0	0	0	7	7
	δ=0.75	1	1	1	1	1	1	1	1	1	1	1	1	1	0	2	-3	2	2	2	2	0	0	2	7	7
	δ=0.95	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	0	2	2	7	2
β=0.95	δ=0.05	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0
	δ=0.25	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0
	δ=0.50	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	2	2	0	0	0	0	7
	δ=0.75	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	2	2	2	2	0	0	7	2	2
	δ=0.95	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	2	2	2	2	0	0	7	2	2

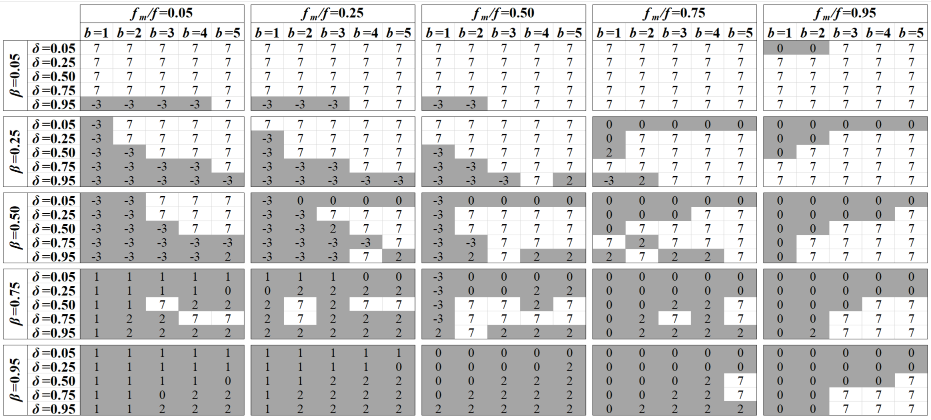
(b) $f = 5$



		f_m/f=0.05					f_m/f=0.25					f_m/f=0.50					f_m/f=0.75					f_m/f=0.95				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	7	7	7	7	7	7	7	7	7	7	0	7	7	7	7	0	0	0	7	7	0	0	0	0	0
	δ=0.25	-3	7	7	7	7	7	7	7	7	7	2	7	7	7	7	7	7	7	7	7	0	7	7	7	7
	δ=0.50	-3	7	7	7	7	-3	7	7	7	7	7	7	7	7	7	2	2	7	7	7	7	2	7	2	7
	δ=0.75	-3	-3	-3	7	7	-3	-3	7	7	7	-3	7	2	2	7	2	2	7	7	7	2	7	2	2	7
	δ=0.95	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	2	7	7	0	2	2	7	2
β=0.25	δ=0.05	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
	δ=0.25	1	1	1	1	1	1	1	1	1	1	1	1	1	0	2	0	0	2	7	2	0	0	0	0	0
	δ=0.50	1	1	1	1	1	1	1	1	1	1	1	0	2	2	2	-3	7	2	7	7	0	0	0	2	2
	δ=0.75	1	1	1	1	1	1	1	1	0	2	1	2	2	2	2	-3	-3	2	7	2	1	0	2	2	2
	δ=0.95	1	1	1	1	0	1	1	1	2	2	0	2	2	2	2	-3	-3	-3	-3	2	3	0	0	2	2
β=0.50	δ=0.05	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
	δ=0.25	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	0	0	0	0
	δ=0.50	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	2	2	3	0	0	0	0
	δ=0.75	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	3	0	0	0	2
	δ=0.95	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	2	2	2	2	3	3	0	0	2
β=0.75	δ=0.05	3	1	1	1	1	3	1	1	1	1	3	1	1	1	1	3	1	1	1	1	3	1	0	0	0
	δ=0.25	3	1	1	1	1	3	1	1	1	1	3	1	1	1	1	3	1	1	1	1	3	1	0	0	0
	δ=0.50	3	1	1	1	1	3	1	1	1	1	3	1	1	1	1	3	1	1	1	1	3	1	0	0	0
	δ=0.75	3	1	1	1	1	3	1	1	1	1	3	1	1	1	1	3	1	1	0	2	3	1	0	0	0
	δ=0.95	3	1	1	1	1	3	1	1	1	1	3	1	1	1	1	3	1	1	2	2	3	3	1	0	0
β=0.95	δ=0.05	3	3	1	1	1	3	3	1	1	1	3	3	1	1	1	3	3	1	1	1	3	3	1	0	0
	δ=0.25	3	3	1	1	1	3	3	1	1	1	3	3	1	1	1	3	3	1	1	1	3	3	0	0	0
	δ=0.50	3	3	1	1	1	3	3	1	1	1	3	3	1	1	1	3	3	1	1	1	3	3	0	0	0
	δ=0.75	3	3	1	1	1	3	3	1	1	1	3	3	1	1	1	3	3	1	1	1	3	3	0	0	0
	δ=0.95	3	3	1	1	1	3	3	1	1	1	3	3	1	1	1	3	3	1	1	0	3	3	1	0	0

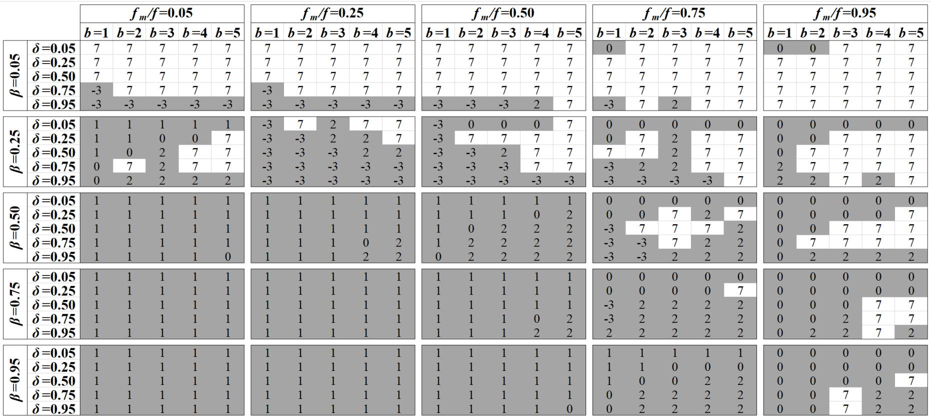
(c) $f = 10$

Figure 5.5: Impact of f on the manufacturer's aftermarket monopolization, where $s_n = 3, \upsilon = 3$, values show the equilibria regions where those with manufacturer's aftermarket monopolization are shown with a gray background, those with shared marked coverage with the independent workshop are shown with a white background, and those with no market coverage are shown with a black background



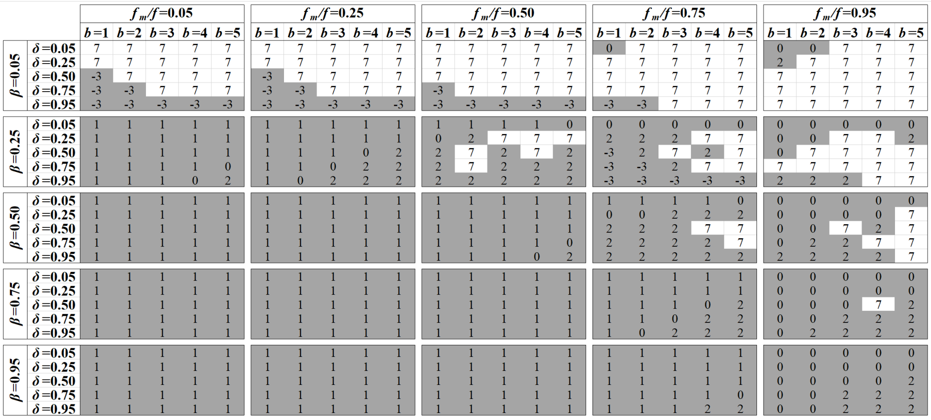
		f_m/f=0.05					f_m/f=0.25					f_m/f=0.50					f_m/f=0.75					f_m/f=0.95				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	0	0	7	7	7
	δ=0.25	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
	δ=0.50	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
	δ=0.75	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
	δ=0.95	-3	-3	-3	-3	7	-3	-3	-3	7	7	-3	-3	7	7	7	7	7	7	7	7	7	7	7	7	7
β=0.25	δ=0.05	-3	7	7	7	7	7	7	7	7	7	7	7	7	7	7	0	0	0	0	0	0	0	0	0	0
	δ=0.25	-3	7	7	7	7	-3	7	7	7	7	7	7	7	7	7	0	7	7	7	7	0	0	7	7	7
	δ=0.50	-3	-3	7	7	7	-3	7	7	7	7	-3	7	7	7	7	2	7	7	7	7	0	7	7	7	7
	δ=0.75	-3	-3	-3	-3	7	-3	-3	-3	7	7	-3	-3	7	7	7	7	7	7	7	7	7	7	7	7	7
	δ=0.95	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	7	2	-3	2	7	7	7	7	7	7	7	7
β=0.50	δ=0.05	-3	-3	7	7	7	-3	0	0	0	0	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	δ=0.25	-3	-3	7	7	7	-3	-3	7	7	7	-3	7	7	7	7	0	0	0	7	7	0	0	0	0	7
	δ=0.50	-3	-3	-3	7	7	-3	-3	2	7	7	-3	7	7	7	7	0	7	7	7	7	0	0	7	7	7
	δ=0.75	-3	-3	-3	-3	-3	-3	-3	-3	-3	7	-3	-3	7	7	7	7	2	7	7	7	0	7	7	7	7
	δ=0.95	-3	-3	-3	-3	2	-3	-3	-3	7	2	-3	2	7	2	2	2	7	2	2	7	0	7	7	7	7
β=0.75	δ=0.05	1	1	1	1	1	1	1	1	0	0	-3	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	δ=0.25	1	1	1	1	0	0	2	2	2	2	-3	0	0	2	2	0	0	0	0	0	0	0	0	0	0
	δ=0.50	1	1	7	2	2	2	7	2	7	7	-3	7	7	2	7	0	0	2	2	7	0	0	0	7	7
	δ=0.75	1	2	2	7	7	2	7	2	2	2	-3	7	7	7	7	0	2	7	2	7	0	0	7	7	7
	δ=0.95	1	2	2	2	2	2	2	2	2	2	2	7	2	2	2	0	2	2	2	2	0	2	7	7	7
β=0.95	δ=0.05	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	δ=0.25	1	1	1	1	1	1	1	1	1	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0
	δ=0.50	1	1	1	1	0	1	1	2	2	2	0	0	2	2	2	0	0	0	2	7	0	0	0	0	7
	δ=0.75	1	1	0	2	2	1	2	2	2	2	0	2	2	2	2	0	0	2	2	7	0	0	7	7	7
	δ=0.95	1	1	2	2	2	1	2	2	2	2	2	2	2	2	2	0	2	2	2	2	0	0	7	7	7

(a) $s_n = 1$



		f_m/f=0.05					f_m/f=0.25					f_m/f=0.50					f_m/f=0.75					f_m/f=0.95				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	0	7	7	7	7	0	0	7	7	7
	δ=0.25	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
	δ=0.50	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
	δ=0.75	-3	7	7	7	7	-3	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
	δ=0.95	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	2	7	-3	7	2	7	7	7	7	7	7	7
β=0.25	δ=0.05	1	1	1	1	1	-3	7	2	7	7	-3	0	0	0	7	0	0	0	0	0	0	0	0	0	0
	δ=0.25	1	1	0	0	7	-3	-3	2	2	7	-3	7	7	7	7	0	7	2	7	7	0	0	7	7	7
	δ=0.50	1	0	2	7	7	-3	-3	-3	2	2	-3	-3	2	7	7	7	7	2	7	7	0	7	7	7	7
	δ=0.75	0	7	2	7	7	-3	-3	-3	-3	-3	-3	-3	-3	7	7	-3	2	2	7	7	2	7	7	7	7
	δ=0.95	0	2	2	2	2	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	7	2	2	7	2	7
β=0.50	δ=0.05	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
	δ=0.25	1	1	1	1	1	1	1	1	1	1	1	1	1	0	2	0	0	7	2	7	0	0	0	0	7
	δ=0.50	1	1	1	1	1	1	1	1	1	1	1	0	2	2	2	-3	7	7	7	2	0	0	7	7	7
	δ=0.75	1	1	1	1	1	1	1	1	0	2	1	2	2	2	2	-3	-3	7	2	2	0	7	7	7	7
	δ=0.95	1	1	1	1	0	1	1	1	2	2	0	2	2	2	2	-3	-3	2	2	2	0	2	2	2	2
β=0.75	δ=0.05	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
	δ=0.25	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	7	0	0	0	0	0
	δ=0.50	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-3	2	2	2	2	0	0	0	7	7
	δ=0.75	1	1	1	1	1	1	1	1	1	1	1	1	1	0	2	-3	2	2	2	2	0	0	2	7	7
	δ=0.95	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	0	2	2	7	2
β=0.95	δ=0.05	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0
	δ=0.25	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0
	δ=0.50	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	2	2	0	0	0	0	7
	δ=0.75	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	2	2	2	2	0	0	7	2	2
	δ=0.95	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	2	2	2	2	0	0	7	2	2

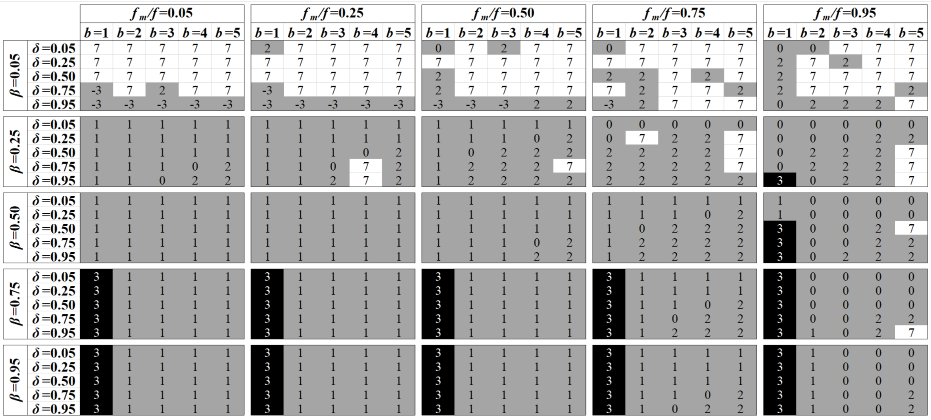
(b) $s_n = 3$



		f_m/f=0.05					f_m/f=0.25					f_m/f=0.50					f_m/f=0.75					f_m/f=0.95				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	0	7	7	7	7	0	0	7	7	7
	δ=0.25	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	2	7	7	7	7
	δ=0.50	-3	7	7	7	7	-3	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
	δ=0.75	-3	-3	7	7	7	-3	-3	7	7	7	-3	7	7	7	7	7	7	7	7	7	7	7	7	7	7
	δ=0.95	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	7	7	7	7	7	7	7	7
β=0.25	δ=0.05	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0
	δ=0.25	1	1	1	1	1	1	1	1	1	1	0	2	7	7	7	2	2	2	7	7	0	0	7	7	2
	δ=0.50	1	1	1	1	1	1	1	1	0	2	2	7	2	7	2	-3	2	7	2	7	0	7	7	7	7
	δ=0.75	1	1	1	1	0	1	1	0	2	2	2	7	2	2	2	-3	-3	2	7	7	7	7	7	7	7
	δ=0.95	1	1	1	0	2	1	0	2	2	2	2	2	2	2	2	-3	-3	-3	-3	-3	2	2	2	7	7
β=0.50	δ=0.05	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0
	δ=0.25	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	2	2	2	0	0	0	0	7
	δ=0.50	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	7	7	0	0	7	2	7
	δ=0.75	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	2	2	2	2	7	0	2	2	7	7
	δ=0.95	1	1	1	1	1	1	1	1	1	1	1	1	1	0	2	2	2	2	2	2	2	2	2	2	7
β=0.75	δ=0.05	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0
	δ=0.25	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0
	δ=0.50	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	2	0	0	0	7	2
	δ=0.75	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	2	2	0	0	2	2	2
	δ=0.95	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	2	2	2	0	2	2	2	2
β=0.95	δ=0.05	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0
	δ=0.25	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0
	δ=0.50	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	2
	δ=0.75	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	2	2	2
	δ=0.95	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	0	0	2	2	2

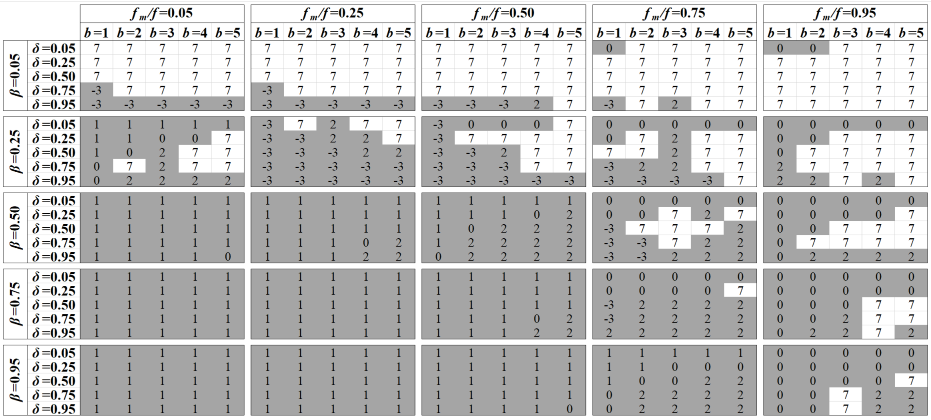
(c) $s_n = 5$

Figure 5.6: Impact of s_n on the manufacturer's aftermarket monopolization, where $f = 5, v = 3$, values show the equilibria regions where those with manufacturer's aftermarket monopolization are shown with a gray background, those with shared marked coverage with the independent workshop are shown with a white background, and those with no market coverage are shown with a black background



		f_m/f=0.05					f_m/f=0.25					f_m/f=0.50					f_m/f=0.75					f_m/f=0.95				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	7	7	7	7	7	2	7	7	7	7	0	7	2	7	7	0	7	7	7	7	0	0	7	7	7
	δ=0.25	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	2	7	2	7	7
	δ=0.50	7	7	7	7	7	7	7	7	7	7	2	7	7	7	7	2	2	7	2	7	2	7	7	7	7
	δ=0.75	-3	7	2	7	7	-3	7	7	7	7	2	7	7	7	7	7	2	7	7	2	2	7	7	7	2
	δ=0.95	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	2	2	-3	2	7	7	7	0	2	2	2	7
β=0.25	δ=0.05	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
	δ=0.25	1	1	1	1	1	1	1	1	1	1	1	1	1	0	2	0	7	2	2	7	0	0	0	2	2
	δ=0.50	1	1	1	1	1	1	1	1	0	2	1	0	2	2	2	2	2	2	2	7	0	2	2	2	7
	δ=0.75	1	1	1	0	2	1	1	0	7	2	1	2	2	2	7	2	2	2	2	7	0	2	2	2	7
	δ=0.95	1	1	0	2	2	1	1	2	7	2	1	2	2	2	2	2	2	2	2	2	3	0	2	2	7
β=0.50	δ=0.05	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
	δ=0.25	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	2	1	0	0	0	0
	δ=0.50	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	2	2	2	3	0	0	2	7
	δ=0.75	1	1	1	1	1	1	1	1	1	1	1	1	1	0	2	1	2	2	2	2	3	0	0	2	2
	δ=0.95	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	1	2	2	2	2	3	0	2	2	2
β=0.75	δ=0.05	3	1	1	1	1	3	1	1	1	1	3	1	1	1	1	3	1	1	1	1	3	0	0	0	0
	δ=0.25	3	1	1	1	1	3	1	1	1	1	3	1	1	1	1	3	1	1	1	1	3	0	0	0	0
	δ=0.50	3	1	1	1	1	3	1	1	1	1	3	1	1	1	1	3	1	1	0	2	3	0	0	0	0
	δ=0.75	3	1	1	1	1	3	1	1	1	1	3	1	1	1	1	3	1	0	2	2	3	0	0	2	2
	δ=0.95	3	1	1	1	1	3	1	1	1	1	3	1	1	1	1	3	1	2	2	2	3	1	0	2	7
β=0.95	δ=0.05	3	1	1	1	1	3	1	1	1	1	3	1	1	1	1	3	1	1	1	1	3	1	0	0	0
	δ=0.25	3	1	1	1	1	3	1	1	1	1	3	1	1	1	1	3	1	1	1	1	3	1	0	0	0
	δ=0.50	3	1	1	1	1	3	1	1	1	1	3	1	1	1	1	3	1	1	1	1	3	1	0	0	0
	δ=0.75	3	1	1	1	1	3	1	1	1	1	3	1	1	1	1	3	1	1	0	2	3	1	0	0	2
	δ=0.95	3	1	1	1	1	3	1	1	1	1	3	1	1	1	1	3	1	0	2	2	3	1	0	0	2

(a) $\upsilon = 1$



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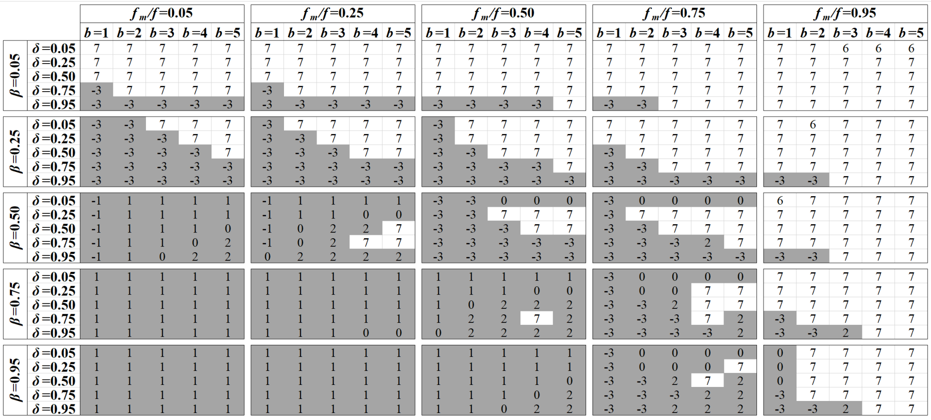
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	0	7	7	7	7	0	0	7	7	7
	δ=0.25	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
	δ=0.50	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
	δ=0.75	-3	7	7	7	7	-3	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
	δ=0.95	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	2	7	-3	7	2	7	7	7	7	7	7	7
β=0.25	δ=0.05	1	1	1	1	1	-3	7	2	7	7	-3	0	0	0	7	0	0	0	0	0	0	0	0	0	0
	δ=0.25	1	1	0	0	7	-3	-3	2	2	7	-3	7	7	7	7	0	7	2	7	7	0	0	7	7	7
	δ=0.50	1	0	2	7	7	-3	-3	-3	2	2	-3	-3	2	7	7	7	7	2	7	7	0	7	7	7	7
	δ=0.75	0	7	2	7	7	-3	-3	-3	-3	-3	-3	-3	-3	7	7	-3	2	2	7	7	2	7	7	7	7
	δ=0.95	0	2	2	2	2	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	7	2	2	7	2	7
β=0.50	δ=0.05	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
	δ=0.25	1	1	1	1	1	1	1	1	1	1	1	1	1	0	2	0	0	7	2	7	0	0	0	0	7
	δ=0.50	1	1	1	1	1	1	1	1	1	1	1	0	2	2	2	-3	7	7	7	2	0	0	7	7	7
	δ=0.75	1	1	1	1	1	1	1	1	0	2	1	2	2	2	2	-3	-3	7	2	2	0	7	7	7	7
	δ=0.95	1	1	1	1	0	1	1	1	2	2	0	2	2	2	2	-3	-3	2	2	2	0	2	2	2	2
β=0.75	δ=0.05	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
	δ=0.25	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	7	0	0	0	0	0
	δ=0.50	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-3	2	2	2	2	0	0	0	7	7
	δ=0.75	1	1	1	1	1	1	1	1	1	1	1	1	1	0	2	-3	2	2	2	2	0	0	2	7	7
	δ=0.95	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	0	2	2	7	2
β=0.95	δ=0.05	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0
	δ=0.25	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0
	δ=0.50	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	2	2	0	0	0	0	7
	δ=0.75	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	2	2	2	2	0	0	7	2	2
	δ=0.95	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	2	2	2	2	0	0	7	2	2

(b) $\upsilon = 3$



		f_m/f=0.05					f_m/f=0.25					f_m/f=0.50					f_m/f=0.75					f_m/f=0.95				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	6	6	6
	δ=0.25	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
	δ=0.50	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
	δ=0.75	-3	7	7	7	7	-3	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
	δ=0.95	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	7	-3	-3	7	7	7	7	7	7	7	7
β=0.25	δ=0.05	-3	-3	7	7	7	-3	7	7	7	7	-3	7	7	7	7	7	7	7	7	7	7	6	7	7	7
	δ=0.25	-3	-3	-3	7	7	-3	-3	7	7	7	-3	7	7	7	7	7	7	7	7	7	7	7	7	7	7
	δ=0.50	-3	-3	-3	-3	7	-3	-3	-3	7	7	-3	-3	7	7	7	-3	7	7	7	7	7	7	7	7	7
	δ=0.75	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	7	-3	-3	7	7	7	7	7	7	7	7
	δ=0.95	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	7	7	7
β=0.50	δ=0.05	-1	1	1	1	1	-1	1	1	1	1	-3	-3	0	0	0	-3	0	0	0	0	6	7	7	7	7
	δ=0.25	-1	1	1	1	1	-1	1	1	0	0	-3	-3	7	7	7	-3	7	7	7	7	7	7	7	7	7
	δ=0.50	-1	1	1	1	0	-1	0	2	2	7	-3	-3	-3	7	7	-3	-3	7	7	7	7	7	7	7	7
	δ=0.75	-1	1	1	0	2	-1	0	2	7	7	-3	-3	-3	-3	-3	-3	-3	-3	2	7	7	7	7	7	7
	δ=0.95	-1	1	0	2	2	0	2	2	2	2	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	-3	7	7	7
β=0.75	δ=0.05	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-3	0	0	0	0	7	7	7	7	7
	δ=0.25	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	-3	0	0	7	7	7	7	7	7	7
	δ=0.50	1	1	1	1	1	1	1	1	1	1	1	0	2	2	2	-3	-3	2	7	7	7	7	7	7	7
	δ=0.75	1	1	1	1	1	1	1	1	1	1	1	2	2	7	2	-3	-3	-3	7	2	-3	7	7	7	7
	δ=0.95	1	1	1	1	1	1	1	1	0	0	0	2	2	2	2	-3	-3	-3	-3	2	-3	-3	2	7	7
β=0.95	δ=0.05	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-3	0	0	0	0	0	7	7	7	7
	δ=0.25	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	-3	0	0	0	7	0	7	7	7	7
	δ=0.50	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	-3	-3	2	7	2	0	7	7	7	7
	δ=0.75	1	1	1	1	1	1	1	1	1	1	1	1	1	0	2	-3	-3	-3	2	2	-3	7	7	7	7
	δ=0.95	1	1	1	1	1	1	1	1	1	1	1	1	0	2	2	-3	-3	2	2	2	-3	-3	2	7	7

(c) $\upsilon = 5$

Figure 5.7: Impact of υ on the manufacturer’s aftermarket monopolization, where $f = 5, s_n = 3$, , values show the equilibria regions where those with manufacturer’s aftermarket monopolization are shown with a gray background, those with shared marked coverage with the independent workshop are shown with a white background, and those with no market coverage are shown with a black background

tendency to monopolize the aftermarket:

For lower values of β, the customer severely underestimates the after-sales costs, both for the retailer and the independent workshop. In this case, since the customer is already more inclined to buy, the manufacturer doesn't have further incentive to lower her wholesale price and drive the retailer to a lower spare part retail price (which would be needed for aftermarket monopolization). Instead, she uses this advantage of higher perceived utility by keeping both the retailer and the independent workshop in the game and serving a larger share of the market as compared to the monopoly model. For higher values of β, firstly, it is more likely that only the high TCO_i area is feasible for a nonnegative product price ($p > 0$), which then guarantees a monopoly result. Secondly, even if the medium and low TCO_i areas are feasible, the manufacturer no longer has the advantage of "hiding" the future costs. In this case, the independent workshop becomes a rival to the existing customer base, instead of helping to serve a larger customer base. Therefore, the manufacturer chooses to bring down the prices, driving the independent workshop out of the market and ensuring that she and the retailer keep all after-sales profits.

Higher total number of repairs (f) is also equivalent to higher number of repairs paid by the customer (f_e) for a given warranty coverage (f_m/f), which increases the customers' after-sales related spending and decreases their tendency to buy. Therefore, they become more sensitive to after-sales pricing while making their buying decision. In these cases, the manufacturer again cannot utilize the independent workshop to increase her customer base, so chooses to increase her aftermarket monopolization and rely on after-sales profits instead.

Decreased warranty coverage (f_m/f) also results in higher number of repairs paid by the customer (f_e) and has the same dynamics described above. However, it is much harder to see its impact, as a change in the fraction of a fixed number of repairs (f) isn't as visible and impactful as a more dramatic change directly in f. In the figures, we only see a slight tendency of decreased aftermarket monopolization for very high warranty coverage scenarios ($f_m/f = 0.95$).

When the non-original spare part price (s_n) is relatively expensive, the presence of the independent workshop does not aid the manufacturer in expanding her market.

Therefore, she has no use for keeping the independent workshop in the market and it is a better strategy for her to set her pricing accordingly, monopolizing the aftermarket.

Again, it is worthwhile to stress that the described effects of increased f_e and increased s_n make the threshold of the high TCO_i area smaller, pushing the model to a guaranteed monopoly result. This effect works the other way around for the positive components of the threshold (hence the positive components of the utility function); namely, the base valuation of the product's utility (v) and customer heterogeneity (b). When the base valuation (v) or the customer heterogeneity (b) decreases, this makes all customers less willing to buy, in turn, making both the product and the after-sales services relatively more expensive. In this case, again either the threshold for the high TCO_i area is pushed to the left, making the non-original alternative too expensive to consider for the customer and relying only on the manufacturer's pricing for affordable but monopolized after-sales services; or the independent services are still affordable (the low and medium TCO_i areas are feasible), but the manufacturer sees no additional market potential from the independent workshop and adjusts (lowers) her prices accordingly towards aftermarket monopolization.

It is also worth mentioning the no market coverage situations: In Figure 5.5(c), we see no market coverage (region ③) for combinations of high customer foresight ($\beta = 0.75, 0.95$) and low customer heterogeneity ($b = 1, 2$), as well as some instances with very high warranty coverage ($f_m/f = 0.95$), again combined with low customer heterogeneity ($b = 1, 2$). In Figure 5.7(a), we also see that very low v values combined with very low b values result in no coverage. In these cases, the manufacturer cannot make a profit with the price the customers are willing to pay, as she either does not have enough utility in the customer base that are willing to pay for a profit-making price for her, or she has to pay too much for warranty repairs (which again she cannot recover from the customers).

Overall, we can say that the manufacturer monopolizes the aftermarket essentially in two cases: (1) When the total cost of ownership of buying and using the after-sales services from the independent workshop is too expensive, (2) When the independent workshop doesn't contribute to increasing the manufacturer's market coverage, but instead would steal part of the customer base if/when given the opportunity.

Before finishing up this section, there is another question to answer: So, what happens when the aftermarket is monopolized? Is it good or bad for the manufacturer, the retailer, or the customer? Cross comparing aftermarket monopolization results (Figures 5.5-5.7) with manufacturer and retailer profits and consumer surplus (Figures 5.2-5.4 and B.1-B.6), we see that monopolization does not always work better for the manufacturer or the retailer, especially when TCO_i is too high. Less aftermarket monopolization is correlated with profits that are less sensitive to changes in parameters, ensuring a steady setup in case significant changes occur.

5.2.2 Model Preference Alignment between the Manufacturer and the Customer

In this section, we compare the results of the downstream competition model (C) studied in the last two chapters with the monopoly model (M) studied in the first part of the thesis. To investigate how the manufacturer's preferences align with the consumer welfare, we identify the preferred business model in terms of manufacturer profit and consumer surplus (CS) in Figures 5.8-5.10. Each zone is represented with two letters; the first letter (M or C) represents the model that produces the higher profit for the manufacturer, and the second letter (M or C) identifies the model that achieves the higher consumer surplus. For example, MC denotes that the manufacturer prefers the monopoly model whereas the downstream competition model produces a higher consumer surplus. For easier reading, the background of each result is highlighted: white for MC, dark gray for MM, light gray for CC, and black for CM. Also, regions without coverage in either model are denoted with "NoCov," again with a white background. When the downstream competition model produces an identical result to the monopoly model in regions 1 or -1, it is reported as an alignment on the monopoly model, MM, which accounts for most of the MM instances.[3]

We see that customers are very likely to prefer the downstream competition model (in 67% of the tested instances), except for the cases of equal results to the monopoly model (in 24% of the tested instances), or the rare cases of actual dominance by the

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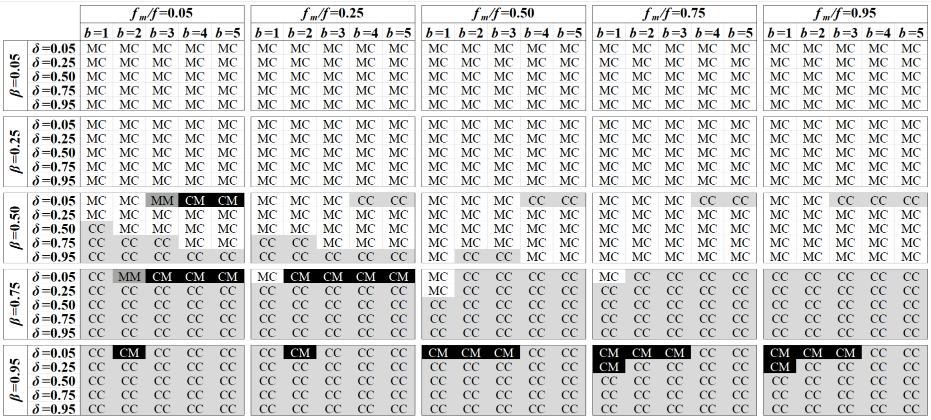
[3] For the interested reader, those MM instances that are *not* due to an identical result can be identified by comparing the MM instances within Figures 5.5-5.7 with the respective equilibrium regions reported within Figures 5.8-5.10.



		f_m/f=0.05					f_m/f=0.25					f_m/f=0.50					f_m/f=0.75					f_m/f=0.95				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.25	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.50	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.75	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.95	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
β=0.25	δ=0.05	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.25	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.50	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
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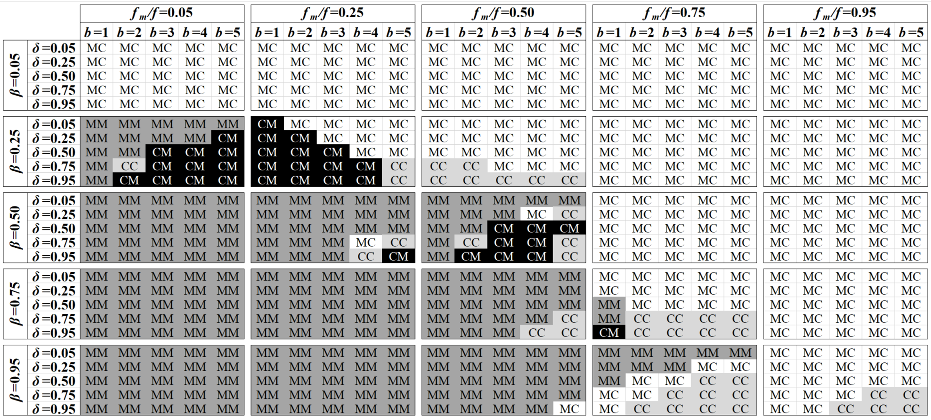
(a) $f = 1, s_n = 3, \upsilon = 3$



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	δ=0.75	MM	MM	MM	MM	MM	MM	MM	MM	MC	CC	MM	CC	CM	CM	CC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
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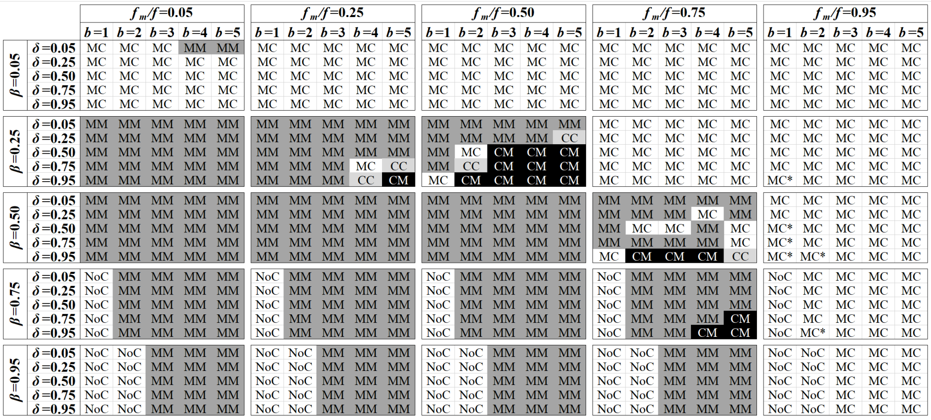
(b) $f = 5, s_n = 3, \upsilon = 3$



		f_m/f=0.05					f_m/f=0.25					f_m/f=0.50					f_m/f=0.75					f_m/f=0.95				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	MC	MC	MC	MM	MM	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.25	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.50	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.75	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.95	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
β=0.25	δ=0.05	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.25	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	CC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
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	δ=0.75	MM	MM	MM	MM	MM	MM	MM	MM	MC	CC	MM	CC	CM	CM	CM	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
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	δ=0.75	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MC	MC*	MC	MC	MC	MC
	δ=0.95	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MC	CM	CM	CM	CC	MC*	MC*	MC	MC	MC
β=0.75	δ=0.05	NoC	MM	MM	MM	MM	NoC	MM	MM	MM	MM	NoC	MM	MM	MM	MM	NoC	MM	MM	MM	MM	NoC	MC	MC	MC	MC
	δ=0.25	NoC	MM	MM	MM	MM	NoC	MM	MM	MM	MM	NoC	MM	MM	MM	MM	NoC	MM	MM	MM	MM	NoC	MC	MC	MC	MC
	δ=0.50	NoC	MM	MM	MM	MM	NoC	MM	MM	MM	MM	NoC	MM	MM	MM	MM	NoC	MM	MM	MM	MM	NoC	MC	MC	MC	MC
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β=0.95	δ=0.05	NoC	NoC	MM	MM	MM	NoC	NoC	MM	MM	MM	NoC	NoC	MM	MM	MM	NoC	NoC	MM	MM	MM	NoC	NoC	MC	MC	MC
	δ=0.25	NoC	NoC	MM	MM	MM	NoC	NoC	MM	MM	MM	NoC	NoC	MM	MM	MM	NoC	NoC	MM	MM	MM	NoC	NoC	MC	MC	MC
	δ=0.50	NoC	NoC	MM	MM	MM	NoC	NoC	MM	MM	MM	NoC	NoC	MM	MM	MM	NoC	NoC	MM	MM	MM	NoC	NoC	MC	MC	MC
	δ=0.75	NoC	NoC	MM	MM	MM	NoC	NoC	MM	MM	MM	NoC	NoC	MM	MM	MM	NoC	NoC	MM	MM	MM	NoC	NoC	MC	MC	MC
	δ=0.95	NoC	NoC	MM	MM	MM	NoC	NoC	MM	MM	MM	NoC	NoC	MM	MM	MM	NoC	NoC	MM	MM	MM	NoC	NoC	MC	MC	MC

(c) $f = 10, s_n = 3, \upsilon = 3$

Figure 5.8: Impact of f on the manufacturer's choice (first letter) and customers' choice (second letter) between the monopoly (M) and the downstream competition (C) models; MM, CC, MC, and CM are highlighted with a dark gray, light gray, white, and black background respectively; "NoC" and "MC*" indicate neither model and only the downstream competition model has no market coverage respectively

monopoly model (in 5% of the tested instances). There is no coverage in either model in the remaining 4% of the instances. Here, note that neither model has coverage in 7,650 instances, but there are 1,050 instances where the monopoly model has positive demand and the downstream competition model has no coverage (as indicated and discussed before). In all of these 1,050 instances, we have the consumer better off with no coverage in downstream competition model with zero consumer surplus (as compared to negative consumer surplus in the monopoly model), and the manufacturer better off with partial coverage and positive profit in the monopoly model (as compared to no coverage and zero profits in the downstream competition model). These few MC instances are marked with an asterisk, which are only observed in Figures 5.8(c) and 5.10(a) amongst the parameter sets we show here.

Reversely, the manufacturer more frequently prefers the monopoly model (in 55% of the tested instances, excluding the equal results in the above mentioned 24%). The manufacturer has higher profits in the downstream competition model for 17% of the tested instances.

In general, we see that the manufacturer and the consumer's interests are not aligned, the manufacturer profit being better with the monopoly model and the consumer surplus being better with the downstream competition model (MC), for lower values of consumer awareness (β) and higher values of warranty coverage (f_m/f). This difference is due to the downstream competition model driving the spare part prices down in order to be able to compete with the independent workshop, hence resulting in increased consumer surplus and decreased manufacturer profit. Instances that result in MC preference account for 54% of all tested instances.

Surrounded by the MC preference, we have alignment on monopoly model (MM) for the instances with high awareness of future costs and low warranty coverage, due to the two models M and C having the same equilibrium. As discussed in section 5.1, this is caused by TCO_i being too high (or, equivalently, the threshold on TCO_i being too low) that it doesn't allow the independent workshop to have a fighting chance at all, and forcing the model to produce equilibria only in the area equivalent to the monopoly model (regions (-1), ①, and ③). Instances that result in MM preference account for 25% of all tested instances, 24% of which arising from the two models producing the same result.

The customer and the manufacturer also show alignment on the competition model (CC), especially for the instances with high awareness of future costs, high warranty coverage, and high consumer heterogeneity. This preference alignment is observed in 13% of all tested instances. We can analyze these instances under two categories: The first category is the two regions where the retailer and the independent workshop are active in the aftermarket (region ⑥ with full market coverage and region ⑦ with partial market coverage). Here, the availability of an independent workshop expands the addressed market by creating a cheaper alternative, but not significantly reducing the original spare part purchases, hence creating a win-win scenario. The second category is comprised of two of the regions where the manufacturer monopolizes the aftermarket. The first of these two regions is the boundary between monopolized aftermarket and shared aftermarket with partial coverage in the medium TCO_i area (region ②), where the manufacturer applies a medium product price p and the retailer applies the maximum possible original spare part price s_o that still keeps the independent workshop at bay. The second one is the region with full market coverage and low product price p (region (-3)).

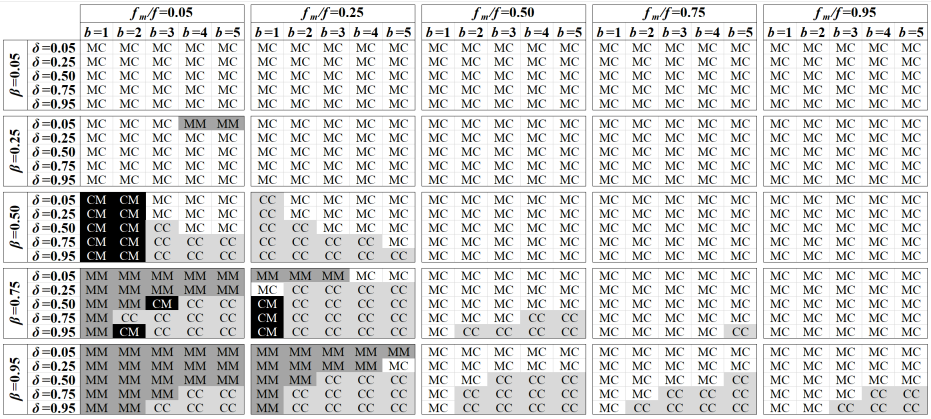
The manufacturer and the customer aligning on the competition model within these two regions with aftermarket monopolization (regions ② and (-3)) are very puzzling at first sight: If the manufacturer can have better profits while still monopolizing the market, why doesn't she already do so in the monopoly model? This is explained by the threat of the independent workshop impacting the retailer's pricing, which is then impacting the manufacturer's pricing. More frequently, it happens at the expense of the retailer: 12,092 instances have reduced retailer profits, out of 13,578 instances with alignment on the downstream competition model and having equilibria in these two regions (89%). However, the manufacturer is able to compensate for her losses and even further improves her profits through p or ω_o. The remaining 1,486 instances (11%) have all three parties (manufacturer, retailer, customer) improving their profit/surplus as compared to the monopoly model, while the manufacturer still monopolizing the aftermarket. One such example from Figure 5.8(b) is for the parameter set $\beta = 0.25$, $\delta = 0.75$, $f_m/f = 0.25$, $b = 5$. For the downstream competition model, we have $s_o^* = 2.875$, $\omega_o^* = 2.286$, $p* = 0.064$, $q_r^* = 1$, $\Pi_r^* = 2.833$, $\Pi_m^* = 5.950$,

$CS^* = -6.283$. For the monopoly model, we have $s_o^* = 6.350$, $\omega_o^* = 4.583$, $p* = 0$, $q_r^* = 0.36$, $\Pi_r^* = 2.628$, $\Pi_m^* = 5.256$, $CS^* = -6.400$. We see that the manufacturer is willing to reduce her unit profit from spare parts in the downstream competition model, in exchange for the lower retail price set by the retailer that results in wider market coverage in the downstream competition model. She also uses the opportunity of the low retailer price that is competing with the independent workshop to capture additional profit from the market by setting a positive product price.

Finally, we have the cases where the manufacturer profit is better with the competition model and the consumer surplus is better with the monopoly model (CM), which account for 3% of the total number of instances tested. Similar to CC cases, we again observe this set of preferences emerging for both of the cases when the independent workshop is also active in the aftermarket, and when the manufacturer monopolizes the aftermarket. The dynamics of why the competition model is working out better for the manufacturer is the same as the argument we have made for the CC cases. However, for the consumer surplus, the reduction in the spare parts retail price in the downstream competition model is offset by the resulting higher demand, which is disproportionately overstated especially when low awareness of future costs (β) is combined with low to medium warranty coverage (f_m/f).

In Figure 5.9, we see the impact of the non-original spare part price, s_n, on the manufacturer and customer's choice across the two models. Similar to the total number of repairs, f, we observe higher alignment on the competition model (CC) when the non-original spare parts are cheaper, and more monopolization of the aftermarket and identical results to the monopoly model as they are more expensive. Unlike in Figure 5.8, the CM cases are bundled more on different customer heterogeneity values (instead of service similarity values).

Figure 5.10 reports the results for different base valuation figures. Similar to our findings before, lower base valuation fosters monopolization and higher base valuation allows for more alignment on the competition model. Additionally, combination of lower β and higher f_m/f values result in the more common MC misalignment, where the manufacturer prefers the monopoly model and the customer prefers the downstream competition model.



		f_m/f=0.05					f_m/f=0.25					f_m/f=0.50					f_m/f=0.75					f_m/f=0.95				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
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	δ=0.75	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.95	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
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	δ=0.75	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.95	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
β=0.50	δ=0.05	CM	CM	MC	MC	MC	CC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
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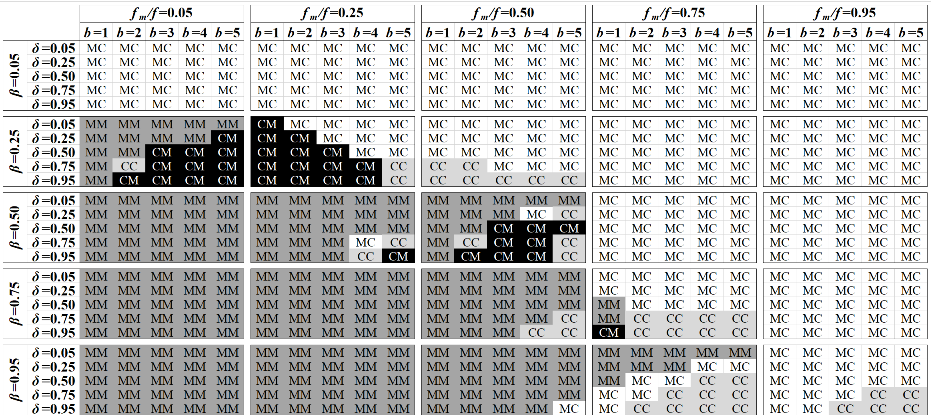
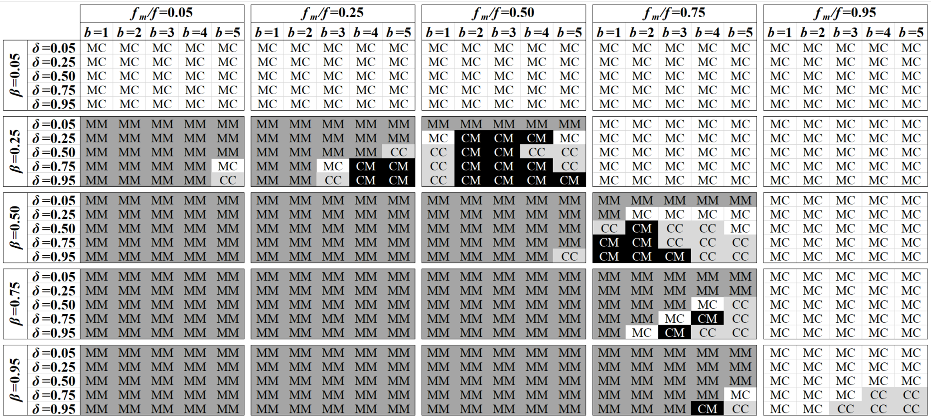
β=0.95	δ=0.05	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.25	MM	MM	MM	MM	MM	MM	MM	MM	MM	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.50	MM	MM	MM	MM	MM	MM	MM	CC	CC	CC	MC	MC	CC	CC	CC	MC	MC	MC	MC	CC	MC	MC	MC	MC	MC
	δ=0.75	MM	MM	MM	CC	CC	MM	CC	CC	CC	CC	MC	CC	CC	CC	CC	MC	MC	CC	CC	CC	MC	MC	MC	CC	CC
	δ=0.95	MM	MM	CC	CC	CC	MM	CC	CC	CC	CC	MC	CC	CC	CC	CC	MC	CC	CC	CC	CC	MC	MC	CC	CC	CC

(a) $s_n = 1, f = 5, \upsilon = 3$



		f_m/f=0.05					f_m/f=0.25					f_m/f=0.50					f_m/f=0.75					f_m/f=0.95				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.25	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.50	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.75	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.95	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
β=0.25	δ=0.05	MM	MM	MM	MM	MM	CM	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.25	MM	MM	MM	MM	CM	CM	CM	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.50	MM	MM	CM	CM	CM	CM	CM	CM	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.75	MM	CC	CM	CM	CM	CM	CM	CM	CM	CC	CC	CC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.95	MM	CM	CM	CM	CM	CM	CM	CM	CM	CC	CC	CC	CC	CC	CC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
β=0.50	δ=0.05	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.25	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MC	CC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.50	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	CM	CM	CM	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.75	MM	MM	MM	MM	MM	MM	MM	MM	MC	CC	MM	CC	CM	CM	CC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.95	MM	MM	MM	MM	MM	MM	MM	MM	CC	CM	MM	CM	CM	CM	CC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
β=0.75	δ=0.05	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.25	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.50	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.75	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	CC	MM	CC	CC	CC	CC	MC	MC	MC	MC	MC
	δ=0.95	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	CC	CC	CM	CC	CC	CC	CC	MC	MC	MC	MC	MC
β=0.95	δ=0.05	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MC	MC	MC	MC	MC
	δ=0.25	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MC	MC	MC	MC	MC	MC	MC
	δ=0.50	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MC	MC	CC	CC	MC	MC	MC	MC	MC
	δ=0.75	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MC	MC	CC	CC	CC	MC	MC	MC	CC	CC
	δ=0.95	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MC	MC	CC	CC	CC	CC	MC	MC	CC	CC	CC

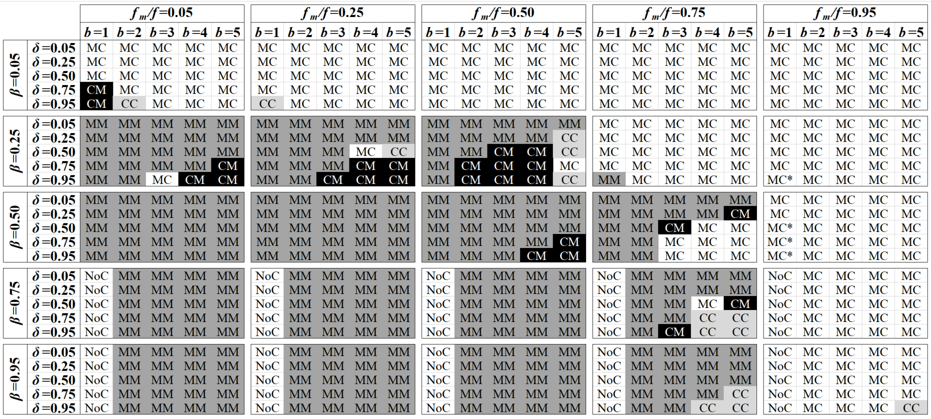
(b) $s_n = 3, f = 5, \upsilon = 3$



		f_m/f=0.05					f_m/f=0.25					f_m/f=0.50					f_m/f=0.75					f_m/f=0.95				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.25	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.50	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.75	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.95	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
β=0.25	δ=0.05	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.25	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MC	CM	CM	CM	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.50	MM	MM	MM	MM	MM	MM	MM	MM	MM	CC	CC	CM	CM	CC	CC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.75	MM	MM	MM	MM	MC	MM	MM	MC	CM	CM	CC	CM	CM	CM	CC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.95	MM	MM	MM	MM	CC	MM	MM	CC	CM	CM	CC	CM	CM	CM	CM	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
β=0.50	δ=0.05	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MC	MC	MC	MC	MC
	δ=0.25	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.50	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	CC	CM	CC	CC	MC	MC	MC	MC	MC	MC
	δ=0.75	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	CM	CM	CC	CC	CC	MC	MC	MC	MC	MC
	δ=0.95	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	CC	CM	CM	CM	CC	CC	MC	MC	MC	MC	MC
β=0.75	δ=0.05	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MC	MC	MC	MC	MC
	δ=0.25	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MC	MC	MC	MC	MC
	δ=0.50	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MC	CC	MC	MC	MC	MC	MC
	δ=0.75	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MC	CM	CC	MC	MC	MC	MC	MC
	δ=0.95	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MC	CM	CC	CC	MC	MC	MC	MC	MC
β=0.95	δ=0.05	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MC	MC	MC	MC	MC
	δ=0.25	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MC	MC	MC	MC	MC
	δ=0.50	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MC	MC	MC	MC	MC
	δ=0.75	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MC	MC	MC	MC	CC	CC
	δ=0.95	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	CM	CC	MC	MC	CC	CC	CC

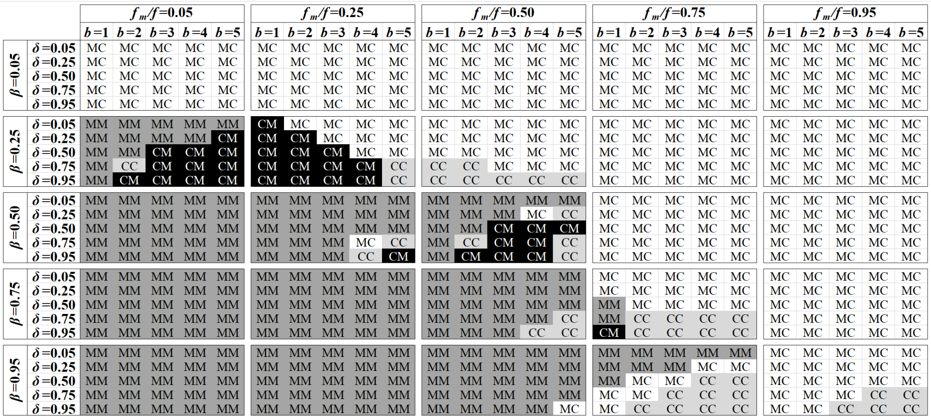
(c) $s_n = 5, f = 5, \upsilon = 3$

Figure 5.9: Impact of s_n on the manufacturer's choice (first letter) and customers' choice (second letter) between the monopoly (M) and the downstream competition (C) models; MM, CC, MC, and CM are highlighted with a dark gray, light gray, white, and black background respectively; "NoC" and "MC*" indicate neither model and only the downstream competition model has no market coverage respectively



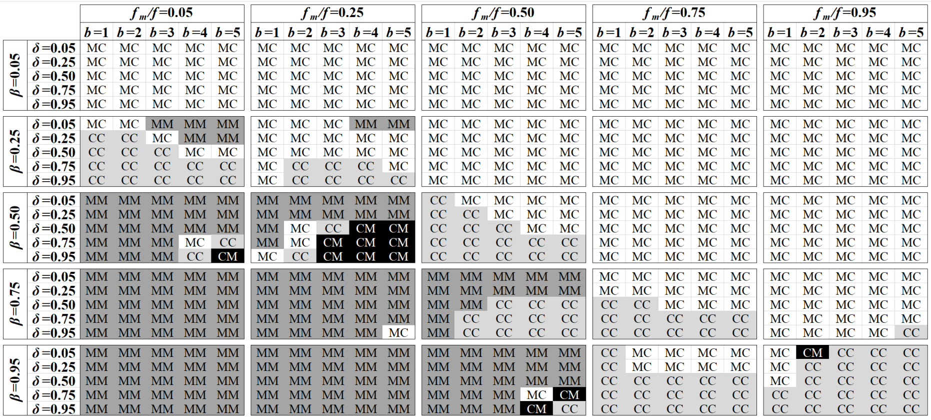
		f_m/f=0.05					f_m/f=0.25					f_m/f=0.50					f_m/f=0.75					f_m/f=0.95				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.25	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.50	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.75	CM	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.95	CM	CC	MC	MC	MC	CC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
β=0.25	δ=0.05	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.25	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	CC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.50	MM	MM	MM	MM	MM	MM	MM	MM	MC	CC	MM	MM	CM	CM	CC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.75	MM	MM	MM	MM	CM	MM	MM	MM	CM	CM	MM	CM	CM	CM	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.95	MM	MM	MC	CM	CM	MM	MM	CM	CM	CM	MM	CM	CM	CM	CC	MM	MC	MC	MC	MC	MC*	MC	MC	MC	MC
β=0.50	δ=0.05	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MC	MC	MC	MC	MC
	δ=0.25	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	CM	MC	MC	MC	MC	MC
	δ=0.50	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	CM	MC	MC	MC*	MC	MC	MC	MC
	δ=0.75	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	CM	MM	MM	MC	MC	MC	MC*	MC	MC	MC	MC
	δ=0.95	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	CM	CM	MM	MM	MC	MC	MC	MC*	MC	MC	MC	MC
β=0.75	δ=0.05	NoC	MM	MM	MM	MM	NoC	MM	MM	MM	MM	NoC	MM	MM	MM	MM	NoC	MM	MM	MM	MM	NoC	MC	MC	MC	MC
	δ=0.25	NoC	MM	MM	MM	MM	NoC	MM	MM	MM	MM	NoC	MM	MM	MM	MM	NoC	MM	MM	MM	MM	NoC	MC	MC	MC	MC
	δ=0.50	NoC	MM	MM	MM	MM	NoC	MM	MM	MM	MM	NoC	MM	MM	MM	MM	NoC	MM	MM	MC	CM	NoC	MC	MC	MC	MC
	δ=0.75	NoC	MM	MM	MM	MM	NoC	MM	MM	MM	MM	NoC	MM	MM	MM	MM	NoC	MM	MM	CC	CC	NoC	MC	MC	MC	MC
	δ=0.95	NoC	MM	MM	MM	MM	NoC	MM	MM	MM	MM	NoC	MM	MM	MM	MM	NoC	MM	CM	CC	CC	NoC	MC	MC	MC	MC
β=0.95	δ=0.05	NoC	MM	MM	MM	MM	NoC	MM	MM	MM	MM	NoC	MM	MM	MM	MM	NoC	MM	MM	MM	MM	NoC	MC	MC	MC	MC
	δ=0.25	NoC	MM	MM	MM	MM	NoC	MM	MM	MM	MM	NoC	MM	MM	MM	MM	NoC	MM	MM	MM	MM	NoC	MC	MC	MC	MC
	δ=0.50	NoC	MM	MM	MM	MM	NoC	MM	MM	MM	MM	NoC	MM	MM	MM	MM	NoC	MM	MM	MM	MM	NoC	MC	MC	MC	MC
	δ=0.75	NoC	MM	MM	MM	MM	NoC	MM	MM	MM	MM	NoC	MM	MM	MM	MM	NoC	MM	MM	MM	CC	NoC	MC	MC	MC	MC
	δ=0.95	NoC	MM	MM	MM	MM	NoC	MM	MM	MM	MM	NoC	MM	MM	MM	MM	NoC	MM	MM	CC	CC	NoC	MC	MC	MC	CC

(a) $\upsilon = 1, f = 5, s_n = 3$



		f_m/f=0.05					f_m/f=0.25					f_m/f=0.50					f_m/f=0.75					f_m/f=0.95				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.25	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.50	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.75	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.95	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
β=0.25	δ=0.05	MM	MM	MM	MM	MM	CM	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.25	MM	MM	MM	MM	CM	CM	CM	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.50	MM	MM	CM	CM	CM	CM	CM	CM	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.75	MM	CC	CM	CM	CM	CM	CM	CM	CM	CC	CC	CC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.95	MM	CM	CM	CM	CM	CM	CM	CM	CM	CC	CC	CC	CC	CC	CC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
β=0.50	δ=0.05	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.25	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MC	CC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.50	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	CM	CM	CM	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.75	MM	MM	MM	MM	MM	MM	MM	MM	MC	CC	MM	CC	CM	CM	CC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.95	MM	MM	MM	MM	MM	MM	MM	MM	CC	CM	MM	CM	CM	CM	CC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
β=0.75	δ=0.05	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.25	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.50	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.75	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	CC	MM	CC	CC	CC	CC	MC	MC	MC	MC	MC
	δ=0.95	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	CC	CC	CM	CC	CC	CC	CC	MC	MC	MC	MC	MC
β=0.95	δ=0.05	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MC	MC	MC	MC	MC
	δ=0.25	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MC	MC	MC	MC	MC	MC	MC
	δ=0.50	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MC	MC	CC	CC	MC	MC	MC	MC	MC
	δ=0.75	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MC	MC	CC	CC	CC	MC	MC	MC	CC	CC
	δ=0.95	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MC	MC	CC	CC	CC	CC	MC	MC	CC	CC	CC

(b) $\upsilon = 3, f = 5, s_n = 3$



		f_m/f=0.05					f_m/f=0.25					f_m/f=0.50					f_m/f=0.75					f_m/f=0.95				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.25	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.50	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.75	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.95	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
β=0.25	δ=0.05	MC	MC	MM	MM	MM	MC	MC	MC	MM	MM	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.25	CC	CC	MC	MM	MM	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.50	CC	CC	CC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.75	CC	CC	CC	CC	CC	MC	CC	CC	CC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.95	CC	CC	CC	CC	CC	MC	CC	CC	CC	CC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
β=0.50	δ=0.05	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	CC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.25	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	CC	CC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.50	MM	MM	MM	MM	MM	MM	MC	CC	CM	CM	CC	CC	CC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.75	MM	MM	MM	MC	CC	MM	MC	CM	CM	CM	CC	CC	CC	CC	CC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.95	MM	MM	MM	CC	CM	MC	CC	CM	CM	CM	CC	CC	CC	CC	CC	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
β=0.75	δ=0.05	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.25	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MC	MC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.50	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	CC	CC	CC	CC	CC	MC	MC	MC	MC	MC	MC	MC	MC
	δ=0.75	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	CC	CC	CC	CC	CC	CC	CC	CC	CC	MC	MC	MC	MC	MC
	δ=0.95	MM	MM	MM	MM	MM	MM	MM	MM	MM	MC	MM	CC	CC	CC	CC	CC	CC	CC	CC	CC	MC	MC	MC	MC	CC
β=0.95	δ=0.05	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	CC	MC	MC	MC	MC	MC	CM	CC	CC	CC
	δ=0.25	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	CC	MC	MC	MC	MC	MC	CC	CC	CC	CC
	δ=0.50	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	CC	CC	CC	CC	CC	MC	CC	CC	CC	CC
	δ=0.75	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MC	CM	CC	CC	CC	CC	CC	CC	CC	CC	CC	CC
	δ=0.95	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	MM	CM	CC	CC	CC	CC	CC	CC	CC	CC	CC	CC	CC

(c) $\upsilon = 5, f = 5, s_n = 3$

Figure 5.10: Impact of υ on the manufacturer's choice (first letter) and customers' choice (second letter) between the monopoly (M) and the downstream competition (C) models; MM, CC, MC, and CM are highlighted with a dark gray, light gray, white, and black background respectively; "NoC" and "MC*" indicate neither model and only the downstream competition model has no market coverage respectively

Overall, we can say that a market environment with low and medium TCO_i brings out the advantages and disadvantages unique to the downstream competition model, as higher TCO_i values (such as with medium to high customer awareness and low to medium warranty coverage) force the equilibria to the monopoly model and the availability of the independent workshop alternative doesn't make a difference; but the low and medium TCO_i are the settings where the customers can still consider the independent workshop as a feasible alternative. A bigger proportion of these simply have misalignment with the manufacturer better off with the monopoly model and the customer better off with the downstream competition model (MC), but also allowing for alignment on the competition model (CC). Although it would be quite valuable to identify, it is somewhat hard to generalize the environment description which results in alignment on the competition model (CC). One thing we can say is that they are usually seen right next to the border of MM alignments, where the we don't have to operate within the high TCO_i area anymore, but the TCO_i isn't threateningly low for the manufacturer yet, which steals customers and/or forces the manufacturer to compete with too low after-sales pricing.

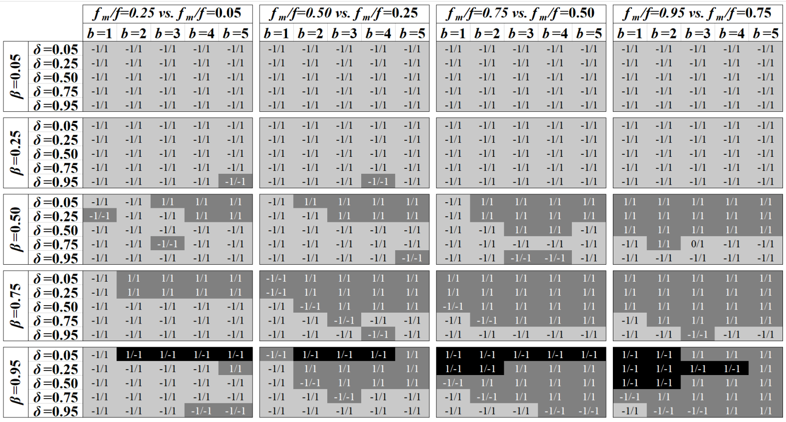
5.2.3 Warranty Decision

In this section, we look into how different levels of warranty coverage affect manufacturer profit and consumer surplus and whether a more comprehensive coverage throughout the lifetime of the product (i.e., greater f_m/f) is actually better for the customer or not. In Section 5.1, we had already observed visually from the reported figures that the manufacturer and the customer were not indifferent to warranty coverage. In this section, we compare the consumer surplus and manufacturer profit difference of each parameter combination with the next higher value of f_m/f. Figures 5.11-5.13 show whether the manufacturer profit and the consumer surplus improve or worsen when compared with the next higher warranty coverage instance. The first number indicates the direction of change for the manufacturer profit, and the second number indicates the direction of change for the consumer surplus, where 1 shows an improvement, -1 shows a worsening, and 0 shows equal value (rounded to the third digit). For example, a 1 under the column "$f_m/f = 0.25$ vs. $f_m/f = 0.05$" means that the corresponding profit function or consumer surplus value is higher for

$f_m/f = 0.25$, as compared to the profit function or consumer value for $f_m/f = 0.05$, all other parameters being unchanged. The figures are highlighted for easier reading where 0/0 are shown in white; 0/1 and -1/1 are shown in light gray; 0/-1, -1/-1, and 1/1 are shown in dark gray; and 1/-1 are shown in black.

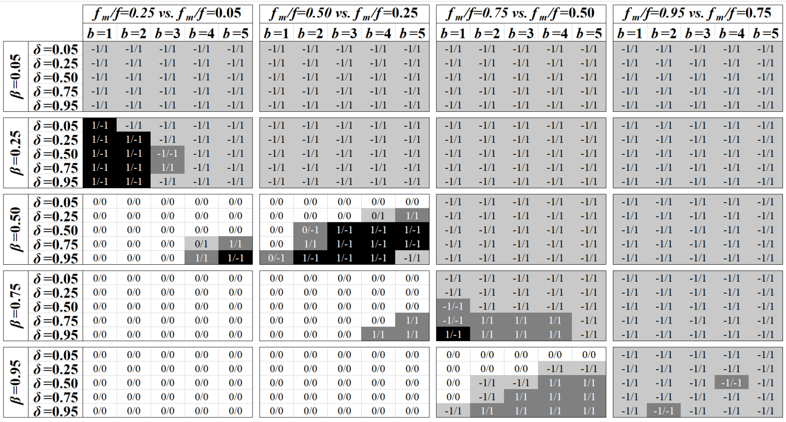
More frequently than not, we observe that a more comprehensive warranty coverage is actually better for the customer and worse for the manufacturer: As f_m/f increases, consumer surplus increases, and manufacturer profit decreases. This is especially true for very low customer foresight ($\beta = 0.05$). With such a low foresight, customers tend to overlook the future costs and more customers who normally don't gain enough utility to buy the product if they had a more accurate view of future costs end up buying the product, hence the demand is overstated. When more customers buy anyways, it's then better for the consumer surplus that some of these costs are covered by the manufacturer. This is actually an intuitive result and the interesting point is why this wasn't the case for the monopoly model. Why does the independent workshop offering an alternative makes a longer warranty period desirable for the customer? They key lies in the pricing. In both models, the customer pays $p+f_e(s_o+\alpha l_r)$ throughout the lifetime of the product. The finding for the monopoly model was that the manufacturer adds any warranty expense to her pricing, so both the manufacturer and the customer are indifferent whether there is less or more warranty coverage. In the downstream competition model, there are two additional factors: (1) Non-original spare parts create a benchmark for original spare part pricing, making the pricing of s_o also dependent on s_n, so the manufacturer cannot always freely increase s_o to reflect the warranty expenses fully to the customer. (2) Some customers alternatively pay $p + f_e(s_n + \alpha l_r)$ throughout the lifetime of the product. As the manufacturer can only impact p in this sum, a lower f_e likely reduces what some customers pay (if the manufacturer cannot fully extract the difference from the product price only), increasing consumer surplus and decreasing the manufacturer profit.

Let us look into one such example of increasing consumer surplus and decreasing manufacturer profit as the warranty coverage increases in detail. Table 5.3 provides the equilibria and relevant key figures of increasing warranty coverage settings from 5% to 95%. As the warranty coverage increases, both s_o and p increase. We also report $f_e(s_o+\alpha l_r)$ to show the lifetime payment of original after-sales services per each



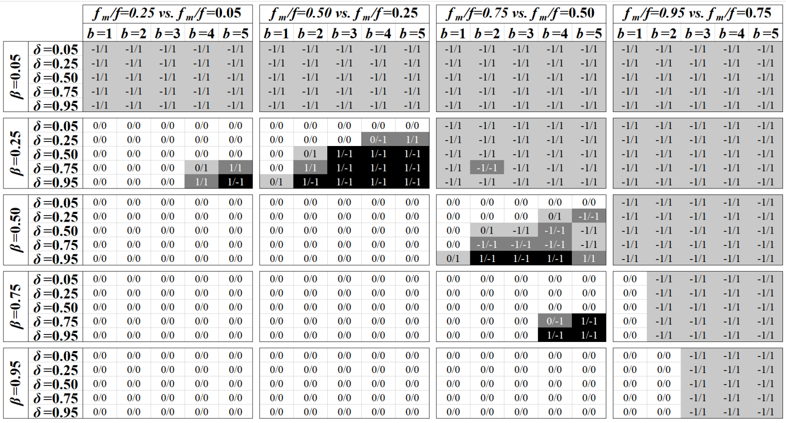
		f_m/f=0.25 vs. f_m/f=0.05					f_m/f=0.50 vs. f_m/f=0.25					f_m/f=0.75 vs. f_m/f=0.50					f_m/f=0.95 vs. f_m/f=0.75				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.25	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.50	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.75	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.95	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
β=0.25	δ=0.05	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.25	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.50	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.75	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.95	-1/1	-1/1	-1/1	-1/1	-1/-1	-1/1	-1/1	-1/1	-1/-1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
β=0.50	δ=0.05	-1/1	-1/1	1/1	1/1	1/1	-1/1	1/1	1/1	1/1	1/1	-1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1
	δ=0.25	-1/-1	-1/1	-1/1	1/1	1/1	-1/1	-1/1	1/1	1/1	1/1	-1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1
	δ=0.50	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	1/1	1/1	-1/1	1/1	1/1	1/1	1/1	1/1
	δ=0.75	-1/1	-1/1	-1/-1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	1/1	0/1	-1/1	-1/1
	δ=0.95	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/-1	-1/1	-1/1	-1/-1	-1/-1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
β=0.75	δ=0.05	-1/1	1/1	1/1	1/1	1/1	-1/-1	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1
	δ=0.25	-1/1	1/1	1/1	1/1	1/1	-1/-1	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1
	δ=0.50	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/-1	1/1	1/1	1/1	-1/-1	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1	1/1
	δ=0.75	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/-1	-1/1	-1/1	-1/1	-1/-1	1/1	1/1	1/1	-1/1	1/1	1/1	1/1	1/1
	δ=0.95	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/-1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/-1	-1/1	-1/1
β=0.95	δ=0.05	-1/1	1/-1	1/-1	1/-1	1/-1	-1/-1	1/-1	1/-1	1/-1	1/1	1/-1	1/-1	1/-1	1/-1	1/-1	1/-1	1/-1	1/1	1/1	1/1
	δ=0.25	-1/1	-1/1	-1/1	-1/1	1/1	-1/1	1/1	1/1	1/1	1/1	1/-1	1/-1	1/1	1/1	1/1	1/-1	1/-1	1/-1	1/-1	1/1
	δ=0.50	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/-1	1/1	1/1	1/1	-1/-1	1/1	1/1	1/1	1/1	1/-1	1/-1	1/1	1/1	1/1
	δ=0.75	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/-1	-1/1	-1/1	-1/1	-1/-1	1/1	1/1	1/1	-1/-1	1/1	1/1	1/1	1/1
	δ=0.95	-1/1	-1/1	-1/1	-1/-1	-1/-1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/-1	-1/-1	-1/1	-1/-1	-1/-1	1/1	1/1

(a) $f = 1, s_n = 3, \upsilon = 3$



		f_m/f=0.25 vs. f_m/f=0.05					f_m/f=0.50 vs. f_m/f=0.25					f_m/f=0.75 vs. f_m/f=0.50					f_m/f=0.95 vs. f_m/f=0.75				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.25	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.50	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.75	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.95	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
β=0.25	δ=0.05	1/-1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.25	1/-1	1/-1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.50	1/-1	1/-1	-1/-1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.75	1/-1	1/-1	1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.95	1/-1	1/-1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
β=0.50	δ=0.05	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.25	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/1	1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.50	0/0	0/0	0/0	0/0	0/0	0/0	0/-1	1/-1	1/-1	1/-1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.75	0/0	0/0	0/0	0/1	1/1	0/0	1/1	1/-1	1/-1	1/-1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.95	0/0	0/0	0/0	1/1	1/-1	0/-1	1/-1	1/-1	1/-1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
β=0.75	δ=0.05	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.25	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.50	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	-1/-1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.75	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	1/1	-1/-1	1/1	1/1	1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.95	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	1/1	1/1	1/-1	1/1	1/1	1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
β=0.95	δ=0.05	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.25	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.50	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	-1/1	-1/1	1/1	1/1	-1/1	-1/1	-1/1	-1/-1	-1/1
	δ=0.75	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	-1/1	1/1	1/1	1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.95	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	-1/1	1/1	1/1	1/1	1/1	-1/1	-1/-1	-1/1	-1/1	-1/1

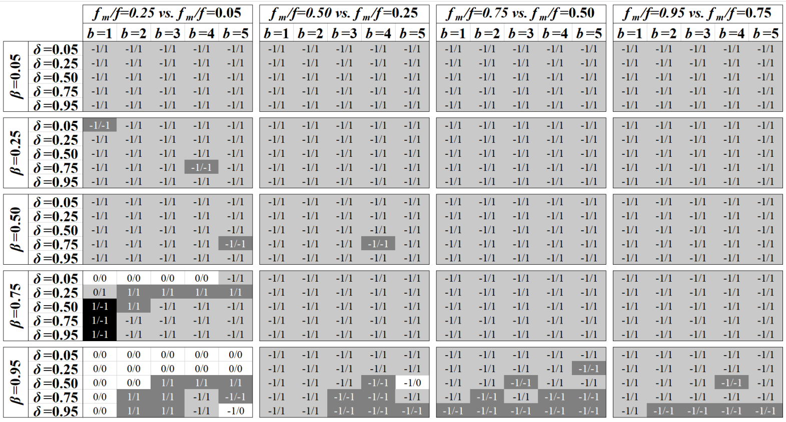
(b) $f = 5, s_n = 3, \upsilon = 3$



		f_m/f=0.25 vs. f_m/f=0.05					f_m/f=0.50 vs. f_m/f=0.25					f_m/f=0.75 vs. f_m/f=0.50					f_m/f=0.95 vs. f_m/f=0.75				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.25	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.50	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.75	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.95	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
β=0.25	δ=0.05	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.25	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/-1	1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.50	0/0	0/0	0/0	0/0	0/0	0/0	0/1	1/-1	1/-1	1/-1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.75	0/0	0/0	0/0	0/1	1/1	0/0	1/1	1/-1	1/-1	1/-1	-1/1	-1/-1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.95	0/0	0/0	0/0	1/1	1/-1	0/1	1/-1	1/-1	1/-1	1/-1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
β=0.50	δ=0.05	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.25	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/1	-1/-1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.50	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/1	-1/1	-1/-1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.75	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	-1/-1	-1/-1	-1/-1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.95	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/1	1/-1	1/-1	1/-1	1/1	-1/1	-1/1	-1/1	-1/1	-1/1
β=0.75	δ=0.05	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	-1/1	-1/1	-1/1	-1/1
	δ=0.25	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	-1/1	-1/1	-1/1	-1/1
	δ=0.50	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	-1/1	-1/1	-1/1	-1/1
	δ=0.75	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/-1	1/-1	0/0	-1/1	-1/1	-1/1	-1/1
	δ=0.95	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	1/-1	1/-1	0/0	-1/1	-1/1	-1/1	-1/1
β=0.95	δ=0.05	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	-1/1	-1/1	-1/1
	δ=0.25	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	-1/1	-1/1	-1/1
	δ=0.50	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	-1/1	-1/1	-1/1
	δ=0.75	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	-1/1	-1/1	-1/1
	δ=0.95	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	-1/1	-1/1	-1/1

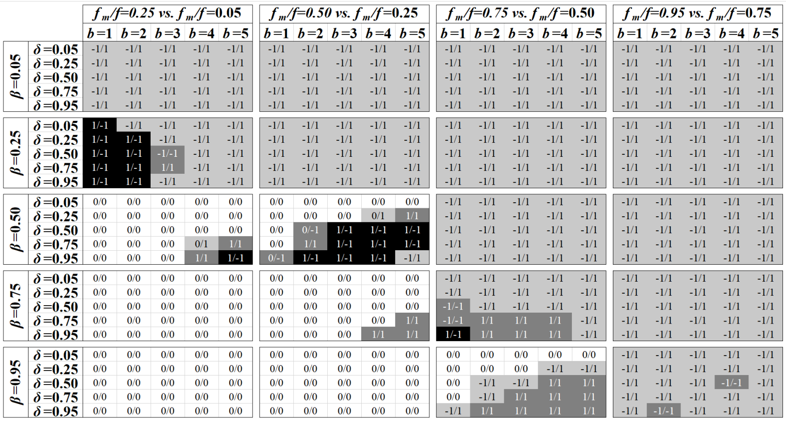
(c) $f = 10, s_n = 3, \upsilon = 3$

Figure 5.11: Impact of f on the manufacturer profit (first number) and the consumer surplus (second number) as f_m/f is increased; where 1, 0, and -1 indicate improvement, equality, and worsening respectively; 0/0 are shown in white; 0/1 and -1/1 are shown in light gray; 0/-1, -1/-1, and 1/1 are shown in dark gray; and 1/-1 are shown in black



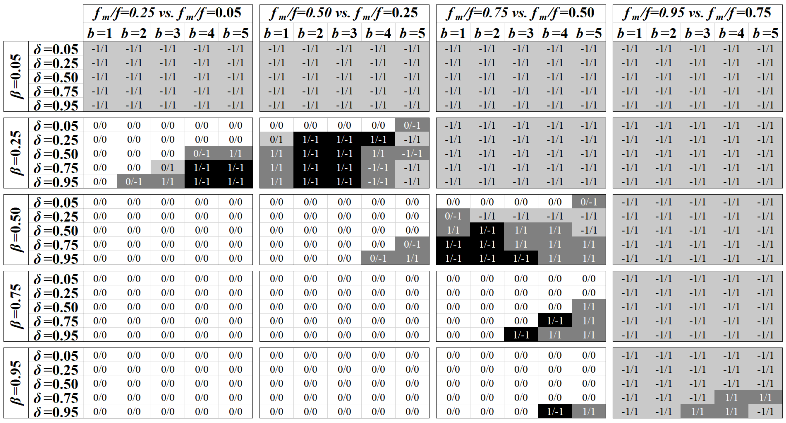
		f_m/f=0.25 vs. f_m/f=0.05					f_m/f=0.50 vs. f_m/f=0.25					f_m/f=0.75 vs. f_m/f=0.50					f_m/f=0.95 vs. f_m/f=0.75				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.25	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.50	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.75	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.95	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
β=0.25	δ=0.05	-1/-1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.25	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.50	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.75	-1/1	-1/1	-1/1	-1/-1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.95	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
β=0.50	δ=0.05	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.25	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.50	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.75	-1/1	-1/1	-1/1	-1/1	-1/-1	-1/1	-1/1	-1/1	-1/-1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.95	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
β=0.75	δ=0.05	0/0	0/0	0/0	0/0	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.25	0/1	1/1	1/1	1/1	1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.50	1/-1	1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.75	1/-1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.95	1/-1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
β=0.95	δ=0.05	0/0	0/0	0/0	0/0	0/0	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.25	0/0	0/0	0/0	0/0	0/0	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/-1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.50	0/0	0/0	1/1	1/1	1/1	-1/1	-1/1	-1/1	-1/-1	-1/0	-1/1	-1/1	-1/-1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/-1	-1/1
	δ=0.75	0/0	1/1	1/1	-1/1	-1/-1	-1/1	-1/1	-1/-1	-1/-1	-1/1	-1/1	-1/-1	-1/1	-1/-1	-1/-1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.95	0/0	1/1	1/1	-1/1	-1/0	-1/1	-1/1	-1/-1	-1/-1	-1/-1	-1/-1	-1/-1	-1/-1	-1/-1	-1/-1	-1/1	-1/-1	-1/-1	-1/-1	-1/-1

(a) $s_n = 1, f = 5, \upsilon = 3$



		f_m/f=0.25 vs. f_m/f=0.05					f_m/f=0.50 vs. f_m/f=0.25					f_m/f=0.75 vs. f_m/f=0.50					f_m/f=0.95 vs. f_m/f=0.75				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.25	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.50	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.75	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.95	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
β=0.25	δ=0.05	1/-1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.25	1/-1	1/-1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.50	1/-1	1/-1	-1/-1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.75	1/-1	1/-1	1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.95	1/-1	1/-1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
β=0.50	δ=0.05	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.25	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/1	1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.50	0/0	0/0	0/0	0/0	0/0	0/0	0/-1	1/-1	1/-1	1/-1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.75	0/0	0/0	0/0	0/1	1/1	0/0	1/1	1/-1	1/-1	1/-1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.95	0/0	0/0	0/0	1/1	1/-1	0/-1	1/-1	1/-1	1/-1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
β=0.75	δ=0.05	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.25	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.50	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	-1/-1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.75	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	1/1	-1/-1	1/1	1/1	1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.95	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	1/1	1/1	1/-1	1/1	1/1	1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
β=0.95	δ=0.05	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.25	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.50	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	-1/1	-1/1	1/1	1/1	-1/1	-1/1	-1/1	-1/-1	-1/1
	δ=0.75	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	-1/1	1/1	1/1	1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.95	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	-1/1	1/1	1/1	1/1	1/1	-1/1	-1/-1	-1/1	-1/1	-1/1

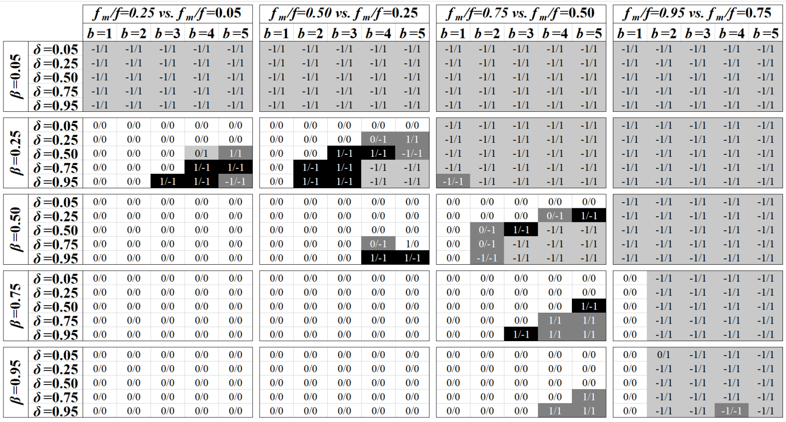
(b) $s_n = 3, f = 5, \upsilon = 3$



		f_m/f=0.25 vs. f_m/f=0.05					f_m/f=0.50 vs. f_m/f=0.25					f_m/f=0.75 vs. f_m/f=0.50					f_m/f=0.95 vs. f_m/f=0.75				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.25	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.50	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.75	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.95	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
β=0.25	δ=0.05	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/-1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.25	0/0	0/0	0/0	0/0	0/0	0/1	1/-1	1/-1	1/-1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.50	0/0	0/0	0/0	0/-1	1/1	1/1	1/-1	1/-1	1/1	-1/-1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.75	0/0	0/0	0/1	1/-1	1/-1	1/1	1/-1	1/-1	-1/-1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.95	0/0	0/-1	1/1	1/-1	1/-1	1/1	1/-1	1/-1	-1/-1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
β=0.50	δ=0.05	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/-1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.25	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/-1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.50	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	1/1	1/-1	1/1	1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.75	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/-1	1/-1	1/-1	1/1	1/1	1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.95	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/-1	1/1	1/-1	1/-1	1/-1	1/1	1/1	-1/1	-1/1	-1/1	-1/1	-1/1
β=0.75	δ=0.05	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.25	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.50	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.75	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	1/-1	1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.95	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	1/-1	1/1	1/1	-1/1	-1/1	-1/1	-1/1	-1/1
β=0.95	δ=0.05	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.25	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.50	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.75	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	-1/1	-1/1	-1/1	1/1	1/1
	δ=0.95	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	1/-1	1/1	-1/1	-1/1	1/1	1/1	-1/1

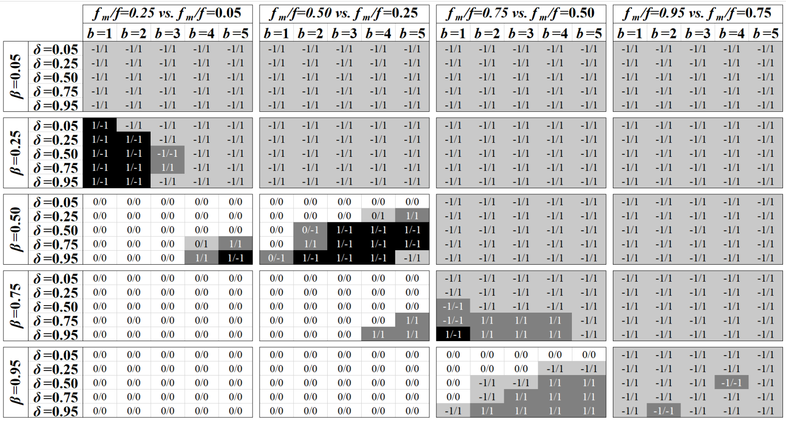
(c) $s_n = 5, f = 5, \upsilon = 3$

Figure 5.12: Impact of s_n on the manufacturer profit (first number) and the consumer surplus (second number) as f_m/f is increased; where 1, 0, and -1 indicate improvement, equality, and worsening respectively; 0/0 are shown in white; 0/1 and -1/1 are shown in light gray; 0/-1, -1/-1, and 1/1 are shown in dark gray; and 1/-1 are shown in black



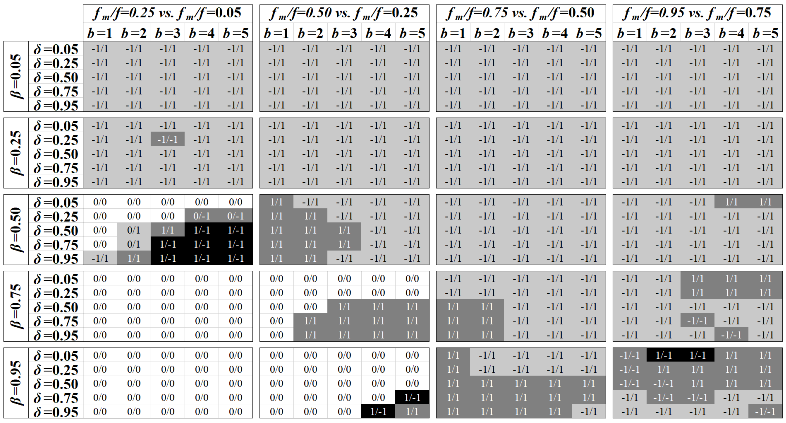
		f_m/f=0.25 vs. f_m/f=0.05					f_m/f=0.50 vs. f_m/f=0.25					f_m/f=0.75 vs. f_m/f=0.50					f_m/f=0.95 vs. f_m/f=0.75				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.25	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.50	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.75	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.95	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
β=0.25	δ=0.05	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.25	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/-1	1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.50	0/0	0/0	0/0	0/1	1/1	0/0	0/0	1/-1	1/-1	-1/-1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.75	0/0	0/0	0/0	1/-1	1/-1	0/0	1/-1	1/-1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.95	0/0	0/0	1/-1	1/-1	-1/-1	0/0	1/-1	1/-1	-1/1	-1/1	-1/-1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
β=0.50	δ=0.05	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.25	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/-1	1/-1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.50	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/-1	1/-1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.75	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/-1	1/0	0/0	0/-1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.95	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	1/-1	1/-1	0/0	-1/-1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
β=0.75	δ=0.05	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	-1/1	-1/1	-1/1	-1/1
	δ=0.25	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	-1/1	-1/1	-1/1	-1/1
	δ=0.50	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	1/-1	0/0	-1/1	-1/1	-1/1	-1/1
	δ=0.75	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	1/1	1/1	0/0	-1/1	-1/1	-1/1	-1/1
	δ=0.95	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	1/-1	1/1	1/1	0/0	-1/1	-1/1	-1/1	-1/1
β=0.95	δ=0.05	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/1	-1/1	-1/1	-1/1
	δ=0.25	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	-1/1	-1/1	-1/1	-1/1
	δ=0.50	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	-1/1	-1/1	-1/1	-1/1
	δ=0.75	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	1/1	0/0	-1/1	-1/1	-1/1	-1/1
	δ=0.95	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	1/1	1/1	0/0	-1/1	-1/1	-1/-1	-1/1

(a) $\upsilon = 1, f = 5, s_n = 3$



		f_m/f=0.25 vs. f_m/f=0.05					f_m/f=0.50 vs. f_m/f=0.25					f_m/f=0.75 vs. f_m/f=0.50					f_m/f=0.95 vs. f_m/f=0.75				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.25	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.50	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.75	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.95	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
β=0.25	δ=0.05	1/-1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.25	1/-1	1/-1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.50	1/-1	1/-1	-1/-1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.75	1/-1	1/-1	1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.95	1/-1	1/-1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
β=0.50	δ=0.05	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.25	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/1	1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.50	0/0	0/0	0/0	0/0	0/0	0/0	0/-1	1/-1	1/-1	1/-1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.75	0/0	0/0	0/0	0/1	1/1	0/0	1/1	1/-1	1/-1	1/-1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.95	0/0	0/0	0/0	1/1	1/-1	0/-1	1/-1	1/-1	1/-1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
β=0.75	δ=0.05	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.25	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.50	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	-1/-1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.75	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	1/1	-1/-1	1/1	1/1	1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.95	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	1/1	1/1	1/-1	1/1	1/1	1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
β=0.95	δ=0.05	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.25	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.50	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	-1/1	-1/1	1/1	1/1	-1/1	-1/1	-1/1	-1/-1	-1/1
	δ=0.75	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	-1/1	1/1	1/1	1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.95	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	-1/1	1/1	1/1	1/1	1/1	-1/1	-1/-1	-1/1	-1/1	-1/1

(b) $\upsilon = 3, f = 5, s_n = 3$



		f_m/f=0.25 vs. f_m/f=0.05					f_m/f=0.50 vs. f_m/f=0.25					f_m/f=0.75 vs. f_m/f=0.50					f_m/f=0.95 vs. f_m/f=0.75				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.25	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.50	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.75	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.95	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
β=0.25	δ=0.05	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.25	-1/1	-1/1	-1/-1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.50	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.75	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.95	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
β=0.50	δ=0.05	0/0	0/0	0/0	0/0	0/0	1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	1/1	1/1
	δ=0.25	0/0	0/0	0/0	0/-1	0/-1	1/1	1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.50	0/0	0/1	1/1	1/-1	1/-1	1/1	1/1	1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.75	0/0	0/1	1/-1	1/-1	1/-1	1/1	1/1	1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.95	-1/1	1/1	1/-1	1/-1	1/-1	1/1	1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
β=0.75	δ=0.05	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	1/1	1/1	1/1
	δ=0.25	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	1/1	1/1	1/1
	δ=0.50	0/0	0/0	0/0	0/0	0/0	0/0	0/0	1/1	1/1	1/1	1/1	1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1
	δ=0.75	0/0	0/0	0/0	0/0	0/0	0/0	1/1	1/1	1/1	1/1	1/1	1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/-1	-1/1	-1/1
	δ=0.95	0/0	0/0	0/0	0/0	0/0	0/0	1/1	1/1	1/1	1/1	1/1	1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/-1	-1/1
β=0.95	δ=0.05	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	1/1	-1/1	-1/1	-1/1	-1/1	-1/-1	1/-1	1/-1	1/1	1/1
	δ=0.25	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	1/1	-1/1	-1/1	-1/1	-1/1	-1/-1	1/1	1/1	1/1	1/1
	δ=0.50	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	1/1	1/1	1/1	1/1	1/1	-1/-1	-1/-1	1/1	1/1	1/1
	δ=0.75	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	1/-1	1/1	1/1	1/1	1/1	1/1	-1/1	-1/-1	-1/-1	-1/1	-1/1
	δ=0.95	0/0	0/0	0/0	0/0	0/0	0/0	0/0	0/0	1/-1	1/1	1/1	1/1	1/1	1/1	-1/1	-1/1	-1/1	-1/1	-1/1	-1/-1

(c) $\upsilon = 5, f = 5, s_n = 3$

Figure 5.13: Impact of υ on the manufacturer profit (first number) and the consumer surplus (second number) as f_m/f is increased; where 1, 0, and -1 indicate improvement, equality, and worsening respectively; 0/0 are shown in white; 0/1 and -1/1 are shown in light gray; 0/-1, -1/-1, and 1/1 are shown in dark gray; and 1/-1 are shown in black

Table 5.3: Comparison of different warranty coverage cases for the parameter set $\beta = 0.05$, $\delta = 0.5$, $v = 3$, $b = 3$, $s_n = 1$, $f = 5$, $\alpha = 0.5$, $c_o = 0.375$, $l_r = 0.5$, $c_r = 0.25$, $\omega_n = 0$, $l_i = 0.25$, $c_i = 0$, $c = 0.5$ from Figure 5.12(a).

f_m/f	0.05	0.25	0.50	0.75	0.95
Equilibrium region	7	7	7	7	7
s_o^*	5.453	6.720	9.721	18.732	90.789
ω_o^*	3.840	4.690	6.692	12.714	60.827
p^*	2.736	2.79	3.21	3.636	3.978
$f_e(s_o^* + \alpha l_r)$	27.088	26.137	24.927	23.728	22.760
$p^* + f_e(s_o^* + \alpha l_r)$	29.824	28.927	28.137	27.364	26.738
$p^*/(p^* + f_e(s_o^* + \alpha l_r))(\%)$	9%	10%	11%	13%	15%
$p^* + \beta f_e(s_o^* + \alpha l_r)$	4.090	4.097	4.456	4.822	5.116
$f_e \omega_o^*$	18.238	17.587	16.730	15.893	15.207
$f_e(s_n^* + \alpha l_i)$	5.344	4.219	2.813	1.406	0.281
$p^* + f_e(s_n^* + \alpha l_i)$	8.080	7.009	6.022	5.042	4.259
$p^*/(p^* + f_e(s_n^* + \alpha l_i))(\%)$	34%	40%	53%	72%	93%
$p^* + \beta f_e(s_n^* + \alpha l_i)$	3.003	3.001	3.351	3.706	3.992
q_i^*	0.723	0.730	0.503	0.273	0.088
q_r^*	0.275	0.269	0.263	0.256	0.251
Π_r^*	2.303	2.333	2.312	2.213	2.087
Π_m^*	6.604	5.867	5.030	4.367	3.962
CS^*	-9.947	-8.811	-7.077	-5.875	-5.311
Direction	n.a.	-1/1	-1/1	-1/1	-1/1
$\Pi_m^*/\Pi_{total}^*(\%)$	74%	72%	69%	66%	65%
$\Pi_{m(sales)}^*$	2.231	2.289	2.077	1.659	1.178
$\Pi_{m(after-sales)}^*$	4.373	3.578	2.954	2.708	2.784
$\Pi_{m(sales)}^*/\Pi_m^*(\%)$	34%	39%	41%	38%	30%

customer who chooses the retailer for non-warranty repairs. Here, we see that what the customer is actually paying over the product's lifetime is decreasing, despite the increase in unit prices. However, due to low awareness of future costs, the customers mistakenly perceive that the total lifetime cost is increasing $(p^* + \beta f_e(s_o^* + \alpha l_r))$. Due to this higher perception of the total lifetime cost, the retailer's demand decreases. The independent workshop's demand decreases even more, because when the customers have low awareness, they cannot fully appreciate the lower lifetime costs that come with higher warranty coverage. Increasing warranty coverage worsens the manufacturer's absolute profit (from 6.604 to 3.962), as well as her share in the total chain profit (from 74% to 65%); whereas the consumer surplus is improved (from -9.947 to -5.311).

From Figures 5.11-5.13, we see it is also possible that the manufacturer and the customer are simultaneously better or worse off with a higher warranty coverage (1/1 and -1/-1, highlighted in dark grey). We observe these alignments are more likely for higher consumer awareness of future costs (β), lower total number of repairs (f), lower non-original spare parts price (s_n), and higher base valuation of the product (v). In Table 5.4, we display another example where we observe an alignment for higher warranty coverage. In this example, we see that the directional changes in the spare part retail and wholesale prices and the product price don't change as compared to Table 5.3: All unit prices increase as the warranty coverage increases. However, unlike the previous example, we see that the independent workshop is increasing his market share as warranty coverage increases. This then works both to the manufacturer's and the consumer's advantage, the increase in market coverage with low prices is good for the customer, and translates into higher product-sales-related profits for the manufacturer. Therefore, we can say that a market environment with high transparency of future costs (high β) and more affordable after-sales services (low f, low s_n, high v) is more likely to generate a better result for both the manufacturer and the customer as the warranty coverage increases.

Table 5.4: Comparison of different warranty coverage cases for the parameter set $\beta = 0.95$, $\delta = 0.5$, $v = 3$, $b = 3$, $s_n = 3$, $f = 1$, $\alpha = 0.5$, $c_o = 0.375$, $l_r = 0.5$, $c_r = 0.25$, $\omega_n = 0$, $l_i = 0.25$, $c_i = 0$, $c = 0.5$ from Figure 5.11(a).

f_m/f	0.05	0.25	0.50	0.75	0.95
Equilibrium region	2	7	7	7	7
s_o^*	3.613	3.811	4.587	6.954	25.910
ω_o^*	2.815	2.767	3.267	4.843	17.491
p^*	0.846	1.44	1.914	2.364	2.856
$f_e(s_o^* + \alpha l_r)$	3.670	3.046	2.419	1.801	1.308
$p^* + f_e(s_o^* + \alpha l_r)$	4.516	4.486	4.333	4.165	4.164
$p^*/(p^* + f_e(s_o^* + \alpha l_r))(\%)$	19%	32%	44%	57%	69%
$p^* + \beta f_e(s_o^* + \alpha l_r)$	4.333	4.333	4.212	4.075	4.099
$f_e\omega_o^*$	2.674	2.075	1.634	1.211	0.875
$f_e(s_n^* + \alpha l_i)$	2.969	2.344	1.563	0.781	0.156
$p^* + f_e(s_n^* + \alpha l_i)$	3.815	3.784	3.477	3.145	3.012
$p^*/(p^* + f_e(s_n^* + \alpha l_i))(\%)$	22%	38%	55%	75%	95%
$p^* + \beta f_e(s_n^* + \alpha l_i)$	3.666	3.667	3.398	3.106	3.004
q_i^*	0.000	0.000	0.277	0.575	0.726
q_r^*	0.556	0.555	0.458	0.354	0.271
Π_r^*	0.491	0.504	0.377	0.285	0.234
Π_m^*	1.463	1.432	1.471	1.692	1.989
CS^*	0.361	0.378	0.485	0.687	0.777
Direction	n.a.	-1/1	1/1	1/1	1/1
$\Pi_m^*/\Pi_{total}^*(\%)$	75%	74%	80%	86%	89%
$\Pi_{m(sales)}^*$	0.192	0.522	1.038	1.732	2.349
$\Pi_{m(after-sales)}^*$	1.271	0.909	0.432	-0.040	-0.360
$\Pi_{m(sales)}^*/\Pi_m^*(\%)$	13%	36%	71%	102%	118%

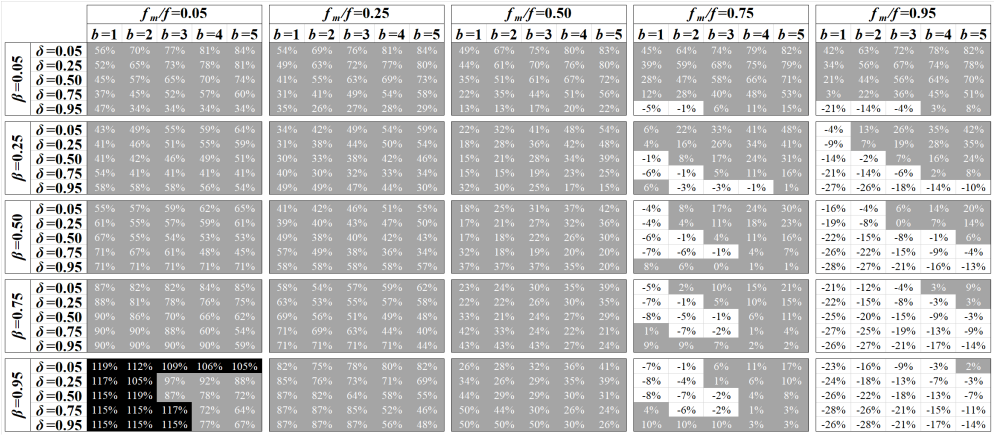
5.2.4 Manufacturer's Pricing and Profit Distribution

In this section, we check the percent of profit that the manufacturer makes from after-sales services (i.e., $(f_m(\alpha l_r + c_o))\, q^C(p, s_o) + f_e(\omega_o - c_o)q_r^C(p, s_o))$ as compared to her total profit. The results are reported in Figures 5.14-5.16. For easier reading, values below 0% are shown with a white background, values between 0% and 100% are shown with a gray background, values greater than 100% are shown with a black background, and all values greater than 100% are shown with a black background, and equilibria with no market coverage are marked as "NoC" with white background.

Firstly, we observe that the manufacturer accepts losses on either end (both for sales or after-sales) depending on the situation. Comparing with Figures 5.5-5.7, we see that the parameter settings that also show aftermarket monopolization also indicate higher than 100% profit from after-sales (hence accepting losses on product sales), parallel to the monopoly model, especially for equilibria in the high TCO_i region. The trends with respect to changes in the parameters are also then follow the same direction. The manufacturer's profits from after-sales services are more likely to be greater than 100% as the total number of repairs (f) increases; the price of non-original spare parts (s_n) is higher; the product base valuation (v) is lower; the consumer awareness of future costs (β) is higher; or the warranty coverage (f_m/f) is lower.

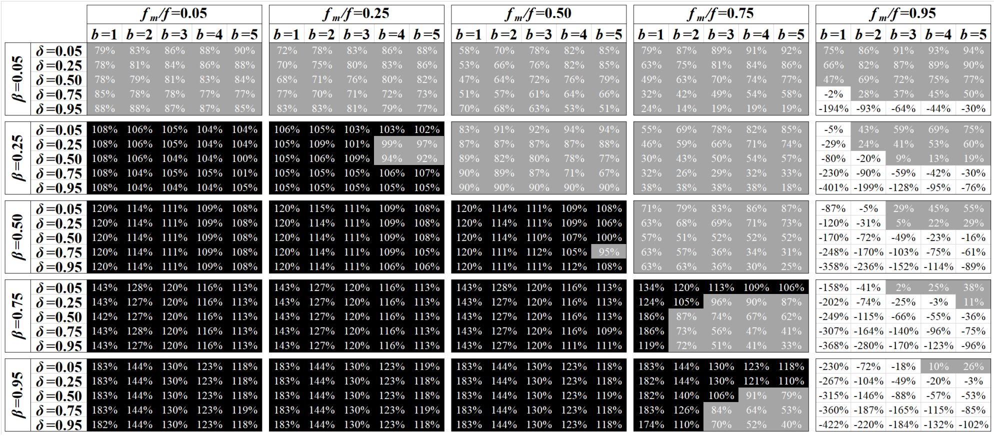
However, the manufacturer doesn't exclusively make a loss in product sales for all instances with aftermarket monopolization. Specifically for high and very high warranty coverage instances, the manufacturer chooses to increase her product price instead and starts making a loss on after-sales services. This strategy keeps TCO_i at still an affordable level by applying a low/moderate product price, but keeps the independent workshop out of the market with low original spare parts prices.

For instances with low to moderate warranty coverage and very low consumer awareness of future costs without aftermarket monopolization, we observe a more balanced strategy, where the manufacturer sets her prices such that she makes a profit in both sales and after-sales services. In Figures 5.2 and B.1-B.2 showing the manufacturer's profit function, we had also seen that these instances also correspond to a higher total profit potential for the manufacturer.



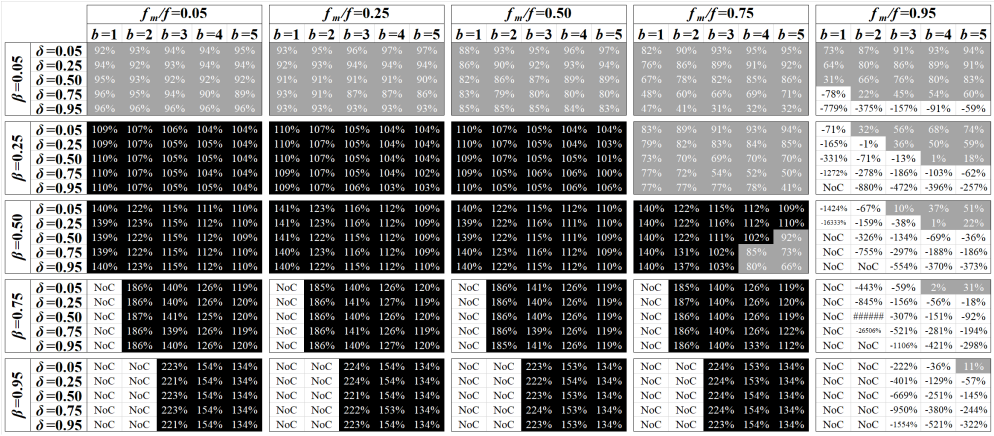
		$f_m/f=0.05$					$f_m/f=0.25$					$f_m/f=0.50$					$f_m/f=0.75$					$f_m/f=0.95$				
		$b=1$	$b=2$	$b=3$	$b=4$	$b=5$	$b=1$	$b=2$	$b=3$	$b=4$	$b=5$	$b=1$	$b=2$	$b=3$	$b=4$	$b=5$	$b=1$	$b=2$	$b=3$	$b=4$	$b=5$	$b=1$	$b=2$	$b=3$	$b=4$	$b=5$
$\beta=0.05$	$\delta=0.05$	56%	70%	77%	81%	84%	54%	69%	76%	81%	84%	49%	67%	75%	80%	83%	45%	64%	74%	79%	82%	42%	63%	72%	78%	82%
	$\delta=0.25$	52%	65%	73%	78%	81%	49%	63%	72%	77%	80%	44%	61%	70%	76%	80%	39%	59%	68%	75%	79%	34%	56%	67%	74%	78%
	$\delta=0.50$	45%	57%	65%	70%	74%	41%	55%	63%	69%	73%	35%	51%	61%	67%	72%	28%	47%	58%	66%	71%	21%	44%	56%	64%	70%
	$\delta=0.75$	37%	45%	52%	57%	60%	31%	41%	49%	54%	58%	22%	35%	44%	51%	56%	12%	28%	40%	48%	53%	3%	22%	36%	45%	51%
	$\delta=0.95$	47%	34%	34%	34%	34%	35%	26%	27%	28%	29%	13%	13%	17%	20%	22%	-5%	-1%	6%	11%	15%	-21%	-14%	-4%	3%	8%
$\beta=0.25$	$\delta=0.05$	43%	49%	55%	59%	64%	34%	42%	49%	54%	59%	22%	32%	41%	48%	54%	6%	22%	33%	41%	48%	-4%	13%	26%	35%	42%
	$\delta=0.25$	41%	46%	51%	55%	59%	31%	38%	44%	50%	54%	18%	28%	36%	42%	48%	4%	16%	26%	34%	41%	-9%	7%	19%	28%	35%
	$\delta=0.50$	41%	42%	46%	49%	51%	30%	33%	38%	42%	46%	15%	21%	28%	34%	39%	-1%	8%	17%	24%	31%	-14%	-2%	7%	16%	24%
	$\delta=0.75$	54%	41%	41%	41%	41%	40%	30%	32%	33%	34%	15%	15%	19%	23%	25%	-6%	-1%	5%	11%	16%	-21%	-14%	-6%	2%	8%
	$\delta=0.95$	58%	58%	58%	56%	54%	49%	49%	47%	44%	30%	32%	30%	25%	17%	15%	6%	-3%	-3%	-1%	1%	-27%	-26%	-18%	-14%	-10%
$\beta=0.50$	$\delta=0.05$	55%	57%	59%	62%	65%	41%	42%	46%	51%	55%	18%	25%	31%	37%	42%	-4%	8%	17%	24%	30%	-16%	-4%	6%	14%	20%
	$\delta=0.25$	61%	55%	57%	59%	61%	39%	40%	43%	47%	50%	17%	21%	27%	32%	36%	-4%	4%	11%	18%	23%	-19%	-8%	0%	7%	14%
	$\delta=0.50$	67%	55%	54%	53%	53%	49%	38%	40%	42%	43%	17%	18%	22%	26%	30%	-6%	-1%	4%	11%	16%	-22%	-15%	-8%	-1%	6%
	$\delta=0.75$	71%	67%	61%	48%	45%	57%	49%	38%	36%	34%	32%	18%	19%	20%	20%	-7%	-6%	-1%	4%	7%	-26%	-22%	-15%	-9%	-4%
	$\delta=0.95$	71%	71%	71%	71%	71%	58%	58%	58%	58%	57%	37%	37%	37%	35%	20%	8%	6%	0%	1%	1%	-28%	-27%	-21%	-16%	-13%
$\beta=0.75$	$\delta=0.05$	87%	82%	82%	84%	85%	58%	54%	57%	59%	62%	23%	24%	30%	35%	39%	-5%	2%	10%	15%	21%	-21%	-12%	-4%	3%	9%
	$\delta=0.25$	88%	81%	78%	76%	75%	63%	53%	55%	57%	58%	22%	22%	26%	30%	35%	-7%	-1%	5%	10%	15%	-22%	-15%	-8%	-3%	3%
	$\delta=0.50$	90%	86%	70%	66%	62%	69%	56%	51%	49%	48%	33%	21%	24%	27%	29%	-8%	-5%	-1%	6%	11%	-25%	-20%	-15%	-9%	-3%
	$\delta=0.75$	90%	90%	88%	60%	54%	71%	69%	63%	44%	40%	42%	33%	24%	22%	21%	1%	-7%	-2%	1%	4%	-27%	-25%	-19%	-13%	-9%
	$\delta=0.95$	90%	90%	90%	90%	59%	71%	71%	71%	71%	44%	43%	43%	43%	27%	24%	9%	9%	7%	2%	2%	-26%	-27%	-21%	-17%	-14%
$\beta=0.95$	$\delta=0.05$	119%	112%	109%	106%	105%	82%	75%	78%	80%	82%	26%	28%	32%	36%	41%	-7%	-1%	6%	11%	17%	-23%	-16%	-9%	-3%	2%
	$\delta=0.25$	117%	105%	97%	92%	88%	85%	76%	73%	71%	69%	34%	26%	29%	35%	39%	-8%	-4%	1%	6%	10%	-24%	-18%	-13%	-7%	-3%
	$\delta=0.50$	115%	119%	87%	78%	72%	87%	82%	64%	58%	55%	44%	29%	29%	30%	31%	-8%	-7%	-2%	4%	8%	-26%	-22%	-18%	-13%	-7%
	$\delta=0.75$	115%	115%	117%	72%	64%	87%	87%	85%	52%	46%	50%	44%	30%	26%	24%	4%	-6%	-2%	1%	3%	-28%	-26%	-21%	-15%	-11%
	$\delta=0.95$	115%	115%	115%	77%	67%	87%	87%	87%	56%	48%	50%	50%	50%	30%	26%	10%	10%	10%	3%	3%	-26%	-28%	-21%	-17%	-14%

(a) $f = 1$



		$f_m/f=0.05$					$f_m/f=0.25$					$f_m/f=0.50$					$f_m/f=0.75$					$f_m/f=0.95$				
		$b=1$	$b=2$	$b=3$	$b=4$	$b=5$	$b=1$	$b=2$	$b=3$	$b=4$	$b=5$	$b=1$	$b=2$	$b=3$	$b=4$	$b=5$	$b=1$	$b=2$	$b=3$	$b=4$	$b=5$	$b=1$	$b=2$	$b=3$	$b=4$	$b=5$
$\beta=0.05$	$\delta=0.05$	79%	83%	86%	88%	90%	72%	78%	83%	86%	88%	58%	70%	78%	82%	85%	79%	87%	89%	91%	92%	75%	86%	91%	93%	94%
	$\delta=0.25$	78%	81%	84%	86%	88%	70%	75%	80%	83%	86%	53%	66%	76%	82%	85%	63%	75%	81%	84%	86%	66%	82%	87%	89%	90%
	$\delta=0.50$	78%	79%	81%	83%	84%	68%	71%	76%	80%	82%	47%	64%	72%	76%	79%	49%	63%	70%	74%	77%	47%	69%	72%	75%	77%
	$\delta=0.75$	85%	78%	78%	77%	77%	77%	70%	71%	72%	73%	51%	57%	61%	64%	66%	32%	42%	49%	54%	58%	-2%	28%	37%	45%	50%
	$\delta=0.95$	88%	88%	87%	87%	85%	83%	83%	81%	79%	77%	70%	68%	63%	53%	51%	24%	14%	19%	19%	19%	-194%	-93%	-64%	-44%	-30%
$\beta=0.25$	$\delta=0.05$	108%	106%	105%	104%	104%	106%	105%	103%	103%	102%	83%	91%	92%	94%	94%	55%	69%	78%	82%	85%	-5%	43%	59%	69%	75%
	$\delta=0.25$	108%	106%	105%	104%	104%	105%	109%	101%	99%	97%	87%	87%	87%	87%	88%	46%	59%	66%	71%	74%	-29%	24%	41%	53%	60%
	$\delta=0.50$	108%	106%	104%	104%	100%	105%	106%	109%	94%	92%	89%	82%	80%	78%	77%	30%	43%	50%	54%	57%	-80%	-20%	9%	13%	19%
	$\delta=0.75$	108%	104%	105%	105%	101%	105%	105%	105%	106%	107%	90%	89%	87%	71%	67%	32%	26%	29%	32%	33%	-230%	-90%	-59%	-42%	-30%
	$\delta=0.95$	108%	104%	104%	104%	105%	105%	105%	105%	105%	105%	90%	90%	90%	90%	90%	38%	38%	38%	38%	18%	-401%	-199%	-128%	-95%	-76%
$\beta=0.50$	$\delta=0.05$	120%	114%	111%	109%	108%	120%	115%	111%	109%	108%	120%	114%	111%	109%	108%	71%	79%	83%	86%	87%	-87%	-5%	29%	45%	55%
	$\delta=0.25$	120%	114%	111%	109%	108%	120%	114%	111%	109%	108%	120%	114%	111%	109%	106%	63%	68%	69%	71%	73%	-120%	-31%	5%	22%	29%
	$\delta=0.50$	120%	114%	111%	109%	108%	120%	114%	111%	109%	108%	120%	114%	110%	107%	100%	57%	51%	52%	52%	52%	-170%	-72%	-49%	-23%	-16%
	$\delta=0.75$	120%	114%	111%	109%	108%	120%	114%	111%	109%	105%	120%	111%	112%	105%	95%	63%	57%	36%	34%	31%	-248%	-170%	-103%	-75%	-61%
	$\delta=0.95$	120%	114%	111%	109%	108%	120%	114%	111%	106%	106%	120%	111%	111%	112%	108%	63%	63%	36%	30%	25%	-358%	-236%	-152%	-114%	-89%
$\beta=0.75$	$\delta=0.05$	143%	128%	120%	116%	113%	143%	127%	120%	116%	113%	143%	128%	120%	116%	113%	134%	120%	113%	109%	106%	-158%	-41%	2%	25%	38%
	$\delta=0.25$	143%	127%	120%	116%	113%	143%	127%	120%	116%	113%	143%	127%	120%	116%	113%	124%	105%	96%	90%	87%	-202%	-74%	-25%	-3%	11%
	$\delta=0.50$	142%	127%	120%	116%	113%	143%	127%	120%	116%	113%	143%	127%	120%	116%	113%	186%	87%	74%	67%	62%	-249%	-115%	-66%	-55%	-36%
	$\delta=0.75$	143%	128%	120%	116%	113%	143%	127%	120%	116%	113%	143%	127%	120%	116%	109%	186%	73%	56%	47%	41%	-307%	-164%	-140%	-96%	-75%
	$\delta=0.95$	143%	127%	120%	116%	113%	143%	127%	120%	116%	113%	143%	127%	120%	111%	111%	119%	72%	51%	41%	33%	-368%	-280%	-170%	-123%	-96%
$\beta=0.95$	$\delta=0.05$	183%	144%	130%	123%	118%	183%	144%	130%	123%	119%	183%	144%	130%	123%	118%	183%	144%	130%	123%	118%	-230%	-72%	-18%	10%	26%
	$\delta=0.25$	183%	144%	130%	123%	118%	183%	144%	130%	123%	118%	183%	144%	130%	123%	118%	182%	144%	130%	121%	110%	-267%	-104%	-49%	-20%	-3%
	$\delta=0.50$	183%	144%	130%	123%	118%	183%	144%	130%	123%	118%	183%	144%	130%	123%	119%	182%	140%	106%	91%	79%	-315%	-146%	-88%	-57%	-53%
	$\delta=0.75$	183%	144%	130%	123%	119%	183%	144%	130%	123%	119%	183%	144%	130%	123%	118%	183%	126%	84%	64%	53%	-360%	-187%	-165%	-115%	-85%
	$\delta=0.95$	182%	144%	130%	123%	118%	183%	144%	130%	123%	118%	183%	144%	130%	123%	118%	174%	110%	70%	52%	40%	-422%	-220%	-184%	-132%	-102%

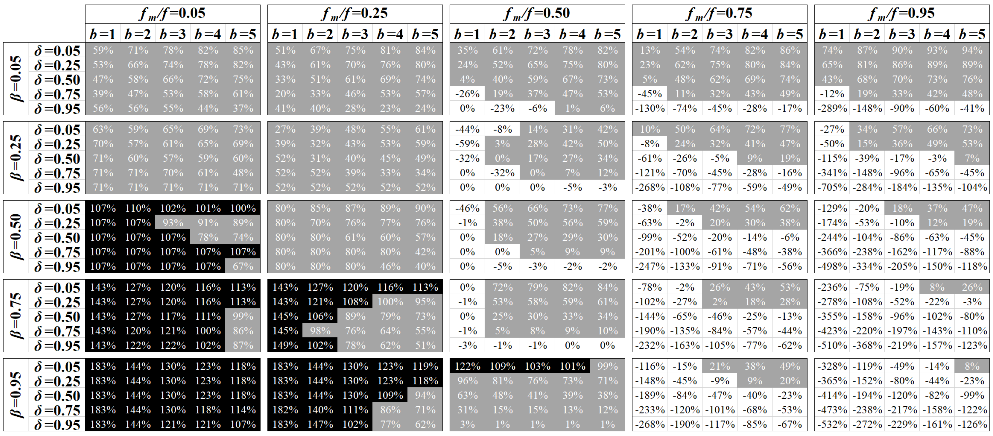
(b) $f = 5$



		$f_m/f=0.05$					$f_m/f=0.25$					$f_m/f=0.50$					$f_m/f=0.75$					$f_m/f=0.95$				
		$b=1$	$b=2$	$b=3$	$b=4$	$b=5$	$b=1$	$b=2$	$b=3$	$b=4$	$b=5$	$b=1$	$b=2$	$b=3$	$b=4$	$b=5$	$b=1$	$b=2$	$b=3$	$b=4$	$b=5$	$b=1$	$b=2$	$b=3$	$b=4$	$b=5$
$\beta=0.05$	$\delta=0.05$	92%	93%	94%	94%	95%	93%	95%	96%	97%	97%	88%	93%	95%	96%	97%	82%	90%	93%	95%	95%	73%	87%	91%	93%	94%
	$\delta=0.25$	94%	92%	93%	94%	94%	92%	93%	94%	94%	94%	86%	90%	92%	93%	94%	76%	86%	89%	91%	92%	64%	80%	86%	89%	91%
	$\delta=0.50$	95%	93%	92%	92%	92%	91%	91%	91%	91%	90%	82%	86%	87%	89%	89%	67%	78%	82%	85%	86%	31%	66%	76%	80%	83%
	$\delta=0.75$	96%	95%	94%	90%	89%	93%	91%	87%	87%	86%	83%	79%	80%	80%	80%	48%	60%	66%	69%	71%	-78%	22%	45%	54%	60%
	$\delta=0.95$	96%	96%	96%	96%	96%	93%	93%	93%	93%	93%	85%	85%	85%	84%	83%	47%	41%	31%	32%	32%	-779%	-375%	-157%	-91%	-59%
$\beta=0.25$	$\delta=0.05$	109%	107%	106%	104%	104%	110%	107%	105%	104%	104%	110%	107%	105%	104%	104%	83%	89%	91%	93%	94%	-71%	32%	56%	68%	74%
	$\delta=0.25$	109%	107%	105%	105%	104%	110%	107%	105%	104%	104%	110%	107%	105%	104%	103%	79%	82%	83%	84%	85%	-165%	-1%	36%	50%	59%
	$\delta=0.50$	110%	107%	105%	105%	104%	110%	107%	105%	104%	104%	109%	107%	105%	105%	101%	73%	70%	69%	70%	70%	-331%	-71%	-13%	1%	18%
	$\delta=0.75$	110%	107%	105%	104%	104%	109%	107%	105%	104%	102%	109%	105%	106%	106%	100%	77%	72%	54%	52%	50%	-1272%	-278%	-186%	-103%	-62%
	$\delta=0.95$	110%	107%	105%	105%	104%	109%	107%	106%	103%	103%	110%	105%	105%	106%	106%	77%	77%	77%	78%	41%	NoC	-880%	-472%	-396%	-257%
$\beta=0.50$	$\delta=0.05$	140%	122%	115%	111%	110%	141%	123%	116%	112%	109%	140%	122%	115%	112%	110%	140%	122%	115%	112%	109%	-1424%	-67%	10%	37%	51%
	$\delta=0.25$	139%	123%	115%	112%	110%	141%	123%	116%	112%	109%	139%	122%	116%	111%	110%	140%	122%	116%	112%	110%	-16333%	-159%	-38%	1%	22%
	$\delta=0.50$	139%	122%	115%	112%	109%	141%	122%	115%	112%	109%	139%	122%	115%	111%	109%	140%	122%	111%	102%	92%	NoC	-326%	-134%	-69%	-36%
	$\delta=0.75$	139%	122%	115%	112%	110%	140%	123%	116%	112%	109%	140%	123%	116%	112%	109%	140%	131%	102%	85%	73%	NoC	-755%	-297%	-188%	-186%
	$\delta=0.95$	140%	123%	115%	112%	110%	140%	122%	115%	112%	110%	140%	122%	115%	112%	110%	140%	137%	103%	80%	66%	NoC	NoC	-554%	-370%	-373%
$\beta=0.75$	$\delta=0.05$	NoC	186%	140%	126%	119%	NoC	185%	140%	126%	120%	NoC	186%	141%	126%	119%	NoC	185%	140%	126%	119%	NoC	-443%	-59%	2%	31%
	$\delta=0.25$	NoC	186%	140%	126%	120%	NoC	186%	141%	127%	119%	NoC	186%	140%	126%	119%	NoC	187%	140%	126%	120%	NoC	-845%	-156%	-56%	-18%
	$\delta=0.50$	NoC	187%	141%	125%	120%	NoC	186%	140%	126%	120%	NoC	186%	140%	126%	119%	NoC	186%	140%	126%	119%	NoC	######	-307%	-151%	-92%
	$\delta=0.75$	NoC	186%	139%	126%	119%	NoC	186%	141%	126%	119%	NoC	186%	139%	126%	119%	NoC	186%	140%	126%	122%	NoC	-26506%	-521%	-281%	-194%
	$\delta=0.95$	NoC	186%	140%	126%	120%	NoC	186%	140%	127%	120%	NoC	185%	141%	126%	119%	NoC	186%	140%	133%	112%	NoC	NoC	-1106%	-421%	-298%
$\beta=0.95$	$\delta=0.05$	NoC	NoC	223%	154%	134%	NoC	NoC	224%	154%	134%	NoC	NoC	223%	153%	134%	NoC	NoC	224%	153%	134%	NoC	NoC	-222%	-36%	11%
	$\delta=0.25$	NoC	NoC	221%	154%	134%	NoC	NoC	224%	154%	134%	NoC	NoC	222%	154%	134%	NoC	NoC	224%	153%	134%	NoC	NoC	-401%	-129%	-57%
	$\delta=0.50$	NoC	NoC	223%	154%	134%	NoC	NoC	221%	154%	134%	NoC	NoC	223%	153%	134%	NoC	NoC	224%	154%	134%	NoC	NoC	-669%	-251%	-145%
	$\delta=0.75$	NoC	NoC	223%	154%	134%	NoC	NoC	222%	153%	134%	NoC	NoC	224%	153%	134%	NoC	NoC	224%	154%	134%	NoC	NoC	-950%	-380%	-244%
	$\delta=0.95$	NoC	NoC	221%	154%	134%	NoC	NoC	223%	154%	134%	NoC	NoC	223%	153%	134%	NoC	NoC	223%	154%	134%	NoC	NoC	-1554%	-521%	-322%

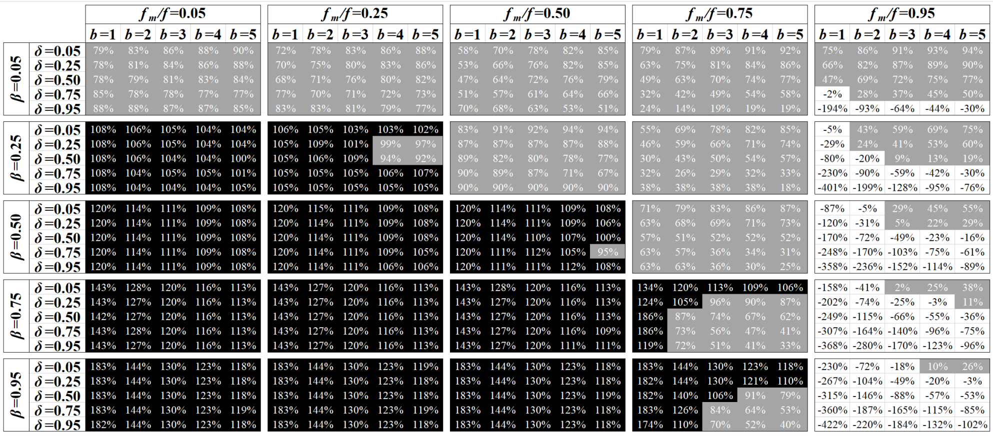
(c) $f = 10$

Figure 5.14: Impact of f on the manufacturer's share of her profits from after-sales services, where $s_n = 3, \upsilon = 3$, values below 0% are shown in white, values between 0% and 100% are shown in gray, values greater than 100% are shown in black, and equilibria with no market coverage are marked as "NoC" in white



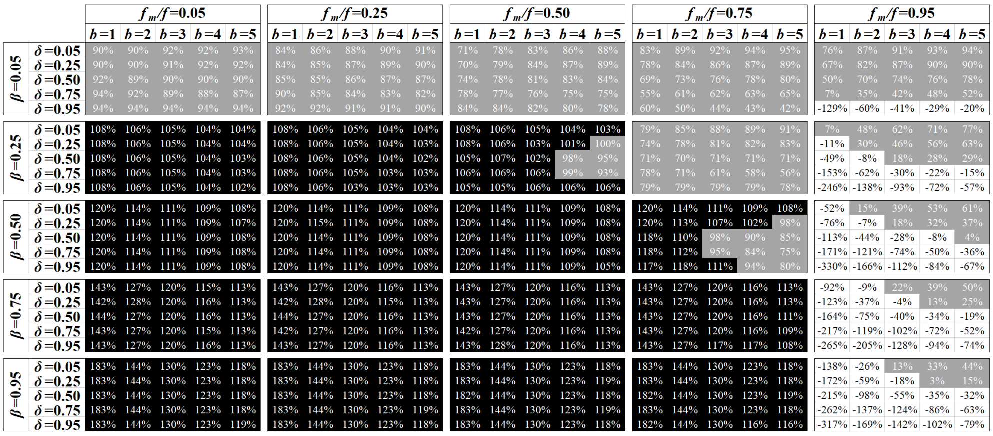
		f_m/f=0.05					f_m/f=0.25					f_m/f=0.50					f_m/f=0.75					f_m/f=0.95				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	59%	71%	78%	82%	85%	51%	67%	75%	81%	84%	35%	61%	72%	78%	82%	13%	54%	74%	82%	86%	74%	87%	90%	93%	94%
	δ=0.25	53%	66%	74%	78%	82%	43%	61%	70%	76%	80%	24%	52%	65%	75%	80%	23%	62%	75%	80%	84%	65%	81%	86%	89%	89%
	δ=0.50	47%	58%	66%	72%	75%	33%	51%	61%	69%	74%	4%	40%	59%	67%	73%	5%	48%	62%	69%	74%	43%	68%	70%	73%	76%
	δ=0.75	39%	47%	53%	58%	61%	20%	33%	46%	53%	57%	-26%	19%	37%	47%	53%	-45%	11%	32%	43%	49%	-12%	19%	33%	42%	48%
	δ=0.95	56%	56%	55%	44%	37%	41%	40%	28%	23%	24%	0%	-23%	-6%	1%	6%	-130%	-74%	-45%	-28%	-17%	-289%	-148%	-90%	-60%	-41%
β=0.25	δ=0.05	63%	59%	65%	69%	73%	27%	39%	48%	55%	61%	-44%	-8%	14%	31%	42%	10%	50%	64%	72%	77%	-27%	34%	57%	66%	73%
	δ=0.25	70%	57%	61%	65%	69%	39%	32%	43%	53%	59%	-59%	3%	28%	42%	50%	-8%	24%	32%	41%	47%	-50%	15%	36%	49%	53%
	δ=0.50	71%	60%	57%	59%	60%	52%	31%	40%	45%	49%	-32%	0%	17%	27%	34%	-61%	-26%	-5%	9%	19%	-115%	-39%	-17%	-3%	7%
	δ=0.75	71%	71%	70%	61%	48%	52%	52%	39%	33%	34%	0%	-32%	0%	7%	12%	-121%	-70%	-45%	-28%	-16%	-341%	-148%	-96%	-65%	-45%
	δ=0.95	71%	71%	71%	71%	71%	52%	52%	52%	52%	52%	0%	0%	0%	-5%	-3%	-268%	-108%	-77%	-59%	-49%	-705%	-284%	-184%	-135%	-104%
β=0.50	δ=0.05	107%	110%	102%	101%	100%	80%	85%	87%	89%	90%	-46%	56%	66%	73%	77%	-38%	17%	42%	54%	62%	-129%	-20%	18%	37%	47%
	δ=0.25	107%	107%	93%	91%	89%	80%	70%	76%	77%	76%	-1%	38%	50%	56%	59%	-63%	-2%	20%	30%	38%	-174%	-53%	-10%	12%	19%
	δ=0.50	107%	107%	107%	78%	74%	80%	80%	61%	60%	57%	0%	18%	27%	29%	30%	-99%	-52%	-20%	-14%	-6%	-244%	-104%	-86%	-63%	-45%
	δ=0.75	107%	107%	107%	107%	107%	80%	80%	80%	80%	42%	0%	0%	5%	9%	9%	-201%	-100%	-61%	-48%	-38%	-366%	-238%	-162%	-117%	-88%
	δ=0.95	107%	107%	107%	107%	67%	80%	80%	80%	46%	40%	0%	-5%	-3%	-2%	-2%	-247%	-133%	-91%	-71%	-56%	-498%	-334%	-205%	-150%	-118%
β=0.75	δ=0.05	143%	127%	120%	116%	113%	143%	127%	120%	116%	113%	0%	72%	79%	82%	84%	-78%	-2%	26%	43%	53%	-236%	-75%	-19%	8%	26%
	δ=0.25	143%	127%	120%	116%	113%	143%	121%	108%	100%	95%	-1%	53%	58%	59%	61%	-102%	-27%	2%	18%	28%	-278%	-108%	-52%	-22%	-3%
	δ=0.50	143%	127%	117%	111%	99%	145%	106%	89%	79%	73%	0%	25%	30%	33%	34%	-144%	-65%	-46%	-25%	-13%	-355%	-158%	-96%	-102%	-80%
	δ=0.75	143%	120%	121%	100%	86%	145%	98%	76%	64%	55%	-1%	5%	8%	9%	10%	-190%	-135%	-84%	-57%	-44%	-423%	-220%	-197%	-143%	-110%
	δ=0.95	143%	122%	122%	102%	87%	149%	102%	78%	62%	51%	-3%	-1%	-1%	0%	0%	-232%	-163%	-105%	-77%	-62%	-510%	-368%	-219%	-157%	-123%
β=0.95	δ=0.05	183%	144%	130%	123%	118%	183%	144%	130%	123%	119%	122%	109%	103%	101%	99%	-116%	-15%	21%	38%	49%	-328%	-119%	-49%	-14%	8%
	δ=0.25	183%	144%	130%	123%	118%	183%	144%	130%	123%	118%	96%	81%	76%	73%	71%	-148%	-45%	-9%	9%	20%	-365%	-152%	-80%	-44%	-23%
	δ=0.50	183%	144%	130%	123%	118%	183%	144%	130%	109%	94%	63%	48%	41%	39%	38%	-189%	-84%	-47%	-40%	-23%	-414%	-194%	-120%	-82%	-99%
	δ=0.75	183%	144%	130%	118%	114%	182%	140%	111%	86%	71%	31%	15%	15%	13%	12%	-233%	-120%	-101%	-68%	-53%	-473%	-238%	-217%	-158%	-122%
	δ=0.95	183%	144%	121%	121%	107%	183%	147%	102%	77%	62%	3%	1%	1%	1%	1%	-268%	-190%	-117%	-85%	-67%	-532%	-272%	-229%	-161%	-126%

(a) $s_n = 1$



		f_m/f=0.05					f_m/f=0.25					f_m/f=0.50					f_m/f=0.75					f_m/f=0.95				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	79%	83%	86%	88%	90%	72%	78%	83%	86%	88%	58%	70%	78%	82%	85%	79%	87%	89%	91%	92%	75%	86%	91%	93%	94%
	δ=0.25	78%	81%	84%	86%	88%	70%	75%	80%	83%	86%	53%	66%	76%	82%	85%	63%	75%	81%	84%	86%	66%	82%	87%	89%	90%
	δ=0.50	78%	79%	81%	83%	84%	68%	71%	76%	80%	82%	47%	64%	72%	76%	79%	49%	63%	70%	74%	77%	47%	69%	72%	75%	77%
	δ=0.75	85%	78%	78%	77%	77%	77%	70%	71%	72%	73%	51%	57%	61%	64%	66%	32%	42%	49%	54%	58%	-2%	28%	37%	45%	50%
	δ=0.95	88%	88%	87%	87%	85%	83%	83%	81%	79%	77%	70%	68%	63%	53%	51%	24%	14%	19%	19%	19%	-194%	-93%	-64%	-44%	-30%
β=0.25	δ=0.05	108%	106%	105%	104%	104%	106%	105%	103%	103%	102%	83%	91%	92%	94%	94%	55%	69%	78%	82%	85%	-5%	43%	59%	69%	75%
	δ=0.25	108%	106%	105%	104%	104%	105%	109%	101%	99%	97%	87%	87%	87%	87%	88%	46%	59%	66%	71%	74%	-29%	24%	41%	53%	60%
	δ=0.50	108%	106%	104%	104%	100%	105%	106%	109%	94%	92%	89%	82%	80%	78%	77%	30%	43%	50%	54%	57%	-80%	-20%	9%	13%	19%
	δ=0.75	108%	104%	105%	105%	101%	105%	105%	105%	106%	107%	90%	89%	87%	71%	67%	32%	26%	29%	32%	33%	-230%	-90%	-59%	-42%	-30%
	δ=0.95	108%	104%	104%	104%	105%	105%	105%	105%	105%	105%	90%	90%	90%	90%	90%	38%	38%	38%	38%	18%	-401%	-199%	-128%	-95%	-76%
β=0.50	δ=0.05	120%	114%	111%	109%	108%	120%	115%	111%	109%	108%	120%	114%	111%	109%	108%	71%	79%	83%	86%	87%	-87%	-5%	29%	45%	55%
	δ=0.25	120%	114%	111%	109%	108%	120%	114%	111%	109%	108%	120%	114%	111%	109%	106%	63%	68%	69%	71%	73%	-120%	-31%	5%	22%	29%
	δ=0.50	120%	114%	111%	109%	108%	120%	114%	111%	109%	108%	120%	114%	110%	107%	100%	57%	51%	52%	52%	52%	-170%	-72%	-49%	-23%	-16%
	δ=0.75	120%	114%	111%	109%	108%	120%	114%	111%	109%	105%	120%	111%	112%	105%	95%	63%	57%	36%	34%	31%	-248%	-170%	-103%	-75%	-61%
	δ=0.95	120%	114%	111%	109%	108%	120%	114%	111%	106%	106%	120%	111%	111%	112%	108%	63%	63%	36%	30%	25%	-358%	-236%	-152%	-114%	-89%
β=0.75	δ=0.05	143%	128%	120%	116%	113%	143%	127%	120%	116%	113%	143%	128%	120%	116%	113%	134%	120%	113%	109%	106%	-158%	-41%	2%	25%	38%
	δ=0.25	143%	127%	120%	116%	113%	143%	127%	120%	116%	113%	143%	127%	120%	116%	113%	124%	105%	96%	90%	87%	-202%	-74%	-25%	-3%	11%
	δ=0.50	142%	127%	120%	116%	113%	143%	127%	120%	116%	113%	143%	127%	120%	116%	113%	186%	87%	74%	67%	62%	-249%	-115%	-66%	-55%	-36%
	δ=0.75	143%	128%	120%	116%	113%	143%	127%	120%	116%	113%	143%	127%	120%	116%	109%	186%	73%	56%	47%	41%	-307%	-164%	-140%	-96%	-75%
	δ=0.95	143%	127%	120%	116%	113%	143%	127%	120%	116%	113%	143%	127%	120%	111%	111%	119%	72%	51%	41%	33%	-368%	-280%	-170%	-123%	-96%
β=0.95	δ=0.05	183%	144%	130%	123%	118%	183%	144%	130%	123%	119%	183%	144%	130%	123%	118%	183%	144%	130%	123%	118%	-230%	-72%	-18%	10%	26%
	δ=0.25	183%	144%	130%	123%	118%	183%	144%	130%	123%	118%	183%	144%	130%	123%	118%	182%	144%	130%	121%	110%	-267%	-104%	-49%	-20%	-3%
	δ=0.50	183%	144%	130%	123%	118%	183%	144%	130%	123%	118%	183%	144%	130%	123%	119%	182%	140%	106%	91%	79%	-315%	-146%	-88%	-57%	-53%
	δ=0.75	183%	144%	130%	123%	119%	183%	144%	130%	123%	119%	183%	144%	130%	123%	118%	183%	126%	84%	64%	53%	-360%	-187%	-165%	-115%	-85%
	δ=0.95	182%	144%	130%	123%	118%	183%	144%	130%	123%	118%	183%	144%	130%	123%	118%	174%	110%	70%	52%	40%	-422%	-220%	-184%	-132%	-102%

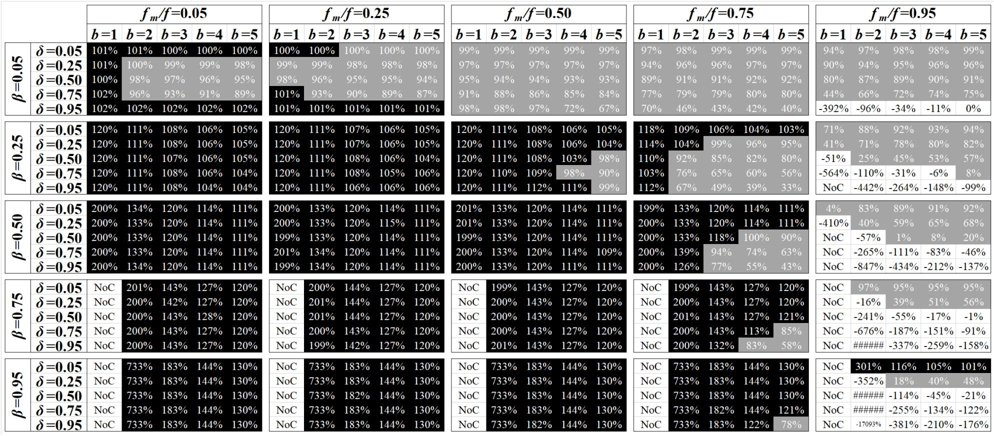
(b) $s_n = 3$



		f_m/f=0.05					f_m/f=0.25					f_m/f=0.50					f_m/f=0.75					f_m/f=0.95				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	90%	90%	92%	92%	93%	84%	86%	88%	90%	91%	71%	78%	83%	86%	88%	83%	89%	92%	94%	95%	76%	87%	91%	93%	94%
	δ=0.25	90%	90%	91%	92%	92%	84%	85%	87%	89%	90%	70%	79%	84%	87%	89%	78%	84%	86%	87%	89%	67%	82%	87%	90%	90%
	δ=0.50	92%	89%	90%	90%	90%	85%	85%	86%	87%	87%	74%	78%	81%	83%	84%	69%	73%	76%	78%	80%	50%	70%	74%	76%	78%
	δ=0.75	94%	92%	89%	88%	87%	90%	85%	84%	83%	82%	78%	77%	76%	75%	75%	55%	61%	62%	63%	65%	7%	35%	42%	48%	52%
	δ=0.95	94%	94%	94%	94%	94%	92%	92%	91%	91%	90%	84%	84%	82%	80%	78%	60%	50%	44%	43%	42%	-129%	-60%	-41%	-29%	-20%
β=0.25	δ=0.05	108%	106%	105%	104%	104%	108%	106%	105%	104%	104%	108%	106%	105%	104%	103%	79%	85%	88%	89%	91%	7%	48%	62%	71%	77%
	δ=0.25	108%	106%	105%	104%	104%	108%	106%	105%	104%	103%	108%	106%	103%	101%	100%	74%	78%	81%	82%	83%	-11%	30%	46%	56%	63%
	δ=0.50	108%	106%	105%	104%	103%	108%	106%	105%	104%	102%	105%	107%	102%	98%	95%	71%	70%	71%	71%	71%	-49%	-8%	18%	28%	29%
	δ=0.75	108%	106%	105%	104%	103%	108%	106%	105%	103%	103%	106%	106%	106%	99%	93%	78%	71%	61%	58%	56%	-153%	-62%	-30%	-22%	-15%
	δ=0.95	108%	106%	105%	104%	102%	108%	106%	103%	103%	103%	105%	105%	106%	106%	106%	79%	79%	79%	79%	78%	-246%	-138%	-93%	-72%	-57%
β=0.50	δ=0.05	120%	114%	111%	109%	108%	120%	114%	111%	109%	108%	120%	114%	111%	109%	108%	120%	114%	111%	109%	108%	-52%	15%	39%	53%	61%
	δ=0.25	120%	114%	111%	109%	107%	120%	115%	111%	109%	108%	120%	114%	111%	109%	108%	120%	113%	107%	102%	98%	-76%	-7%	18%	32%	37%
	δ=0.50	120%	114%	111%	109%	108%	120%	114%	111%	109%	108%	120%	114%	111%	109%	108%	118%	110%	98%	90%	85%	-113%	-44%	-28%	-8%	4%
	δ=0.75	120%	114%	111%	109%	108%	120%	114%	111%	109%	108%	120%	114%	111%	109%	108%	118%	112%	95%	84%	75%	-171%	-121%	-74%	-50%	-36%
	δ=0.95	120%	114%	111%	109%	108%	120%	114%	111%	109%	108%	120%	114%	111%	109%	105%	117%	118%	111%	94%	80%	-330%	-166%	-112%	-84%	-67%
β=0.75	δ=0.05	143%	127%	120%	115%	113%	143%	127%	120%	116%	113%	143%	127%	120%	116%	113%	143%	127%	120%	116%	113%	-92%	-9%	22%	39%	50%
	δ=0.25	142%	128%	120%	116%	113%	142%	128%	120%	115%	113%	143%	127%	120%	116%	113%	143%	127%	120%	116%	113%	-123%	-37%	-4%	13%	25%
	δ=0.50	144%	127%	120%	116%	113%	144%	127%	120%	116%	113%	143%	127%	120%	116%	113%	143%	127%	120%	116%	111%	-164%	-75%	-40%	-34%	-19%
	δ=0.75	143%	127%	120%	115%	113%	142%	127%	120%	116%	113%	142%	127%	120%	116%	113%	143%	127%	120%	116%	109%	-217%	-119%	-102%	-72%	-52%
	δ=0.95	143%	127%	120%	116%	113%	143%	127%	120%	116%	113%	143%	128%	120%	116%	113%	143%	127%	117%	117%	108%	-265%	-205%	-128%	-94%	-74%
β=0.95	δ=0.05	183%	144%	130%	123%	118%	183%	144%	130%	123%	118%	183%	144%	130%	123%	118%	183%	144%	130%	123%	118%	-138%	-26%	13%	33%	44%
	δ=0.25	183%	144%	130%	123%	118%	183%	144%	130%	123%	118%	183%	144%	130%	123%	119%	183%	144%	130%	123%	118%	-172%	-59%	-18%	3%	15%
	δ=0.50	183%	144%	130%	123%	118%	183%	144%	130%	123%	118%	182%	144%	130%	123%	118%	182%	144%	130%	123%	119%	-215%	-98%	-55%	-35%	-32%
	δ=0.75	183%	144%	130%	123%	118%	183%	144%	130%	123%	119%	183%	144%	130%	123%	119%	183%	144%	130%	123%	118%	-262%	-137%	-124%	-86%	-63%
	δ=0.95	183%	144%	130%	123%	119%	183%	144%	130%	123%	118%	183%	144%	130%	123%	118%	182%	144%	130%	116%	116%	-317%	-169%	-142%	-102%	-79%

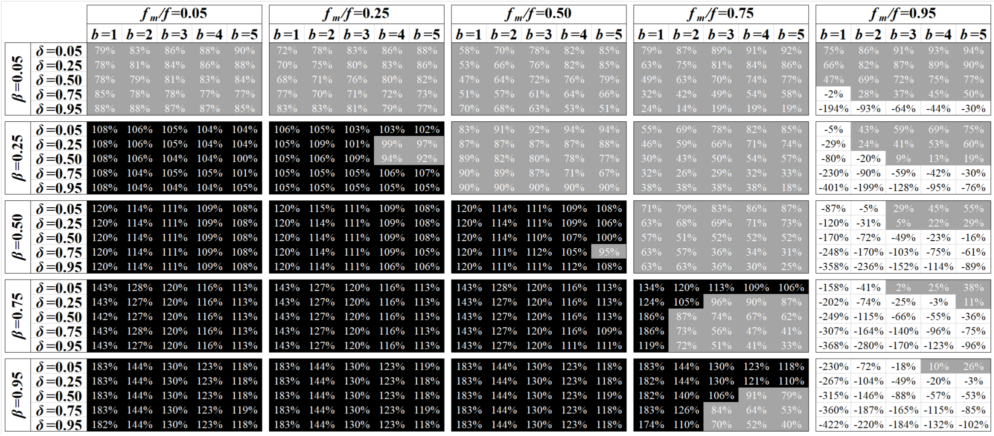
(c) $s_n = 5$

Figure 5.15: Impact of s_n on the manufacturer's share of her profits from after-sales services, where $s_n = 3, \upsilon = 3$, values below 0% are shown in white, values between 0% and 100% are shown in gray, values greater than 100% are shown in black, and equilibria with no market coverage are marked as "NoC" in white



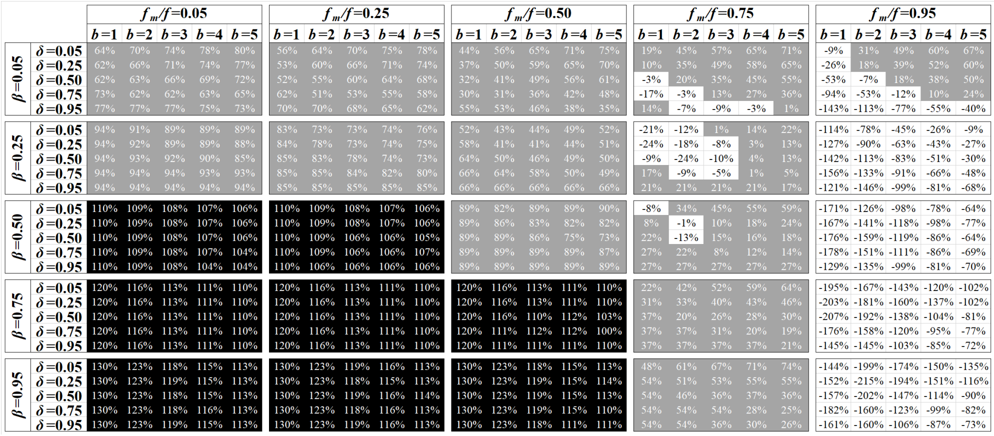
		f_m/f=0.05					f_m/f=0.25					f_m/f=0.50					f_m/f=0.75					f_m/f=0.95				
		b=1	*b*=2	*b*=3	*b*=4	*b*=5	*b*=1	*b*=2	*b*=3	*b*=4	*b*=5	*b*=1	*b*=2	*b*=3	*b*=4	*b*=5	*b*=1	*b*=2	*b*=3	*b*=4	*b*=5	*b*=1	*b*=2	*b*=3	*b*=4	*b*=5
β=0.05	δ=0.05	101%	101%	100%	100%	100%	100%	100%	100%	100%	100%	99%	99%	99%	99%	99%	97%	98%	99%	99%	99%	94%	97%	98%	98%	99%
	δ=0.25	101%	100%	99%	99%	98%	99%	99%	98%	98%	98%	97%	97%	97%	97%	97%	94%	96%	96%	97%	97%	90%	94%	95%	96%	96%
	δ=0.50	100%	98%	97%	96%	95%	98%	96%	95%	95%	94%	95%	94%	94%	93%	93%	89%	91%	91%	92%	92%	80%	87%	89%	90%	91%
	δ=0.75	102%	96%	93%	91%	89%	101%	93%	90%	89%	87%	91%	88%	86%	85%	84%	77%	79%	79%	80%	80%	44%	66%	72%	74%	75%
	δ=0.95	102%	102%	102%	102%	102%	101%	101%	101%	101%	101%	98%	98%	97%	72%	67%	70%	46%	43%	42%	40%	-392%	-96%	-34%	-11%	0%
β=0.25	δ=0.05	120%	111%	108%	106%	105%	120%	111%	107%	106%	105%	120%	111%	108%	106%	105%	118%	109%	106%	104%	103%	71%	88%	92%	93%	94%
	δ=0.25	120%	111%	108%	106%	105%	120%	111%	107%	106%	105%	120%	111%	108%	106%	104%	114%	104%	99%	96%	95%	41%	71%	78%	80%	82%
	δ=0.50	120%	111%	107%	106%	105%	120%	111%	108%	106%	104%	120%	111%	108%	103%	98%	110%	92%	85%	82%	80%	-51%	25%	45%	53%	57%
	δ=0.75	120%	111%	108%	106%	104%	120%	111%	108%	105%	106%	120%	110%	109%	98%	90%	103%	76%	65%	60%	56%	-564%	-110%	-31%	-6%	8%
	δ=0.95	120%	111%	108%	104%	104%	120%	111%	106%	106%	106%	120%	111%	112%	111%	99%	112%	67%	49%	39%	33%	NoC	-442%	-264%	-148%	-99%
β=0.50	δ=0.05	200%	134%	120%	114%	111%	200%	133%	120%	114%	111%	201%	133%	120%	114%	111%	199%	133%	120%	114%	111%	4%	83%	89%	91%	92%
	δ=0.25	200%	133%	120%	114%	111%	200%	133%	120%	115%	111%	201%	133%	120%	114%	111%	200%	133%	120%	114%	111%	-410%	40%	59%	65%	68%
	δ=0.50	200%	133%	120%	114%	111%	199%	133%	120%	114%	111%	199%	133%	120%	114%	111%	200%	133%	118%	100%	90%	NoC	-57%	1%	8%	20%
	δ=0.75	200%	133%	120%	114%	111%	201%	134%	120%	114%	111%	200%	133%	120%	114%	109%	200%	139%	94%	74%	63%	NoC	-265%	-111%	-83%	-46%
	δ=0.95	200%	134%	120%	114%	111%	199%	134%	120%	114%	111%	200%	133%	120%	111%	111%	200%	126%	77%	55%	43%	NoC	-847%	-434%	-212%	-137%
β=0.75	δ=0.05	NoC	201%	143%	127%	120%	NoC	200%	144%	127%	120%	NoC	199%	143%	127%	120%	NoC	199%	143%	127%	120%	NoC	97%	95%	95%	95%
	δ=0.25	NoC	200%	142%	127%	120%	NoC	201%	144%	127%	120%	NoC	200%	143%	127%	120%	NoC	200%	143%	127%	120%	NoC	-16%	39%	51%	56%
	δ=0.50	NoC	200%	143%	128%	120%	NoC	201%	144%	127%	120%	NoC	200%	143%	127%	120%	NoC	201%	143%	127%	121%	NoC	-241%	-55%	-17%	-1%
	δ=0.75	NoC	200%	143%	127%	120%	NoC	200%	143%	127%	120%	NoC	200%	143%	127%	120%	NoC	200%	143%	113%	85%	NoC	-676%	-187%	-151%	-91%
	δ=0.95	NoC	200%	143%	127%	120%	NoC	199%	142%	127%	120%	NoC	201%	143%	127%	120%	NoC	200%	132%	83%	58%	NoC	######	-337%	-259%	-158%
β=0.95	δ=0.05	NoC	733%	183%	144%	130%	NoC	733%	183%	144%	130%	NoC	733%	183%	144%	130%	NoC	733%	183%	144%	130%	NoC	301%	116%	105%	101%
	δ=0.25	NoC	733%	183%	144%	130%	NoC	733%	183%	144%	130%	NoC	733%	183%	144%	130%	NoC	733%	183%	144%	130%	NoC	-352%	18%	40%	48%
	δ=0.50	NoC	733%	183%	144%	130%	NoC	733%	182%	144%	130%	NoC	733%	183%	144%	130%	NoC	733%	183%	144%	130%	NoC	######	-114%	-45%	-21%
	δ=0.75	NoC	733%	183%	144%	130%	NoC	733%	183%	144%	130%	NoC	733%	183%	144%	130%	NoC	733%	182%	144%	121%	NoC	######	-255%	-134%	-122%
	δ=0.95	NoC	733%	183%	144%	130%	NoC	733%	183%	144%	130%	NoC	733%	182%	144%	130%	NoC	733%	183%	122%	78%	NoC	-17093%	-381%	-210%	-176%

(a) $\upsilon = 1$



		f_m/f=0.05					f_m/f=0.25					f_m/f=0.50					f_m/f=0.75					f_m/f=0.95				
		b=1	*b*=2	*b*=3	*b*=4	*b*=5	*b*=1	*b*=2	*b*=3	*b*=4	*b*=5	*b*=1	*b*=2	*b*=3	*b*=4	*b*=5	*b*=1	*b*=2	*b*=3	*b*=4	*b*=5	*b*=1	*b*=2	*b*=3	*b*=4	*b*=5
β=0.05	δ=0.05	79%	83%	86%	88%	90%	72%	78%	83%	86%	88%	58%	70%	78%	82%	85%	79%	87%	89%	91%	92%	75%	86%	91%	93%	94%
	δ=0.25	78%	81%	84%	86%	88%	70%	75%	80%	83%	86%	53%	66%	76%	82%	85%	63%	75%	81%	84%	86%	66%	82%	87%	89%	90%
	δ=0.50	78%	79%	81%	83%	84%	68%	71%	76%	80%	82%	47%	64%	72%	76%	79%	49%	63%	70%	74%	77%	47%	69%	72%	75%	77%
	δ=0.75	85%	78%	78%	77%	77%	77%	70%	71%	72%	73%	51%	57%	61%	64%	66%	32%	42%	49%	54%	58%	-2%	28%	37%	45%	50%
	δ=0.95	88%	88%	87%	87%	85%	83%	83%	81%	79%	77%	70%	68%	63%	53%	51%	24%	14%	19%	19%	19%	-194%	-93%	-64%	-44%	-30%
β=0.25	δ=0.05	108%	106%	105%	104%	104%	106%	105%	103%	103%	102%	83%	91%	92%	94%	94%	55%	69%	78%	82%	85%	-5%	43%	59%	69%	75%
	δ=0.25	108%	106%	105%	104%	104%	105%	109%	101%	99%	97%	87%	87%	87%	87%	88%	46%	59%	66%	71%	74%	-29%	24%	41%	53%	60%
	δ=0.50	108%	106%	104%	104%	100%	105%	106%	109%	94%	92%	89%	82%	80%	78%	77%	30%	43%	50%	54%	57%	-80%	-20%	9%	13%	19%
	δ=0.75	108%	104%	105%	105%	101%	105%	105%	105%	106%	107%	90%	89%	87%	71%	67%	32%	26%	29%	32%	33%	-230%	-90%	-59%	-42%	-30%
	δ=0.95	108%	104%	104%	104%	105%	105%	105%	105%	105%	105%	90%	90%	90%	90%	90%	38%	38%	38%	38%	18%	-401%	-199%	-128%	-95%	-76%
β=0.50	δ=0.05	120%	114%	111%	109%	108%	120%	115%	111%	109%	108%	120%	114%	111%	109%	108%	71%	79%	83%	86%	87%	-87%	-5%	29%	45%	55%
	δ=0.25	120%	114%	111%	109%	108%	120%	114%	111%	109%	108%	120%	114%	111%	109%	106%	63%	68%	69%	71%	73%	-120%	-31%	5%	22%	29%
	δ=0.50	120%	114%	111%	109%	108%	120%	114%	111%	109%	108%	120%	114%	110%	107%	100%	57%	51%	52%	52%	52%	-170%	-72%	-49%	-23%	-16%
	δ=0.75	120%	114%	111%	109%	108%	120%	114%	111%	109%	105%	120%	111%	112%	105%	95%	63%	57%	36%	34%	31%	-248%	-170%	-103%	-75%	-61%
	δ=0.95	120%	114%	111%	109%	108%	120%	114%	111%	106%	106%	120%	111%	111%	112%	108%	63%	63%	36%	30%	25%	-358%	-236%	-152%	-114%	-89%
β=0.75	δ=0.05	143%	128%	120%	116%	113%	143%	127%	120%	116%	113%	143%	128%	120%	116%	113%	134%	120%	113%	109%	106%	-158%	-41%	2%	25%	38%
	δ=0.25	143%	127%	120%	116%	113%	143%	127%	120%	116%	113%	143%	127%	120%	116%	113%	124%	105%	96%	90%	87%	-202%	-74%	-25%	-3%	11%
	δ=0.50	142%	127%	120%	116%	113%	143%	127%	120%	116%	113%	143%	127%	120%	116%	113%	186%	87%	74%	67%	62%	-249%	-115%	-66%	-55%	-36%
	δ=0.75	143%	128%	120%	116%	113%	143%	127%	120%	116%	113%	143%	127%	120%	116%	109%	186%	73%	56%	47%	41%	-307%	-164%	-140%	-96%	-75%
	δ=0.95	143%	127%	120%	116%	113%	143%	127%	120%	116%	113%	143%	127%	120%	111%	111%	119%	72%	51%	41%	33%	-368%	-280%	-170%	-123%	-96%
β=0.95	δ=0.05	183%	144%	130%	123%	118%	183%	144%	130%	123%	119%	183%	144%	130%	123%	118%	183%	144%	130%	123%	118%	-230%	-72%	-18%	10%	26%
	δ=0.25	183%	144%	130%	123%	118%	183%	144%	130%	123%	118%	183%	144%	130%	123%	118%	182%	144%	130%	121%	110%	-267%	-104%	-49%	-20%	-3%
	δ=0.50	183%	144%	130%	123%	118%	183%	144%	130%	123%	118%	183%	144%	130%	123%	119%	182%	140%	106%	91%	79%	-315%	-146%	-88%	-57%	-53%
	δ=0.75	183%	144%	130%	123%	119%	183%	144%	130%	123%	119%	183%	144%	130%	123%	118%	183%	126%	84%	64%	53%	-360%	-187%	-165%	-115%	-85%
	δ=0.95	182%	144%	130%	123%	118%	183%	144%	130%	123%	118%	183%	144%	130%	123%	118%	174%	110%	70%	52%	40%	-422%	-220%	-184%	-132%	-102%

(b) $\upsilon = 3$



		f_m/f=0.05					f_m/f=0.25					f_m/f=0.50					f_m/f=0.75					f_m/f=0.95				
		b=1	*b*=2	*b*=3	*b*=4	*b*=5	*b*=1	*b*=2	*b*=3	*b*=4	*b*=5	*b*=1	*b*=2	*b*=3	*b*=4	*b*=5	*b*=1	*b*=2	*b*=3	*b*=4	*b*=5	*b*=1	*b*=2	*b*=3	*b*=4	*b*=5
β=0.05	δ=0.05	64%	70%	74%	78%	80%	56%	64%	70%	75%	78%	44%	56%	65%	71%	75%	19%	45%	57%	65%	71%	-9%	31%	49%	60%	67%
	δ=0.25	62%	66%	71%	74%	77%	53%	60%	66%	71%	74%	37%	50%	59%	65%	70%	10%	35%	49%	58%	65%	-26%	18%	39%	52%	60%
	δ=0.50	62%	63%	66%	69%	72%	52%	55%	60%	64%	68%	32%	41%	49%	56%	61%	-3%	20%	35%	45%	55%	-53%	-7%	18%	38%	50%
	δ=0.75	73%	62%	62%	63%	65%	62%	51%	53%	55%	58%	30%	31%	36%	42%	48%	-17%	-3%	13%	27%	36%	-94%	-53%	-12%	10%	24%
	δ=0.95	77%	77%	77%	75%	73%	70%	70%	68%	65%	62%	55%	53%	46%	38%	35%	14%	-7%	-9%	-3%	1%	-143%	-113%	-77%	-55%	-40%
β=0.25	δ=0.05	94%	91%	89%	89%	89%	83%	73%	73%	74%	76%	52%	43%	44%	49%	52%	-21%	-12%	1%	14%	22%	-114%	-78%	-45%	-26%	-9%
	δ=0.25	94%	92%	89%	89%	88%	84%	78%	73%	74%	75%	58%	41%	41%	44%	51%	-24%	-18%	-8%	3%	13%	-127%	-90%	-63%	-43%	-27%
	δ=0.50	94%	93%	92%	90%	85%	85%	83%	78%	74%	73%	64%	50%	46%	49%	50%	-9%	-24%	-10%	4%	13%	-142%	-113%	-83%	-51%	-30%
	δ=0.75	94%	94%	94%	93%	93%	85%	85%	84%	82%	80%	66%	64%	58%	50%	49%	17%	-9%	-5%	1%	5%	-156%	-133%	-91%	-66%	-48%
	δ=0.95	94%	94%	94%	94%	94%	85%	85%	85%	85%	85%	66%	66%	66%	66%	66%	21%	21%	21%	21%	17%	-121%	-146%	-99%	-81%	-68%
β=0.50	δ=0.05	110%	109%	108%	107%	106%	110%	109%	108%	107%	106%	89%	82%	89%	89%	90%	-8%	34%	45%	55%	59%	-171%	-126%	-98%	-78%	-64%
	δ=0.25	110%	109%	108%	107%	106%	110%	109%	108%	107%	106%	89%	86%	83%	82%	82%	8%	-1%	10%	18%	24%	-167%	-141%	-118%	-98%	-77%
	δ=0.50	110%	109%	108%	107%	106%	110%	109%	106%	106%	105%	89%	89%	86%	75%	73%	22%	-13%	15%	16%	18%	-176%	-159%	-119%	-86%	-64%
	δ=0.75	110%	109%	108%	107%	104%	110%	109%	106%	106%	107%	89%	89%	89%	89%	87%	27%	22%	8%	12%	14%	-178%	-151%	-111%	-86%	-69%
	δ=0.95	110%	109%	108%	104%	104%	110%	106%	106%	106%	106%	89%	89%	89%	89%	89%	27%	27%	27%	27%	27%	-129%	-135%	-99%	-81%	-70%
β=0.75	δ=0.05	120%	116%	113%	111%	110%	120%	116%	113%	111%	110%	120%	116%	113%	111%	110%	22%	42%	52%	59%	64%	-195%	-167%	-143%	-120%	-102%
	δ=0.25	120%	116%	113%	111%	110%	120%	116%	113%	111%	110%	120%	116%	113%	111%	110%	31%	33%	40%	43%	46%	-203%	-181%	-160%	-137%	-102%
	δ=0.50	120%	116%	113%	111%	110%	120%	116%	113%	111%	110%	120%	116%	110%	112%	103%	37%	20%	26%	28%	30%	-207%	-192%	-138%	-104%	-81%
	δ=0.75	120%	116%	113%	111%	110%	120%	116%	113%	111%	110%	120%	111%	112%	112%	100%	37%	37%	31%	20%	19%	-176%	-158%	-120%	-95%	-77%
	δ=0.95	120%	116%	113%	111%	110%	120%	116%	113%	111%	110%	120%	111%	111%	111%	110%	37%	37%	37%	37%	21%	-145%	-145%	-103%	-85%	-72%
β=0.95	δ=0.05	130%	123%	118%	115%	113%	130%	123%	119%	116%	113%	130%	123%	118%	115%	113%	48%	61%	67%	71%	74%	-144%	-199%	-174%	-150%	-135%
	δ=0.25	130%	123%	119%	115%	113%	130%	123%	118%	115%	113%	130%	123%	119%	115%	114%	54%	51%	53%	55%	55%	-152%	-215%	-194%	-151%	-116%
	δ=0.50	130%	123%	118%	115%	113%	130%	123%	119%	116%	114%	130%	123%	119%	115%	113%	54%	46%	36%	37%	36%	-157%	-202%	-147%	-114%	-90%
	δ=0.75	130%	123%	118%	116%	113%	130%	123%	118%	116%	113%	130%	123%	119%	115%	110%	54%	54%	54%	28%	25%	-182%	-160%	-123%	-99%	-82%
	δ=0.95	130%	123%	119%	115%	113%	130%	123%	119%	116%	113%	130%	123%	118%	111%	111%	54%	54%	36%	30%	26%	-161%	-160%	-106%	-87%	-73%

(c) $\upsilon = 5$

Figure 5.16: Impact of υ on the manufacturer's share of her profits from after-sales services, where $s_n = 3, \upsilon = 3$, values below 0% are shown in white, values between 0% and 100% are shown in gray, values greater than 100% are shown in black, and equilibria with no market coverage are marked as "NoC" in white

5.2.5 Double Marginalization and Manufacturer Profit

In this section, we look into how the manufacturer's share of the total chain profit behaves in the downstream competition model. As reported in the monopoly model, the manufacturer captured 67% of the total chain profit when the market was partially covered and she captured 75% of the total chain profit when the market was fully covered. We have a much wider range in the downstream competition model, from 2% to 100% (with rounding). In Figures 5.17- 5.19, we report the manufacturer's share of chain's total profit, where values below 50% are shown with a white background, values between 50% and 65% are shown with a light gray background, values between 65% and 75% are shown with a dark gray background, and values above 75% are shown with a black background.

Although we don't make a one-to-one comparison between the two models, we can use the color coding to interpret as the lighter background colors (less than 65%) showing instances where the manufacturer falls back from his potential share of chain profits as compared to the monopoly model, and the black background instances (greater than 75%) indicating the manufacturer increasing his potential share of chain profits as compared to the monopoly model.

We observe that the manufacturer rarely captures less than 50% of the total chain profits, all of which are observed within the parameter settings with very high warranty coverage ($f_m/f = 0.95$) under the shown combinations of $f, s_n,$ and v. In general, we see that the manufacturer's share of chain profits tends to decrease as the warranty coverage increases.

We also see that the manufacturer has an increased potential of capturing more than 75% of the chain profits as the total number of repairs (f) decreases, the non-original spare part price (s_n) decreases, or the consumer's base product valuation (v) increases; all of which are factors we had identified before for other trends, as well. An interesting result specific to this research question is that we see the manufacturer has a higher potential of increasing her share of chain profits for instances with very high level of similarity of the independent workshop, δ. Checking the demand structure, we see that these cases correspond to full market coverage with low product price and low spare part prices. Apparently, the retailer decreases his prices due facing harsher competition from the independent workshop, while the manufacturer sells more units covering the whole market, but also not compromising from her after-sales profits.

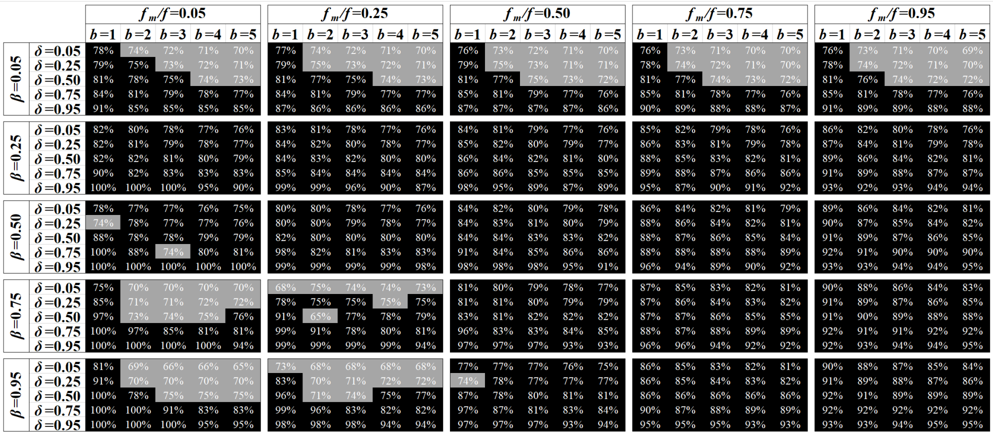
5.3 Conclusion

In this chapter, we conducted a numerical study of the downstream competition model and shared the observations and insights we derived from the results.

We first identify under what conditions the manufacturer (and the retailer) monopolize the after-sales market. We observe that having an independent workshop selling non-original spare parts in the market doesn't create a difference if the perceived total cost of ownership for buying a product and receiving after-sales services for non-warranty repairs from the independent workshop is too high. In such cases, the downstream competition model produces results equivalent to the monopoly model. We also find that our experimental design doesn't produce any equilibria where the independent workshop monopolizes the after-sales market.

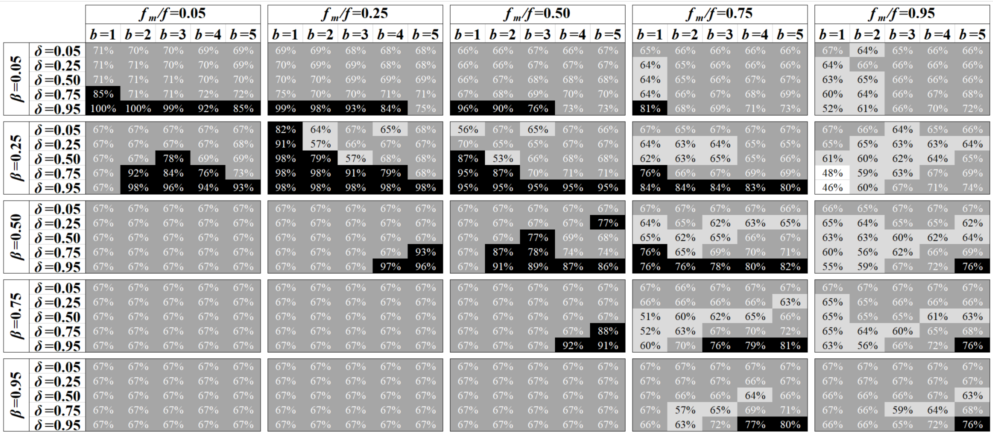
Next, we study under which conditions the manufacturer benefits from the existence of an independent workshop. We see that although the independent workshop increases her market coverage in some cases, the manufacturer is still better off mostly with the monopoly model from a profit comparison perspective. Reversely, having the independent workshop as an alternative usually improves the consumer surplus as compared to the monopoly model, unless the total cost of ownership of the non-original alternative is too high and the monopoly model becomes the only feasible alternative. The consumer and the manufacturer align on the downstream competition model for some cases, mostly when the total cost of ownership alternative is not too low to force the manufacturer to bring her prices down, but helps to expand her market instead.

Unlike the monopoly model, we find that a higher warranty coverage is better for the customer and worse for the manufacturer in general, especially for cases where the independent workshop is also active in the market. Therefore, the authorities mandating a minimum warranty coverage would likely improve the consumer surplus



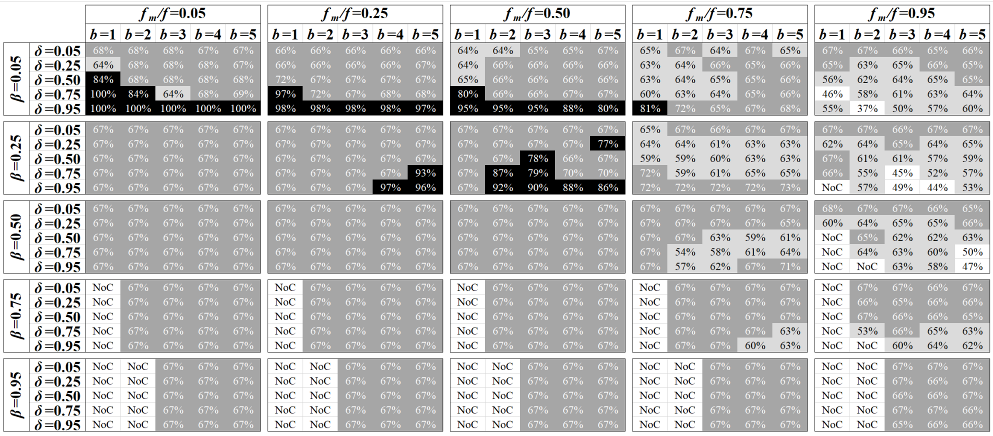
		f_m/f=0.05					f_m/f=0.25					f_m/f=0.50					f_m/f=0.75					f_m/f=0.95				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	78%	74%	72%	71%	70%	77%	74%	72%	71%	70%	76%	73%	72%	71%	70%	76%	73%	71%	70%	70%	76%	73%	71%	70%	69%
	δ=0.25	79%	75%	73%	72%	71%	79%	75%	73%	72%	71%	79%	75%	73%	71%	71%	78%	74%	72%	71%	70%	78%	74%	72%	71%	70%
	δ=0.50	81%	78%	75%	74%	73%	81%	77%	75%	74%	73%	81%	77%	75%	73%	72%	81%	77%	74%	73%	72%	81%	76%	74%	72%	72%
	δ=0.75	84%	81%	79%	78%	77%	84%	81%	79%	77%	77%	85%	81%	79%	77%	76%	85%	81%	78%	77%	76%	85%	81%	78%	77%	76%
	δ=0.95	91%	85%	85%	85%	85%	87%	86%	86%	86%	86%	87%	87%	87%	87%	86%	90%	89%	88%	88%	87%	91%	89%	89%	88%	88%
β=0.25	δ=0.05	82%	80%	78%	77%	76%	83%	81%	78%	77%	76%	84%	81%	79%	77%	76%	85%	82%	79%	78%	76%	86%	82%	80%	78%	76%
	δ=0.25	82%	81%	79%	78%	77%	84%	82%	80%	78%	77%	85%	82%	80%	79%	77%	86%	83%	81%	79%	78%	87%	84%	81%	79%	78%
	δ=0.50	82%	82%	81%	80%	79%	84%	83%	82%	80%	80%	86%	84%	82%	81%	80%	88%	85%	83%	82%	81%	89%	86%	84%	82%	81%
	δ=0.75	90%	82%	83%	83%	83%	85%	84%	84%	84%	84%	86%	86%	85%	85%	85%	89%	88%	87%	86%	86%	91%	89%	88%	87%	87%
	δ=0.95	100%	100%	100%	95%	90%	99%	99%	96%	90%	87%	98%	95%	89%	87%	89%	95%	87%	90%	91%	92%	93%	92%	93%	94%	94%
β=0.50	δ=0.05	78%	77%	77%	76%	75%	80%	80%	78%	77%	76%	84%	82%	80%	79%	78%	86%	84%	82%	81%	79%	89%	86%	84%	82%	81%
	δ=0.25	74%	78%	77%	77%	76%	80%	80%	79%	78%	77%	84%	83%	81%	80%	79%	88%	86%	84%	82%	81%	90%	87%	85%	84%	82%
	δ=0.50	88%	78%	78%	79%	79%	82%	80%	80%	80%	80%	84%	84%	83%	83%	82%	88%	87%	86%	85%	84%	91%	89%	87%	86%	85%
	δ=0.75	100%	88%	74%	80%	81%	98%	82%	81%	83%	83%	91%	84%	85%	86%	86%	88%	88%	88%	88%	89%	92%	91%	90%	90%	90%
	δ=0.95	100%	100%	100%	100%	100%	99%	99%	99%	99%	98%	98%	98%	98%	95%	91%	96%	94%	89%	90%	92%	93%	93%	94%	94%	95%
β=0.75	δ=0.05	75%	70%	70%	70%	70%	68%	75%	74%	74%	73%	81%	80%	79%	78%	77%	87%	85%	83%	82%	81%	90%	88%	86%	84%	83%
	δ=0.25	85%	71%	71%	72%	72%	78%	75%	75%	75%	75%	81%	81%	80%	79%	79%	87%	86%	84%	83%	82%	91%	89%	87%	86%	85%
	δ=0.50	97%	73%	74%	75%	76%	91%	65%	77%	78%	79%	83%	81%	82%	82%	82%	87%	87%	86%	85%	85%	91%	90%	89%	88%	88%
	δ=0.75	100%	97%	85%	81%	81%	99%	91%	78%	80%	81%	96%	83%	83%	84%	85%	88%	87%	88%	89%	89%	92%	91%	91%	92%	92%
	δ=0.95	100%	100%	100%	100%	94%	99%	99%	99%	99%	94%	97%	97%	97%	93%	93%	96%	96%	94%	92%	92%	92%	93%	94%	94%	95%
β=0.95	δ=0.05	81%	69%	66%	66%	65%	73%	68%	68%	68%	68%	77%	77%	77%	76%	75%	86%	85%	83%	82%	81%	90%	88%	87%	85%	84%
	δ=0.25	91%	70%	70%	70%	70%	83%	70%	71%	72%	72%	74%	78%	77%	77%	77%	86%	85%	84%	83%	82%	91%	89%	88%	87%	86%
	δ=0.50	100%	78%	75%	75%	75%	96%	71%	74%	75%	77%	87%	78%	80%	81%	81%	86%	86%	86%	86%	86%	92%	91%	89%	89%	89%
	δ=0.75	100%	100%	91%	83%	83%	99%	96%	83%	82%	82%	97%	87%	81%	83%	84%	90%	87%	88%	89%	89%	92%	92%	92%	92%	92%
	δ=0.95	100%	100%	100%	95%	95%	98%	98%	98%	94%	94%	97%	97%	97%	93%	94%	95%	95%	95%	93%	93%	93%	93%	94%	95%	95%

(a) $f = 1$



		f_m/f=0.05					f_m/f=0.25					f_m/f=0.50					f_m/f=0.75					f_m/f=0.95				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	71%	70%	70%	69%	69%	69%	69%	68%	68%	68%	66%	66%	67%	66%	67%	65%	66%	66%	66%	66%	67%	64%	65%	66%	66%
	δ=0.25	71%	71%	70%	70%	69%	70%	69%	69%	68%	68%	66%	66%	67%	67%	67%	64%	65%	66%	66%	66%	64%	66%	66%	66%	66%
	δ=0.50	71%	71%	71%	70%	70%	70%	70%	69%	69%	69%	66%	67%	68%	68%	68%	64%	65%	66%	67%	67%	63%	65%	66%	66%	66%
	δ=0.75	85%	71%	71%	72%	72%	75%	70%	70%	71%	71%	67%	68%	69%	70%	70%	64%	66%	67%	68%	69%	60%	64%	66%	67%	68%
	δ=0.95	100%	100%	99%	92%	85%	99%	98%	93%	84%	75%	96%	90%	76%	73%	73%	81%	68%	69%	71%	73%	52%	61%	66%	70%	72%
β=0.25	δ=0.05	67%	67%	67%	67%	67%	82%	64%	67%	65%	68%	56%	67%	65%	67%	66%	67%	65%	67%	67%	67%	67%	66%	64%	65%	66%
	δ=0.25	67%	67%	67%	67%	68%	91%	57%	66%	67%	67%	70%	65%	65%	67%	67%	64%	63%	64%	65%	65%	65%	65%	63%	63%	64%
	δ=0.50	67%	67%	78%	69%	69%	98%	79%	57%	68%	68%	87%	53%	66%	68%	68%	62%	63%	65%	65%	66%	61%	60%	62%	64%	65%
	δ=0.75	67%	92%	84%	76%	73%	98%	98%	91%	79%	68%	95%	87%	70%	71%	71%	76%	66%	67%	69%	69%	48%	59%	63%	67%	69%
	δ=0.95	67%	98%	96%	94%	93%	98%	98%	98%	98%	98%	95%	95%	95%	95%	95%	84%	84%	84%	83%	80%	46%	60%	67%	71%	74%
β=0.50	δ=0.05	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	66%	66%	65%	67%	67%	67%
	δ=0.25	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	77%	64%	65%	62%	63%	65%	65%	64%	65%	65%	62%
	δ=0.50	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	77%	69%	68%	65%	62%	65%	66%	67%	63%	63%	60%	62%	64%
	δ=0.75	67%	67%	67%	67%	67%	67%	67%	67%	67%	93%	67%	87%	78%	74%	74%	76%	65%	69%	70%	71%	60%	56%	62%	66%	69%
	δ=0.95	67%	67%	67%	67%	67%	67%	67%	67%	97%	96%	67%	91%	89%	87%	86%	76%	76%	78%	80%	82%	55%	59%	67%	72%	76%
β=0.75	δ=0.05	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	66%	67%	67%	67%	67%	67%	67%	67%	67%
	δ=0.25	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	66%	66%	66%	66%	63%	65%	65%	66%	66%	66%
	δ=0.50	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	51%	60%	62%	65%	66%	65%	65%	65%	61%	63%
	δ=0.75	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	88%	52%	63%	67%	70%	72%	65%	64%	60%	65%	68%
	δ=0.95	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	92%	91%	60%	70%	76%	79%	81%	63%	56%	66%	72%	76%
β=0.95	δ=0.05	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%
	δ=0.25	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	66%	67%	67%	67%	67%	67%	67%
	δ=0.50	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	66%	66%	64%	66%	66%	66%	66%	67%	63%
	δ=0.75	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	57%	65%	69%	71%	67%	66%	59%	64%	68%
	δ=0.95	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	66%	63%	72%	77%	80%	66%	66%	65%	72%	76%

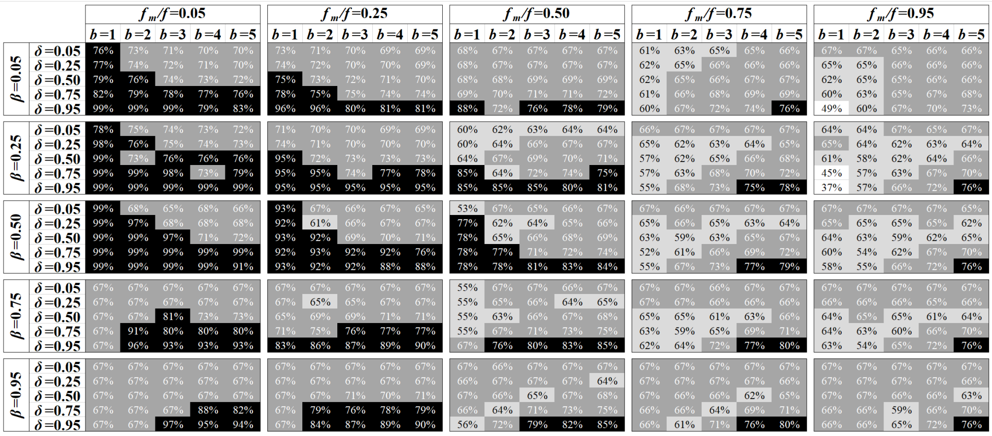
(b) $f = 5$



		f_m/f=0.05					f_m/f=0.25					f_m/f=0.50					f_m/f=0.75					f_m/f=0.95				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	68%	68%	68%	67%	67%	66%	66%	66%	66%	66%	64%	64%	65%	65%	67%	65%	67%	64%	67%	65%	67%	67%	66%	65%	66%
	δ=0.25	64%	68%	68%	68%	67%	66%	66%	66%	66%	67%	64%	66%	66%	66%	66%	63%	64%	66%	65%	66%	65%	63%	65%	66%	65%
	δ=0.50	84%	68%	68%	68%	68%	72%	67%	67%	67%	67%	65%	66%	66%	66%	66%	63%	64%	65%	65%	66%	56%	62%	64%	65%	65%
	δ=0.75	100%	84%	64%	68%	69%	97%	72%	67%	68%	68%	80%	66%	66%	67%	67%	60%	63%	64%	65%	66%	46%	58%	61%	63%	64%
	δ=0.95	100%	100%	100%	100%	100%	98%	98%	98%	98%	97%	95%	95%	95%	88%	80%	81%	72%	65%	67%	68%	55%	37%	50%	57%	60%
β=0.25	δ=0.05	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	65%	67%	66%	67%	67%	67%	67%	66%	67%	67%
	δ=0.25	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	77%	64%	64%	61%	63%	63%	62%	64%	65%	64%	65%
	δ=0.50	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	78%	66%	67%	59%	59%	60%	63%	63%	67%	61%	61%	57%	59%
	δ=0.75	67%	67%	67%	67%	67%	67%	67%	67%	67%	93%	67%	87%	79%	70%	70%	72%	59%	61%	65%	65%	66%	55%	45%	52%	57%
	δ=0.95	67%	67%	67%	67%	67%	67%	67%	67%	97%	96%	67%	92%	90%	88%	86%	72%	72%	72%	72%	73%	NoC	57%	49%	44%	53%
β=0.50	δ=0.05	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	68%	67%	67%	66%	65%
	δ=0.25	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	65%	60%	64%	65%	65%	66%
	δ=0.50	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	63%	59%	61%	NoC	65%	62%	62%	63%
	δ=0.75	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	54%	58%	61%	64%	NoC	64%	63%	60%	50%
	δ=0.95	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	57%	62%	67%	71%	NoC	NoC	63%	58%	47%
β=0.75	δ=0.05	NoC	67%	67%	67%	67%	NoC	67%	67%	67%	67%	NoC	67%	67%	67%	67%	NoC	67%	67%	67%	67%	NoC	67%	67%	66%	67%
	δ=0.25	NoC	67%	67%	67%	67%	NoC	67%	67%	67%	67%	NoC	67%	67%	67%	67%	NoC	67%	67%	67%	67%	NoC	66%	65%	66%	66%
	δ=0.50	NoC	67%	67%	67%	67%	NoC	67%	67%	67%	67%	NoC	67%	67%	67%	67%	NoC	67%	67%	67%	67%	NoC	67%	66%	66%	65%
	δ=0.75	NoC	67%	67%	67%	67%	NoC	67%	67%	67%	67%	NoC	67%	67%	67%	67%	NoC	67%	67%	67%	63%	NoC	53%	66%	65%	63%
	δ=0.95	NoC	67%	67%	67%	67%	NoC	67%	67%	67%	67%	NoC	67%	67%	67%	67%	NoC	67%	67%	60%	63%	NoC	NoC	60%	64%	62%
β=0.95	δ=0.05	NoC	NoC	67%	67%	67%	NoC	NoC	67%	67%	67%	NoC	NoC	67%	67%	67%	NoC	NoC	67%	67%	67%	NoC	NoC	67%	67%	67%
	δ=0.25	NoC	NoC	67%	67%	67%	NoC	NoC	67%	67%	67%	NoC	NoC	67%	67%	67%	NoC	NoC	67%	67%	67%	NoC	NoC	67%	66%	67%
	δ=0.50	NoC	NoC	67%	67%	67%	NoC	NoC	67%	67%	67%	NoC	NoC	67%	67%	67%	NoC	NoC	67%	67%	67%	NoC	NoC	66%	66%	67%
	δ=0.75	NoC	NoC	67%	67%	67%	NoC	NoC	67%	67%	67%	NoC	NoC	67%	67%	67%	NoC	NoC	67%	67%	67%	NoC	NoC	67%	67%	66%
	δ=0.95	NoC	NoC	67%	67%	67%	NoC	NoC	67%	67%	67%	NoC	NoC	67%	67%	67%	NoC	NoC	67%	67%	67%	NoC	NoC	65%	66%	66%

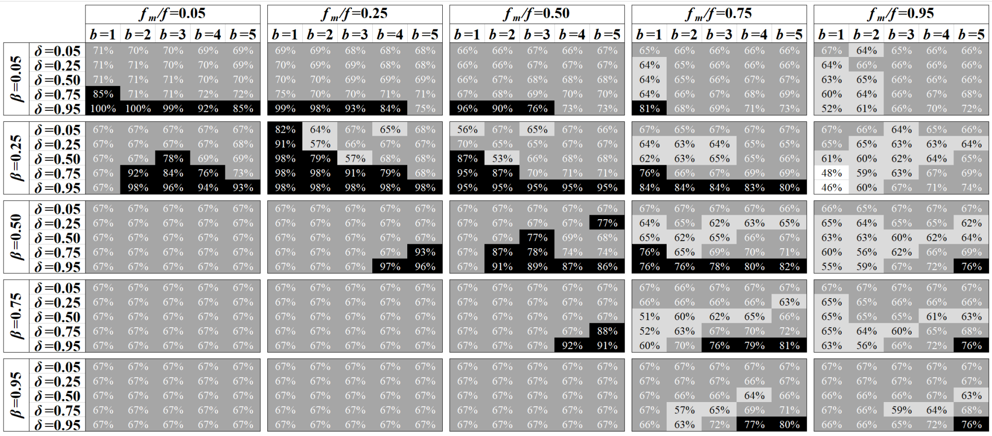
(c) $f = 10$

Figure 5.17: Impact of f on the manufacturer's share of chain's total profit, where the parameters are set to $s_n = 3, \upsilon = 3$, values below 50% are shown in white, values between 50% and 65% are shown in light gray, values between 65% and 75% are shown in dark gray, and values above 70% are shown in black



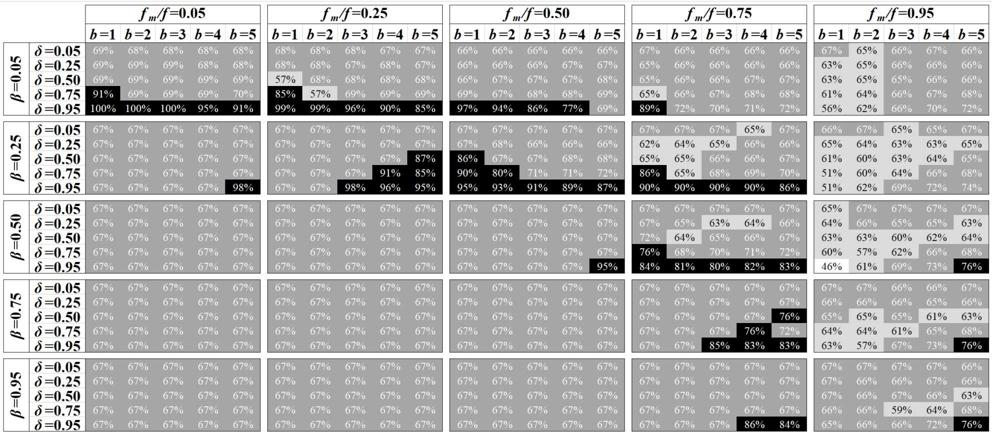
		f_m/f=0.05					f_m/f=0.25					f_m/f=0.50					f_m/f=0.75					f_m/f=0.95				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	76%	73%	71%	70%	70%	73%	71%	70%	69%	69%	68%	67%	67%	67%	67%	61%	63%	65%	65%	66%	67%	67%	65%	66%	66%
	δ=0.25	77%	74%	72%	71%	70%	74%	72%	70%	70%	69%	68%	67%	67%	67%	67%	62%	65%	66%	66%	66%	65%	65%	66%	66%	66%
	δ=0.50	79%	76%	74%	73%	72%	75%	73%	72%	71%	70%	68%	68%	69%	69%	69%	62%	65%	66%	67%	67%	62%	65%	65%	66%	66%
	δ=0.75	82%	79%	78%	77%	76%	78%	75%	75%	74%	74%	69%	70%	71%	71%	72%	61%	66%	68%	69%	69%	60%	63%	65%	67%	68%
	δ=0.95	99%	99%	99%	79%	83%	96%	96%	80%	81%	81%	88%	72%	76%	78%	79%	60%	67%	72%	74%	76%	49%	60%	67%	70%	73%
β=0.25	δ=0.05	78%	75%	74%	73%	72%	71%	70%	70%	69%	69%	60%	62%	63%	64%	64%	66%	67%	67%	67%	67%	64%	64%	67%	65%	67%
	δ=0.25	98%	76%	75%	74%	73%	74%	71%	70%	70%	70%	60%	64%	66%	67%	67%	65%	62%	63%	64%	65%	65%	64%	62%	63%	64%
	δ=0.50	99%	73%	76%	76%	76%	95%	72%	73%	73%	73%	64%	67%	69%	70%	71%	57%	62%	65%	66%	68%	61%	58%	62%	64%	66%
	δ=0.75	99%	99%	98%	73%	79%	95%	95%	74%	77%	78%	85%	64%	72%	74%	75%	57%	63%	68%	70%	72%	45%	57%	63%	67%	70%
	δ=0.95	99%	99%	99%	99%	99%	95%	95%	95%	95%	95%	85%	85%	85%	80%	81%	55%	68%	73%	75%	78%	37%	57%	66%	72%	76%
β=0.50	δ=0.05	99%	68%	65%	68%	66%	93%	67%	66%	67%	65%	53%	67%	65%	66%	67%	67%	66%	67%	67%	67%	67%	67%	67%	67%	65%
	δ=0.25	99%	97%	68%	68%	68%	92%	61%	66%	67%	67%	77%	62%	64%	65%	66%	65%	66%	65%	63%	64%	65%	65%	65%	65%	62%
	δ=0.50	99%	99%	97%	71%	72%	93%	92%	69%	70%	71%	78%	65%	66%	68%	69%	63%	59%	63%	65%	67%	64%	63%	59%	62%	65%
	δ=0.75	99%	99%	99%	99%	99%	92%	93%	92%	92%	76%	78%	77%	71%	72%	74%	52%	61%	66%	69%	72%	60%	54%	62%	67%	70%
	δ=0.95	99%	99%	99%	99%	91%	93%	92%	92%	88%	88%	78%	78%	81%	83%	84%	55%	67%	73%	77%	79%	58%	55%	66%	72%	76%
β=0.75	δ=0.05	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	55%	67%	67%	66%	67%	67%	67%	66%	67%	67%	67%	67%	67%	67%	67%
	δ=0.25	67%	67%	67%	67%	67%	67%	65%	65%	67%	67%	55%	65%	66%	64%	65%	67%	67%	66%	66%	66%	66%	66%	66%	65%	66%
	δ=0.50	67%	67%	81%	73%	73%	65%	69%	69%	71%	71%	55%	63%	66%	67%	68%	65%	65%	61%	63%	66%	64%	65%	65%	61%	64%
	δ=0.75	67%	91%	80%	80%	80%	71%	75%	76%	77%	77%	55%	67%	71%	73%	75%	63%	59%	65%	69%	71%	64%	63%	60%	66%	70%
	δ=0.95	67%	96%	93%	93%	93%	83%	86%	87%	89%	90%	67%	76%	80%	83%	85%	62%	64%	72%	77%	80%	63%	54%	65%	72%	76%
β=0.95	δ=0.05	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	66%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%
	δ=0.25	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	66%	67%	67%	67%	64%	67%	67%	67%	67%	66%	67%	67%	67%	67%	66%
	δ=0.50	67%	67%	67%	67%	67%	67%	67%	71%	70%	71%	67%	66%	65%	67%	68%	67%	66%	66%	62%	65%	67%	67%	66%	66%	63%
	δ=0.75	67%	67%	67%	88%	82%	67%	79%	76%	78%	79%	66%	64%	71%	73%	75%	66%	66%	64%	69%	71%	66%	66%	59%	66%	70%
	δ=0.95	67%	67%	97%	95%	94%	67%	84%	87%	89%	90%	56%	72%	79%	82%	85%	66%	61%	71%	76%	80%	66%	66%	65%	72%	76%

(a) $s_n = 1$



		f_m/f=0.05					f_m/f=0.25					f_m/f=0.50					f_m/f=0.75					f_m/f=0.95				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	71%	70%	70%	69%	69%	69%	69%	68%	68%	68%	66%	66%	67%	66%	67%	65%	66%	66%	66%	66%	67%	64%	65%	66%	66%
	δ=0.25	71%	71%	70%	70%	69%	70%	69%	69%	68%	68%	66%	66%	67%	67%	67%	64%	65%	66%	66%	66%	64%	66%	66%	66%	66%
	δ=0.50	71%	71%	71%	70%	70%	70%	70%	69%	69%	69%	66%	67%	68%	68%	68%	64%	65%	66%	67%	67%	63%	65%	66%	66%	66%
	δ=0.75	85%	71%	71%	72%	72%	75%	70%	70%	71%	71%	67%	68%	69%	70%	70%	64%	66%	67%	68%	69%	60%	64%	66%	67%	68%
	δ=0.95	100%	100%	99%	92%	85%	99%	98%	93%	84%	75%	96%	90%	76%	73%	73%	81%	68%	69%	71%	73%	52%	61%	66%	70%	72%
β=0.25	δ=0.05	67%	67%	67%	67%	67%	82%	64%	67%	65%	68%	56%	67%	65%	67%	66%	67%	65%	67%	67%	67%	67%	66%	64%	65%	66%
	δ=0.25	67%	67%	67%	67%	68%	91%	57%	66%	67%	67%	70%	65%	65%	67%	67%	64%	63%	64%	65%	65%	65%	65%	63%	63%	64%
	δ=0.50	67%	67%	78%	69%	69%	98%	79%	57%	68%	68%	87%	53%	66%	68%	68%	62%	63%	65%	65%	66%	61%	60%	62%	64%	65%
	δ=0.75	67%	92%	84%	76%	73%	98%	98%	91%	79%	68%	95%	87%	70%	71%	71%	76%	66%	67%	69%	69%	48%	59%	63%	67%	69%
	δ=0.95	67%	98%	96%	94%	93%	98%	98%	98%	98%	98%	95%	95%	95%	95%	95%	84%	84%	84%	83%	80%	46%	60%	67%	71%	74%
β=0.50	δ=0.05	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	66%	66%	65%	67%	67%	67%
	δ=0.25	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	77%	64%	65%	62%	63%	65%	65%	64%	65%	65%	62%
	δ=0.50	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	77%	69%	68%	65%	62%	65%	66%	67%	63%	63%	60%	62%	64%
	δ=0.75	67%	67%	67%	67%	67%	67%	67%	67%	67%	93%	67%	87%	78%	74%	74%	76%	65%	69%	70%	71%	60%	56%	62%	66%	69%
	δ=0.95	67%	67%	67%	67%	67%	67%	67%	67%	97%	96%	67%	91%	89%	87%	86%	76%	76%	78%	80%	82%	55%	59%	67%	72%	76%
β=0.75	δ=0.05	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	66%	67%	67%	67%	67%	67%	67%	67%	67%
	δ=0.25	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	66%	66%	66%	66%	63%	65%	65%	66%	66%	66%
	δ=0.50	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	51%	60%	62%	65%	66%	65%	65%	65%	61%	63%
	δ=0.75	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	88%	52%	63%	67%	70%	72%	65%	64%	60%	65%	68%
	δ=0.95	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	92%	91%	60%	70%	76%	79%	81%	63%	56%	66%	72%	76%
β=0.95	δ=0.05	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%
	δ=0.25	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	66%	67%	67%	67%	67%	67%	67%
	δ=0.50	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	66%	66%	64%	66%	66%	66%	66%	67%	63%
	δ=0.75	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	57%	65%	69%	71%	67%	66%	59%	64%	68%
	δ=0.95	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	66%	63%	72%	77%	80%	66%	66%	65%	72%	76%

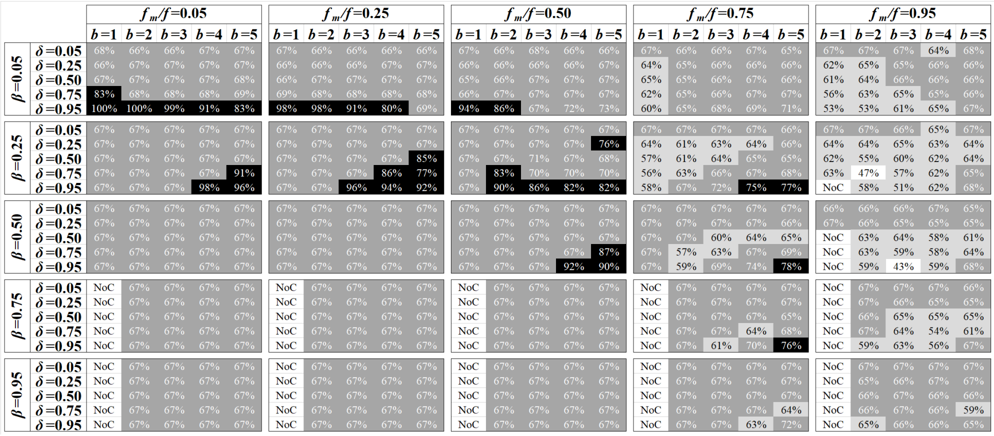
(b) $s_n = 3$



		f_m/f=0.05					f_m/f=0.25					f_m/f=0.50					f_m/f=0.75					f_m/f=0.95				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	69%	68%	68%	68%	68%	68%	68%	68%	67%	67%	66%	66%	66%	66%	66%	67%	66%	66%	66%	66%	67%	65%	66%	67%	66%
	δ=0.25	69%	69%	69%	68%	68%	68%	68%	67%	68%	67%	66%	66%	66%	67%	67%	65%	66%	66%	66%	66%	63%	65%	66%	66%	66%
	δ=0.50	69%	69%	69%	69%	69%	57%	68%	68%	68%	68%	66%	67%	67%	67%	68%	65%	66%	66%	67%	67%	63%	65%	65%	66%	66%
	δ=0.75	91%	69%	69%	69%	70%	85%	57%	69%	69%	69%	69%	67%	68%	68%	69%	65%	66%	67%	68%	68%	61%	64%	66%	67%	68%
	δ=0.95	100%	100%	100%	95%	91%	99%	99%	96%	90%	85%	97%	94%	86%	77%	69%	89%	72%	70%	71%	72%	56%	62%	66%	70%	72%
β=0.25	δ=0.05	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	65%	67%	66%	67%	65%	65%	67%
	δ=0.25	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	68%	66%	66%	66%	62%	64%	65%	66%	66%	65%	64%	63%	63%	65%
	δ=0.50	67%	67%	67%	67%	67%	67%	67%	67%	67%	87%	86%	67%	67%	68%	68%	65%	65%	66%	66%	67%	61%	60%	63%	64%	65%
	δ=0.75	67%	67%	67%	67%	67%	67%	67%	67%	91%	85%	90%	80%	71%	71%	72%	86%	65%	68%	69%	70%	51%	60%	64%	66%	68%
	δ=0.95	67%	67%	67%	67%	98%	67%	67%	98%	96%	95%	95%	93%	91%	89%	87%	90%	90%	90%	90%	86%	51%	62%	69%	72%	74%
β=0.50	δ=0.05	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	65%	67%	67%	67%	67%
	δ=0.25	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	65%	63%	64%	66%	64%	66%	65%	65%	63%
	δ=0.50	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	72%	64%	65%	66%	67%	63%	63%	60%	62%	64%
	δ=0.75	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	76%	68%	70%	71%	72%	60%	57%	62%	66%	68%
	δ=0.95	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	95%	84%	81%	80%	82%	83%	46%	61%	69%	73%	76%
β=0.75	δ=0.05	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	66%	66%	67%
	δ=0.25	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	66%	66%	66%	65%	66%
	δ=0.50	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	76%	65%	65%	65%	61%	63%
	δ=0.75	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	76%	72%	64%	64%	61%	65%	68%
	δ=0.95	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	85%	83%	83%	63%	57%	67%	73%	76%
β=0.95	δ=0.05	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	66%
	δ=0.25	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	66%	66%	67%	66%
	δ=0.50	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	66%	67%	66%	63%
	δ=0.75	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	66%	66%	59%	64%	68%
	δ=0.95	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	86%	84%	65%	66%	66%	72%	76%

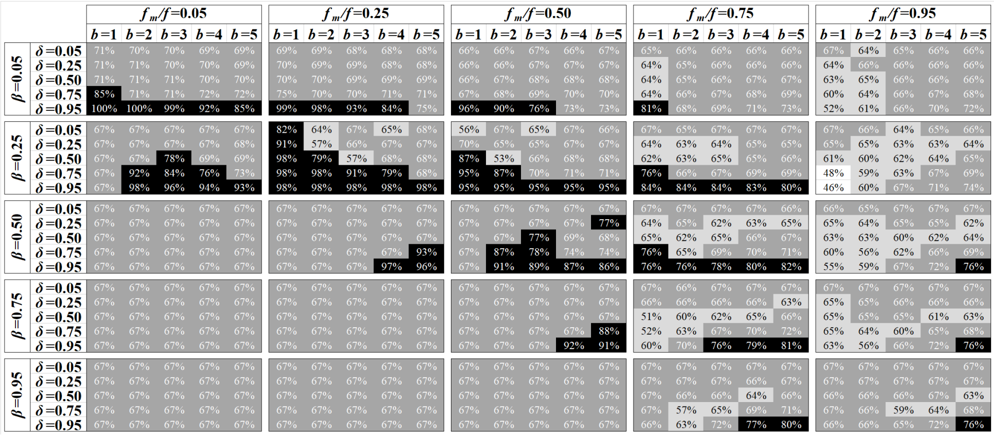
(c) $s_n = 5$

Figure 5.18: Impact of s_n on the manufacturer's share of chain's total profit, where the parameters are set to $f = 5, v = 3$, values below 50% are shown in white, values between 50% and 65% are shown in light gray, values between 65% and 75% are shown in dark gray, and values above 70% are shown in black



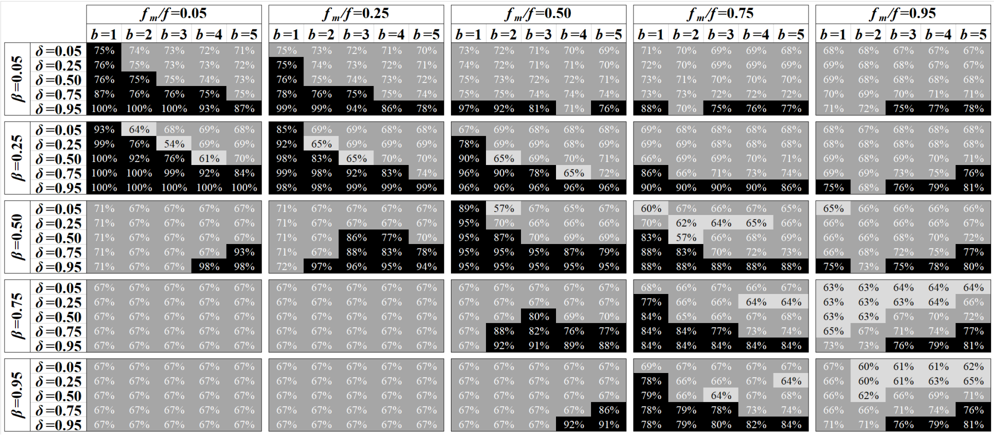
		f_m/f=0.05					f_m/f=0.25					f_m/f=0.50					f_m/f=0.75					f_m/f=0.95				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	68%	66%	66%	67%	67%	67%	66%	66%	66%	66%	67%	66%	68%	66%	66%	67%	66%	66%	67%	65%	67%	67%	67%	64%	68%
	δ=0.25	66%	67%	67%	67%	67%	66%	66%	66%	67%	67%	66%	66%	66%	66%	67%	64%	65%	66%	66%	66%	62%	65%	65%	66%	66%
	δ=0.50	67%	67%	67%	67%	68%	66%	67%	67%	67%	67%	65%	66%	67%	67%	67%	65%	65%	66%	67%	66%	61%	64%	66%	66%	66%
	δ=0.75	83%	68%	68%	68%	69%	69%	68%	68%	68%	68%	66%	67%	67%	67%	67%	62%	65%	66%	67%	67%	56%	63%	65%	65%	66%
	δ=0.95	100%	100%	99%	91%	83%	98%	98%	91%	80%	69%	94%	86%	67%	72%	73%	60%	65%	68%	69%	71%	53%	53%	61%	65%	67%
β=0.25	δ=0.05	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	66%	67%	67%	66%	65%	67%
	δ=0.25	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	76%	64%	61%	63%	64%	66%	64%	64%	65%	63%	64%
	δ=0.50	67%	67%	67%	67%	67%	67%	67%	67%	67%	85%	67%	67%	71%	67%	68%	57%	61%	64%	65%	65%	62%	55%	60%	62%	64%
	δ=0.75	67%	67%	67%	67%	91%	67%	67%	67%	86%	77%	67%	83%	70%	70%	70%	56%	63%	66%	67%	68%	63%	47%	57%	62%	65%
	δ=0.95	67%	67%	67%	98%	96%	67%	67%	96%	94%	92%	67%	90%	86%	82%	82%	58%	67%	72%	75%	77%	NoC	58%	51%	62%	68%
β=0.50	δ=0.05	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	66%	66%	66%	67%	65%
	δ=0.25	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	66%	67%	66%	65%	65%	65%
	δ=0.50	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	60%	64%	65%	NoC	63%	64%	58%	61%
	δ=0.75	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	87%	67%	57%	63%	67%	69%	NoC	63%	59%	58%	64%
	δ=0.95	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	92%	90%	67%	59%	69%	74%	78%	NoC	59%	43%	59%	68%
β=0.75	δ=0.05	NoC	67%	67%	67%	67%	NoC	67%	67%	67%	67%	NoC	67%	67%	67%	67%	NoC	67%	67%	67%	67%	NoC	67%	67%	67%	66%
	δ=0.25	NoC	67%	67%	67%	67%	NoC	67%	67%	67%	67%	NoC	67%	67%	67%	67%	NoC	67%	67%	67%	67%	NoC	67%	66%	65%	65%
	δ=0.50	NoC	67%	67%	67%	67%	NoC	67%	67%	67%	67%	NoC	67%	67%	67%	67%	NoC	67%	67%	67%	65%	NoC	66%	65%	65%	65%
	δ=0.75	NoC	67%	67%	67%	67%	NoC	67%	67%	67%	67%	NoC	67%	67%	67%	67%	NoC	67%	67%	64%	68%	NoC	67%	64%	54%	61%
	δ=0.95	NoC	67%	67%	67%	67%	NoC	67%	67%	67%	67%	NoC	67%	67%	67%	67%	NoC	67%	61%	70%	76%	NoC	59%	63%	56%	67%
β=0.95	δ=0.05	NoC	67%	67%	67%	67%	NoC	67%	67%	67%	67%	NoC	67%	67%	67%	67%	NoC	67%	67%	67%	67%	NoC	67%	67%	67%	67%
	δ=0.25	NoC	67%	67%	67%	67%	NoC	67%	67%	67%	67%	NoC	67%	67%	67%	67%	NoC	67%	67%	67%	67%	NoC	65%	66%	67%	67%
	δ=0.50	NoC	67%	67%	67%	67%	NoC	67%	67%	67%	67%	NoC	67%	67%	67%	67%	NoC	67%	67%	67%	67%	NoC	66%	67%	66%	66%
	δ=0.75	NoC	67%	67%	67%	67%	NoC	67%	67%	67%	67%	NoC	67%	67%	67%	67%	NoC	67%	67%	67%	64%	NoC	66%	67%	66%	59%
	δ=0.95	NoC	67%	67%	67%	67%	NoC	67%	67%	67%	67%	NoC	67%	67%	67%	67%	NoC	67%	67%	63%	72%	NoC	65%	66%	66%	65%

(a) $\upsilon = 1$



		f_m/f=0.05					f_m/f=0.25					f_m/f=0.50					f_m/f=0.75					f_m/f=0.95				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	71%	70%	70%	69%	69%	69%	69%	68%	68%	68%	66%	66%	67%	66%	67%	65%	66%	66%	66%	66%	67%	64%	65%	66%	66%
	δ=0.25	71%	71%	70%	70%	69%	70%	69%	69%	68%	68%	66%	66%	67%	67%	67%	64%	65%	66%	66%	66%	64%	66%	66%	66%	66%
	δ=0.50	71%	71%	71%	70%	70%	70%	70%	69%	69%	69%	66%	67%	68%	68%	68%	64%	65%	66%	67%	67%	63%	65%	66%	66%	66%
	δ=0.75	85%	71%	71%	72%	72%	75%	70%	70%	71%	71%	67%	68%	69%	70%	70%	64%	66%	67%	68%	69%	60%	64%	66%	67%	68%
	δ=0.95	100%	100%	99%	92%	85%	99%	98%	93%	84%	75%	96%	90%	76%	73%	73%	81%	68%	69%	71%	73%	52%	61%	66%	70%	72%
β=0.25	δ=0.05	67%	67%	67%	67%	67%	82%	64%	67%	65%	68%	56%	67%	65%	67%	66%	67%	65%	67%	67%	67%	67%	66%	64%	65%	66%
	δ=0.25	67%	67%	67%	67%	68%	91%	57%	66%	67%	67%	70%	65%	65%	67%	67%	64%	63%	64%	65%	65%	65%	65%	63%	63%	64%
	δ=0.50	67%	67%	78%	69%	69%	98%	79%	57%	68%	68%	87%	53%	66%	68%	68%	62%	63%	65%	65%	66%	61%	60%	62%	64%	65%
	δ=0.75	67%	92%	84%	76%	73%	98%	98%	91%	79%	68%	95%	87%	70%	71%	71%	76%	66%	67%	69%	69%	48%	59%	63%	67%	69%
	δ=0.95	67%	98%	96%	94%	93%	98%	98%	98%	98%	98%	95%	95%	95%	95%	95%	84%	84%	84%	83%	80%	46%	60%	67%	71%	74%
β=0.50	δ=0.05	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	66%	66%	65%	67%	67%	67%
	δ=0.25	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	77%	64%	65%	62%	63%	65%	65%	64%	65%	65%	62%
	δ=0.50	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	77%	69%	68%	65%	62%	65%	66%	67%	63%	63%	60%	62%	64%
	δ=0.75	67%	67%	67%	67%	67%	67%	67%	67%	67%	93%	67%	87%	78%	74%	74%	76%	65%	69%	70%	71%	60%	56%	62%	66%	69%
	δ=0.95	67%	67%	67%	67%	67%	67%	67%	67%	97%	96%	67%	91%	89%	87%	86%	76%	76%	78%	80%	82%	55%	59%	67%	72%	76%
β=0.75	δ=0.05	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	66%	67%	67%	67%	67%	67%	67%	67%	67%
	δ=0.25	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	66%	66%	66%	66%	63%	65%	65%	66%	66%	66%
	δ=0.50	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	51%	60%	62%	65%	66%	65%	65%	65%	61%	63%
	δ=0.75	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	88%	52%	63%	67%	70%	72%	65%	64%	60%	65%	68%
	δ=0.95	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	92%	91%	60%	70%	76%	79%	81%	63%	56%	66%	72%	76%
β=0.95	δ=0.05	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%
	δ=0.25	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	66%	67%	67%	67%	67%	67%	67%
	δ=0.50	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	66%	66%	64%	66%	66%	66%	66%	67%	63%
	δ=0.75	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	57%	65%	69%	71%	67%	66%	59%	64%	68%
	δ=0.95	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	66%	63%	72%	77%	80%	66%	66%	65%	72%	76%

(b) $\upsilon = 3$



		f_m/f=0.05					f_m/f=0.25					f_m/f=0.50					f_m/f=0.75					f_m/f=0.95				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	75%	74%	73%	72%	71%	75%	73%	72%	71%	70%	73%	72%	71%	70%	69%	71%	70%	69%	69%	68%	68%	68%	67%	67%	67%
	δ=0.25	76%	75%	73%	73%	72%	75%	74%	73%	72%	71%	74%	72%	71%	71%	70%	72%	70%	69%	69%	69%	69%	68%	68%	67%	67%
	δ=0.50	76%	75%	75%	74%	73%	76%	75%	74%	73%	72%	75%	73%	72%	72%	71%	73%	71%	70%	70%	70%	69%	68%	68%	68%	68%
	δ=0.75	87%	76%	76%	75%	75%	78%	76%	75%	75%	74%	75%	75%	74%	74%	74%	73%	73%	72%	72%	72%	70%	69%	70%	71%	71%
	δ=0.95	100%	100%	100%	93%	87%	99%	99%	94%	86%	78%	97%	92%	81%	71%	76%	88%	70%	75%	76%	77%	71%	72%	75%	77%	78%
β=0.25	δ=0.05	93%	64%	68%	69%	68%	85%	69%	69%	68%	68%	67%	69%	68%	68%	68%	69%	68%	68%	68%	68%	68%	67%	68%	68%	68%
	δ=0.25	99%	76%	54%	69%	69%	92%	65%	69%	69%	69%	78%	69%	69%	68%	69%	69%	69%	68%	68%	68%	68%	68%	68%	68%	68%
	δ=0.50	100%	92%	76%	61%	70%	98%	83%	65%	70%	70%	90%	65%	69%	70%	71%	66%	69%	69%	70%	71%	69%	68%	69%	70%	71%
	δ=0.75	100%	100%	99%	92%	84%	99%	98%	92%	83%	74%	96%	90%	78%	65%	72%	86%	66%	71%	73%	74%	69%	69%	73%	75%	76%
	δ=0.95	100%	100%	100%	100%	100%	98%	98%	99%	99%	99%	96%	96%	96%	96%	96%	90%	90%	90%	90%	86%	75%	68%	76%	79%	81%
β=0.50	δ=0.05	71%	67%	67%	67%	67%	71%	67%	67%	67%	67%	89%	57%	67%	65%	67%	60%	67%	66%	67%	65%	65%	66%	66%	66%	66%
	δ=0.25	71%	67%	67%	67%	67%	71%	67%	67%	67%	67%	95%	70%	66%	66%	66%	70%	62%	64%	65%	66%	66%	66%	66%	66%	67%
	δ=0.50	71%	67%	67%	67%	67%	71%	67%	86%	77%	70%	95%	87%	70%	69%	69%	83%	57%	66%	68%	69%	66%	66%	68%	70%	72%
	δ=0.75	71%	67%	67%	67%	93%	71%	67%	88%	83%	78%	95%	95%	95%	87%	79%	88%	83%	70%	72%	73%	66%	68%	72%	75%	77%
	δ=0.95	71%	67%	67%	98%	98%	72%	97%	96%	95%	94%	95%	95%	95%	95%	95%	88%	88%	88%	88%	88%	75%	73%	75%	78%	80%
β=0.75	δ=0.05	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	68%	66%	67%	66%	67%	63%	63%	64%	64%	64%
	δ=0.25	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	77%	66%	66%	64%	64%	63%	63%	63%	64%	66%
	δ=0.50	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	80%	69%	70%	84%	65%	66%	67%	68%	63%	63%	67%	70%	72%
	δ=0.75	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	88%	82%	76%	77%	84%	84%	77%	73%	74%	65%	67%	71%	74%	77%
	δ=0.95	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	92%	91%	89%	88%	84%	84%	84%	84%	84%	73%	73%	76%	79%	81%
β=0.95	δ=0.05	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	69%	67%	67%	67%	67%	67%	60%	61%	61%	62%
	δ=0.25	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	78%	66%	66%	67%	64%	66%	60%	61%	63%	65%
	δ=0.50	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	79%	66%	64%	67%	68%	66%	62%	66%	69%	71%
	δ=0.75	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	86%	78%	79%	78%	73%	74%	66%	66%	71%	74%	76%
	δ=0.95	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	67%	92%	91%	78%	79%	80%	82%	84%	71%	71%	76%	79%	81%

(c) $\upsilon = 5$

Figure 5.19: Impact of υ on the manufacturer's share of chain's total profit, where the parameters are set to $f = 5, s_n = 3$, values below 50% are shown in white, values between 50% and 65% are shown in light gray, values between 65% and 75% are shown in dark gray, and values above 70% are shown in black

(or make no difference, rarely resulting in a worse consumer surplus value).

We see that the manufacturer's utilizes different pricing strategies depending on the market conditions, shifting her profitability between her product sales and after-sales services. When the situation calls for it, she accepts losses on either side; focusing on after-sales profits for instances that revert to the monopoly model, and focusing on product sales profits for high warranty coverage scenarios. However, she benefits most from the instances where she can rely on both streams as profit generators.

Downstream competition increases the manufacturer's potential to capture more of the chain profits, especially for lower values of total number of repairs and higher values of independent workshop similarity.

For especially the situations with very low consumer awareness of future costs, the consumer surplus can become very negative. Therefore, similar to the monopoly model, it is important for regulatory authorities to take measures in increasing the after-sales cost transparency. On the other hand, the manufacturers would be advised to withhold this information.

Considering our experimental setting, the parameter that has the biggest scale of impact is the total number of repairs, which can be related with the designed durability of the product. A lower value protects the customers from extremely negative consumer surplus values to some extent and also smoothes the manufacturer's profit function potential across different parameter values. This is to the manufacturer's disadvantage when consumers have low awareness of future costs, but to her advantage when they have high awareness of future costs. We would again advise the regulatory authorities to encourage the manufacturers to change their designs such that fewer repair and maintenances are needed. The manufacturers would already be willing to do so when the consumers' awareness of future costs is high, so it would be a good combination with the previous measure we recommended.

CHAPTER 6

CONCLUSION AND FUTURE RESEARCH DIRECTIONS

In this thesis, we have mainly studied two alternative models and their extensions for the after-sales service channel structure of a durable goods manufacturer, the first considering a monopolized aftermarket and the second considering downstream competition for after-sales goods and services. The major contribution of these new models is that the customers evaluate the total cost of ownership of a product while making their purchasing decision. We identified optimal pricing of the product and its after-sales services analytically where possible, and derived insights based on numerical studies otherwise.

For the monopoly model, we have shown that the manufacturer extracts all of her profit from the after-sales services of the product. This has two implications: (1) having guaranteed the after-sales revenues and profits, the manufacturer gives away the product itself for free; (2) the extent of the warranty coverage offered by the manufacturer does not have any effect on the equilibrium results, because the customer essentially pays for it throughout the remaining after-sales services she receives. This result is mainly due to the lack of full foresight of the after-sales costs for the customer. For the downstream competition model, this is among the possible results, especially when the total cost of ownership of the independent workshop alternative is not cheap enough. But, in other cases, the manufacturer is more flexible in utilizing both sales and after-sales channel profits. Especially under very high warranty coverage scenarios, the manufacturer tends to make a loss in after-sales and compensate this by profits from product sales.

As a benchmark to the monopoly model, we have analyzed the third party model where the after-sales services are provided by an independent retailer and investigated when the manufacturer would prefer being a monopolistic provider of after-sales services. We have found that the manufacturer prefers to control the after-sales market when customers are myopic. When customer foresight is moderate to high, manufacturer's efficiency in after-sales services, the base product valuation in the market and customer heterogeneity in valuation affect the manufacturer's preferences as well. In these cases, the manufacturer favors the decentralized system if she is sufficiently efficient in after-sales services as compared to the independent retailer option. From the customer perspective, we see that third party is the model that is mostly favored, especially in markets with high product valuation and high heterogeneity. Decentralized model may be preferable in rare cases only, when both models perform badly with very limited market coverage. As a consequence, we observe that the manufacturer's and the consumer's preferences align occasionally, and mostly on the third party model, when customers are strategic, and especially if the manufacturer is not efficient in after-sales services.

As an extension to the monopoly model, we also studied a model where the market is composed of two distinct segments: a fraction of the customers are myopic with low foresight of future costs, and the rest are strategic with high foresight of future costs. In this model, we mainly see that the manufacturer and the retailer follow a pricing strategy that chooses to focus on the myopic customer segment, and address the strategic segment only when they can capture the additional potential it offers without significantly lowering the prices. We also found that most results of the decentralized model also hold when the customers are heterogeneous in their foresight of future costs: First of all, the manufacturer makes the majority of her revenues and profits from after-sales rather than product sales, except for a few instances where the product price is positive. Secondly, the manufacturer captures two-thirds of the total chain profit in the majority of the instances, consistent with the results of the decentralized model. Thirdly, as the average customer foresight increases, the spare parts wholesale and retail prices, as well as the manufacturer and retailer profits decrease, and the consumer surplus increases; except for rare cases with high disparity in consumer foresight. Finally, we found that the profits and consumer surplus are not sensitive to different levels of warranty coverage offered by the manufacturer.

Comparing the downstream competition model with the monopoly model, we studied under which conditions the manufacturer benefits from the existence of an independent workshop. We found that although the independent workshop increases her market coverage in some cases, the manufacturer is still better off mostly with the monopoly model from a profit comparison perspective. Reversely, having the independent workshop as an alternative usually improves the consumer surplus as compared to the monopoly model, unless the total cost of ownership of the non-original alternative is too high and the monopoly model becomes the only feasible alternative. The consumer and the manufacturer align on the downstream competition model for some cases, mostly when the total cost of ownership alternative is not too low to force the manufacturer to bring her prices down, but helps to expand her market instead.

In both models, the manufacturer captures more of the chain profits. Downstream competition increases the manufacturer's potential to capture more of the chain profits, especially for lower values of total number of repairs and higher values of independent workshop similarity.

Based on our findings and insights for all alternative models, the most influential market parameter is the consumer awareness of future costs. We would advise a manufacturer to avoid measures that would actively increase her after-sales cost transparency. We would advise the regulatory authorities and non-governmental organizations to focus their effort on increasing the transparency of after-sales costs, to consequently increase the consumer surplus in the market. Additionally, they should be aware that when both the sales and the after-sales markets are a monopoly, a law-mandated warranty coverage is irrelevant in any case. From a regulatory authority's perspective, one should be careful when mandating a warranty coverage and forcing the manufacturer to participate in the after-sales market, especially when the sales market is a monopoly. In majority of the cases, that could work to the disadvantage of the consumer. The competitive structure in the after-sales market also needs to be taken into account, along with other market characteristics, since it will influence the effect of a regulation. Unlike the monopoly model, we find that a higher warranty coverage is better for the customer and worse for the manufacturer under the downstream competition model, especially for cases where the independent workshop is also active in the market. Therefore, the authorities mandating a minimum warranty coverage would likely improve the consumer surplus in these scenarios (or make no difference, rarely resulting in a worse consumer surplus value).

The second most influential market parameter is the total number of repairs, which can be related with the designed durability or reliability of the product, and also reflects to the total after-sales costs. Reducing this value improves both manufacturer and retailer profits, as well as consumer surplus, creating a win-win scenario. For the downstream competition model, lower number of lifetime repairs protects the customers from extremely negative consumer surplus values to some extent and also smoothes the manufacturer's profit function potential across different parameter values. This is to the manufacturer's disadvantage when consumers have low awareness of future costs, but to her advantage when they have high awareness of future costs. We would again advise the regulatory authorities to encourage the manufacturers to change their designs such that fewer repair and maintenances are needed. The manufacturers would already be willing to do so when the consumers' awareness of future costs is high, so it would be a good combination with the previous measure we recommended.

As future research directions, the models can be extended and brought closer to the real life applications. First of all, the retailers also commonly sell non-original spare parts. After this change is introduced, the next step could also consider including non-original spare part manufacturers/wholesalers and their pricing in the modeling. Secondly, a competition dimension on the product level could be added. Thirdly, the awareness of future costs can be handled differently, maybe also allowing for overestimation.

Another approach could be considering the complete lifecycle of the product, integrating planned lifetime, replacement decisions, and used product markets. This kind of approach has been taken in a limited fashion in the literature, but no study is available where the after-sales decisions are also taken into consideration along with these decisions.

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Appendix A

ADDITIONAL MATERIAL FOR MONOPOLY MODEL

A.1 Additional Theoretical Results

Lemma 9 *The following statements are true regarding the profit function of the retailer for the decentralized model,*

$$\Pi_r^D(p,\omega_o,s_o) = \begin{cases} \Pi_{r0}^D(p,\omega_o,s_o) = f\alpha(l_r - c_r) + f_e(s_o - \omega_o), \\ \hfill \textit{if } s_o \le \frac{v-p}{\beta f_e} - \alpha l_r, \\ \Pi_{r1}^D(p,\omega_o,s_o) = (f\alpha(l_r - c_r) + f_e(s_o - \omega_o))\left(1 - \frac{\beta f_e(s_o+\alpha l_r)-v+p}{b}\right), \\ \hfill \textit{if } \frac{v-p}{\beta f_e} - \alpha l_r < s_o \le \frac{v-p+b}{\beta f_e} - \alpha l_r, \\ \Pi_{r2}^D(p,\omega_o,s_o) = 0, \\ \hfill \textit{if } s_o > \frac{v-p+b}{\beta f_e} - \alpha l_r\text{:} \end{cases}$$

1. $\Pi_r^D(p,\omega_o,s_o)$ *is continuous in* s_o.

2. $\Pi_{r0}^D(p,\omega_o,s_o)$ *is increasing in* s_o.

3. $\Pi_{r1}^D(p,\omega_o,s_o)$ *is concave in* s_o.

Lemma 10 *The following statements are true regarding the manufacturer's profit* *function for the decentralized model given the retailer's best response,*

$$\Pi_m^D(p,\omega_o,s_o^*(p,\omega_o))$$
$$= \begin{cases} \Pi_{m0}^D(p,\omega_o) = p - c + f_e\omega_o - fc_o - f_m\alpha l_r, \\ \qquad \textit{if}\, \omega_o \le \frac{\upsilon-p-b}{\beta f_e} + \frac{f_m\alpha(l_r-c_r)}{f_e} - \alpha c_r, \\ \Pi_{m1}^D(p,\omega_o) = \frac{1}{2b}(p - c + f_e\omega_o - fc_o - f_m\alpha l_r) \\ \qquad\qquad \times (b + \upsilon - p + \beta(f_m\alpha(l_r - c_r) - f_e(\omega_o + \alpha c_r)))\,, \\ \qquad \textit{if}\, \frac{\upsilon-p-b}{\beta f_e} + \frac{f_m\alpha(l_r-c_r)}{f_e} - \alpha c_r < \omega_o \le \frac{\upsilon-p+b}{\beta f_e} + \frac{f_m\alpha(l_r-c_r)}{f_e} - \alpha c_r, \\ \Pi_{m2}^D(p,\omega_o) = 0, \\ \qquad\qquad \textit{if}\, \omega_o > \frac{\upsilon-p+b}{\beta f_e} + \frac{f_m\alpha(l_r-c_r)}{f_e} - \alpha c_r : \end{cases}$$

1. $\Pi_m^D(p,\omega_o,s_o^*(p,\omega_o))$ *is continuous in* ω_o*.*

2. $\Pi_{m0}^D(p,\omega_o,s_o^*(p,\omega_o))$ *is increasing in* ω_o*.*

3. $\Pi_{m1}^D(p,\omega_o,s_o^*(p,\omega_o))$ *is concave in* ω_o*.*

Lemma 11 *The following statements are true regarding the manufacturer's profit function for the decentralized model for a given value of the product retail price,*

$$\Pi_m^D(p,\omega_o^*(p),s_o^*(p))$$
$$= \begin{cases} \Pi_{m0}^D(p) = \frac{\upsilon-p-b-\beta(f(c_o+\alpha c_r)-p+c)}{\beta}, \\ \qquad\qquad \textit{if}\, p \le \frac{\upsilon-3b-\beta(f(c_o+\alpha c_r)+c)}{1-\beta}, \\ \Pi_{m1}^D(p) = \frac{(\upsilon-p+b-\beta(f(c_o+\alpha c_r)-p+c))^2}{8\beta b}, \\ \qquad \textit{if}\, \frac{\upsilon-3b-\beta(f(c_o+\alpha c_r)+c)}{1-\beta} < p \le \frac{\upsilon+b-\beta(f(c_o+\alpha c_r)+c)}{1-\beta}, \\ \Pi_{m2}^D(p) = 0, \\ \qquad\qquad \textit{if}\, p > \frac{\upsilon+b-\beta(f(c_o+\alpha c_r)+c)}{1-\beta} : \end{cases}$$

1. $\Pi_m^D(p,\omega_o^*(p),s_o^*(p))$ *is continuous in p.*

2. $\Pi_{m0}^{D1}(p,\omega_o^*(p),s_o^*(p))$ *is decreasing in p.*

3. $\Pi_{m1}^{D1}(p,\omega_o^*(p),s_o^*(p))$ *is convex in p.*

Lemma 12 *The following statements are true regarding the profit function of the* *system under the centralized model,*

$$\Pi^{CM}(p, s_o)$$

$$= \begin{cases} \Pi_0^{CM}(p, s_o) = p - c - f_m(\alpha c_r + c_o) + f_e(s_o - c_o + \alpha(l_r - c_r)), \\ \hfill \textit{if } s_o \leq \frac{\upsilon - p}{\beta f_e} - \alpha l_r, \\ \Pi_1^{CM}(p, s_o) = (p - c - f_m(\alpha c_r + c_o) + f_e(s_o - c_o + \alpha(l_r - c_r))) \\ \hfill \times \left(1 - \frac{\beta f_e(\alpha l_r + s_o) - \upsilon + p}{b}\right), \\ \hfill \textit{if } \frac{\upsilon - p}{\beta f_e} - \alpha l_r < s_o \leq \frac{\upsilon - p + b}{\beta f_e} - \alpha l_r, \\ \Pi_2^{CM}(p, s_o) = 0, \\ \hfill \textit{if } s_o > \frac{\upsilon - p + b}{\beta f_e} - \alpha l_r: \end{cases}$$

1. *$\Pi^{CM}(p, s_o)$ is continuous in s_o.*

2. *$\Pi_0^{CM}(p, s_o)$ is increasing in s_o.*

3. *$\Pi_1^{CM}(p, s_o)$ is concave in s_o.*

Proposition 11 *The optimal price decision for the system, s_o^* for a given value of p under the centralized model, is as follows:*

$$s_o^*(p) = \begin{cases} \frac{\upsilon - p}{\beta f_e} - \alpha l_r, \\ \hfill \textit{if } \beta(f(c_o + \alpha c_r) - p + c) \leq \upsilon - p - b, \\ \frac{\upsilon - p + b + \beta(f(c_o + \alpha c_r) - p + c)}{2\beta f_e} - \alpha l_r, \\ \hfill \textit{if } \upsilon - p - b < \beta(f(c_o + \alpha c_r) - p + c) \leq \upsilon - p + b, \\ \left[\frac{\upsilon - p + b}{\beta f_e} - \alpha l_r \text{ , } \infty\right), \\ \hfill \textit{if } \beta(f(c_o + \alpha c_r) - p + c) > \upsilon - p + b : \end{cases}$$

Lemma 13 *The following statements are true regarding the system's profit function given the spare parts sales price under the centralized model:*

$$\Pi^{CM}(p, s_o^*(p))$$

$$= \begin{cases} \Pi_0^{CM}(p) = \frac{\upsilon - p - \beta(f(c_o + \alpha c_r) - p + c)}{\beta}, \\ \hfill \textit{if } p \leq \frac{\upsilon - b - \beta(f(c_o + \alpha c_r) + c)}{1 - \beta}, \\ \Pi_1^{CM}(p) = \frac{(\upsilon - p + b - \beta F)^2}{4\beta b}, \\ \hfill \textit{if } \frac{\upsilon - b - \beta(f(c_o + \alpha c_r) + c)}{1 - \beta} < p \leq \frac{\upsilon + b - \beta(f(c_o + \alpha c_r) + c)}{1 - \beta}, \\ \Pi_2^{CM}(p) = 0, \\ \hfill \textit{if } p > \frac{\upsilon + b - \beta(f(c_o + \alpha c_r) + c)}{1 - \beta} : \end{cases}$$

1. $\Pi^{CM}(p, s_o^*(p))$ *is continuous in* p*.*

2. $\Pi_0^{CM}(p, s_o^*(p))$ *is decreasing in* p*.*

3. $\Pi_1^{CM}(p, s_o^*(p))$ *is convex in* p*.*

Proposition 12 *The equilibrium product retail price of the manufacturer (*p^**) under the centralized model is the minimum possible value for* p*, which is 0.*

Lemma 14 *The following statements are true regarding the manufacturer's profit function under the third party model,*

$$\Pi_m^T(p) = \begin{cases} \Pi_{m0}^T(p) = (p-c), \\ \qquad\qquad\qquad\qquad \textit{if } p < v - \beta f(s_t + \alpha l_t), \\ \Pi_{m1}^T(p) = (p-c)\left(1 - \frac{\beta f(\alpha l_t + s_t) - v + p}{b}\right), \\ \qquad\qquad \textit{if } v - \beta f(s_t + \alpha l_t) \le p \le v + b - \beta f(s_t + \alpha l_t), \\ \Pi_{m2}^T(p) = 0, \\ \qquad\qquad\qquad\qquad \textit{if } p > v + b - \beta f(s_t + \alpha l_t) : \end{cases}$$

1. $\Pi_m^T(p)$ *is continuous in* p*.*

2. $\Pi_{m0}^T(p)$ *is increasing in* p*.*

3. $\Pi_{m1}^T(p)$ *is concave in* p*.*

A.2 Verification of Manufacturer's Decisions When She Determines Wholesale Price for Spare Parts First

The alternative sequence of events is as follows:

1. Manufacturer sets the original spare parts price to the retailer, ω_o
2. Manufacturer sets the product's retail price to end customer, p
3. Retailer sets the original spare parts price to the end customer, s_o

Lemma 15 *The manufacturer's profit function given the retailer's best response, $\Pi_m^D(\omega_o, p, s_o^*(\omega_o, p))$, is as follows, where $T = \beta\left(f_m\alpha(l_r - c_r) - f_e(\omega_o + \alpha c_r)\right)$:*

$$\Pi_m^D(\omega_o, p, s_o^*) = \begin{cases} \Pi_{m0}^D(\omega_o, p) = p - c + f_e\omega_o - fc_o - f_m\alpha l_r, \\ \hfill \textit{if } p < v - b + T, \\ \Pi_{m1}^D(\omega_o, p) = \frac{1}{2b}(p - c + f_e\omega_o - fc_o - f_m\alpha l_r) \\ \hfill \times\left(v + b + T - p\right), \\ \hfill \textit{if } v - b + T < p < v + b + T, \\ \Pi_{m2}^D(\omega_o) = 0, \\ \hfill \textit{if } p > v + b + T. \end{cases}$$

Furthermore, the following statements are true regarding $\Pi_m^D(\omega_o, p, s_o^*)$:

1. *$\Pi_m^D(\omega_o, p, s_o^*)$ is continuous in p.*
2. *$\Pi_{m0}^D(\omega_o, p, s_o^*)$ is increasing in p.*
3. *$\Pi_{m1}^D(\omega_o, p, s_o^*)$ is concave in p.*

Proposition 13 *The manufacturer's best response given her wholesale price decision, $p^*(\omega_o)$, is as follows, where $U = \frac{f_m\alpha(l_r - c_r)}{f_e} + \frac{f(c_o + \alpha c_r) + c}{(1-\beta)f_e}$:*

$$p^*(\omega_o) = \begin{cases} v - b + \beta\left(f_m\alpha(l_r - c_r) - f_e(\omega_o + \alpha c_r)\right), \\ \hfill \textit{if } \omega_o > U - \frac{v - 3b}{(1-\beta)f_e} - \alpha c_r, \\ \frac{1}{2}\left(v + b + c + f_m\left(\alpha(l_r - c_r) + c_o + \alpha c_r\right) - f_e(\omega_o - c_o)\right) \\ \hfill +\frac{\beta}{2}\left(f_m\alpha(l_r - c_r) - f_e(\omega_o + \alpha c_r)\right), \\ \hfill \textit{if } U - \frac{v+b}{(1-\beta)f_e} - \alpha c_r < \omega_o < U - \frac{v-3b}{(1-\beta)f_e} - \alpha c_r, \\ \left[v + b + \beta\left(f_m\alpha(l_r - c_r) - f_e(\omega_o + \alpha c_r)\right), \infty\right), \\ \hfill \textit{if } \omega_o < U - \frac{v+b}{(1-\beta)f_e} - \alpha c_r. \end{cases}$$

Proposition 14 *The manufacturer's profit function given the best responses of the retailer for the retail price of original spare parts ($s_o^*(\omega_o, p)$) and the manufacturer for the retail price of the product ($p^*(\omega_o)$), $\Pi_m^D(\omega_o, s_o^*, p^*)$ is as follows, where $U =$*

$\frac{f_m\alpha(l_r-c_r)}{f_e}+\frac{f(c_o+\alpha c_r)+c}{(1-\beta)f_e}$:

$$\Pi_m^D(\omega_o, s_o^*, p^*)$$

$$= \begin{cases} \Pi_{m0}^D(\omega_o) = v - b - c - f(c_o+\alpha c_r) \\ \qquad -(1-\beta)\left(f_m\alpha(l_r-c_r) - f_e(\omega_o+\alpha c_r)\right), \\ \qquad \textit{if}\, \omega_o > U - \frac{v-3b}{(1-\beta)f_e} - \alpha c_r, \\ \Pi_{m1}^D(p) = \frac{(v+b-c-f(c_o+\alpha c_r)-(1-\beta)(f_m\alpha(l_r-c_r)-f_e(\omega_o+\alpha c_r)))^2}{8b}, \\ \qquad \textit{if}\, U - \frac{v+b}{(1-\beta)f_e} - \alpha c_r < \omega_o < U - \frac{v-3b}{(1-\beta)f_e} - \alpha c_r, \\ \Pi_{m2}^D(p) = 0, \\ \qquad \textit{if}\, \omega_o < U - \frac{v+b}{(1-\beta)f_e} - \alpha c_r. \end{cases}$$

Furthermore, the following statements are true regarding $\Pi_m^D(\omega_o, s_o^*, p^*)$:

1. *$\Pi_m^D(\omega_o, s_o^*, p^*)$ is continuous in ω_o.*
2. *$\Pi_{m0}^D(\omega_o)$ is increasing in ω_o.*
3. *$\Pi_{m1}^D(\omega_o)$ is convex in ω_o.*

Therefore, the equilibrium spare parts wholesale price of the manufacturer (ω^*) is the maximum possible value for ω_o.

Although there is no apparent upper bound for ω_o, we observe that a bound is imposed due to p^*. The $p^*(\omega_o)$ corresponding to the valid ω_o region equals $v - b + \beta\left(f_m\alpha(l_r-c_r) - f_e(\omega_o+\alpha c_r)\right)$, where ω_o appears with a multiplier of $-\beta f_e$. Therefore, the more ω_o is increased, the more p should be decreased. As p has a lower bound of zero, ω_o is bound by this condition, equaling to $\frac{v-b+\beta f_m\alpha(l_r-c_r)}{\beta f_e} - \alpha c_r$. Hence we see that the model converges to the same solution found in the previous case where the manufacturer decided p before ω_o.

A.3 Special Case $\beta = 1$

The case of full customer foresight, i.e., $\beta = 1$, deserves discussion as a special case. When $\beta = 1$, the manufacturer's objective function can be rewritten such that both of her decision variables (ω_o and p) can be represented jointly within a single variable,

$P = p + f_e(\omega_o + \alpha c_r)$. P shows that the manufacturer could do a joint pricing on the product retail price and the lifetime cost of after-sales services to the retailer. This way, she can actually adjust the sales and after-sales prices depending on how she wishes to communicate her TCO pricing to the end customer.

Lemma 16 *The following statements are true regarding the manufacturer's objective function given the retailer's best response for the base model with $\beta = 1$ (which is the equivalent of the equation within Lemma A.2. of the paper and rearranged using P accordingly),*

$$\Pi_m^D(P, s_o^*(P)) = \begin{cases} \Pi_{m0}^D(P) = P - c - f_e(c_o + \alpha c_r) - f_m(c_o + \alpha l_r), \\ \hfill \textit{if } P \leq \upsilon - b + f_m\alpha(l_r - c_r), \\ \Pi_{m1}^D(P) = \frac{1}{2b}(P - c - f_e(c_o + \alpha c_r) - f_m(c_o + \alpha l_r)) \\ \hfill \times \left(b + \upsilon + f_m\alpha(l_r - c_r) - P\right), \\ \hfill \textit{if } \upsilon - b + f_m\alpha(l_r - c_r) < P \leq \upsilon + b + f_m\alpha(l_r - c_r), \\ \Pi_{m2}^D(P) = 0, \\ \hfill \textit{if } P > \upsilon + b + f_m\alpha(l_r - c_r) : \end{cases}$$

1. $\Pi_m^D(P, s_o^*(P))$ *is continuous in* P.
2. $\Pi_{m0}^D(P, s_o^*(P))$ *is increasing in* P.
3. $\Pi_{m1}^D(P, s_o^*(P))$ *is concave in* P.

Proposition 15 *When $\beta = 1$, in equilibrium, the manufacturer sets the price $P = p + f_e(\omega_o + \alpha c_r)$ as follows:*

$$P^* = \begin{cases} \upsilon - b + f_m\alpha(l_r - c_r), & \textit{if } c + f(c_o + \alpha c_r) \leq \upsilon - 3b, \\ \frac{\upsilon + b + c + f(c_o + \alpha c_r)}{2} + f_m\alpha(l_r - c_r), & \textit{if } \upsilon - 3b < c + f(c_o + \alpha c_r) \leq \upsilon + b, \\ \left[\upsilon + b + f_m\alpha(l_r - c_r)\ , \infty\right), & \textit{if } c + f(c_o + \alpha c_r) > \upsilon + b. \end{cases}$$

When we look at this equilibrium, we see that it is almost the same as the manufacturer's best response given in Proposition 2 of the paper; except that the manufacturer's pricing decision is now defined over the total lifecycle cost of the product, instead of the cost per each after-sales service request. The equation also carries the same maximum and minimum willingness-to-pay values as thresholds, with the exclusion of p since it became part of the decision variable P. The manufacturer still makes the warranty-period labor costs part of her pricing, extracting back what she would pay, same as when she would make the ω_o and p decisions sequentially.

A.4 Heterogeneity in Customer Foresight

Here, we assume the market is split into two distinct segments regarding the customer foresight: λ fraction of the customers have low foresight (β_L) and $1 - \lambda$ fraction of the customers have high foresight (β_H), where $0 < \beta_L < \beta_H < 1$. In each segment, customer type θ is uniformly distributed on $[0, b]$.

Then, the product sales quantity can be characterized as given in Table A.1.

Table A.1: Heterogeneity in Customer Foresight Model: Product Sales Quantity ($\hat{q}$)

When $(v-p)\beta_H - (v-p+b)\beta_L < 0$	
Conditions	$q^D(p, s_o)$
$s_o \leq \frac{v-p}{\beta_H f_e} - \alpha l_r$	1
$\frac{v-p}{\beta_H f_e} - \alpha l_r < s_o \leq \frac{v-p}{\beta_L f_e} - \alpha l_r$	$\lambda + (1-\lambda)(1 - \frac{\beta_H f_e(s_o+\alpha l_r)-(v-p)}{b})$
$\frac{v-p}{\beta_L f_e} - \alpha l_r < s_o \leq \frac{v-p+b}{\beta_H f_e} - \alpha l_r$	$\lambda(1 - \frac{\beta_L f_e(s_o+\alpha l_r)-(v-p)}{b}) + (1-\lambda)(1 - \frac{\beta_H f_e(s_o+\alpha l_r)-(v-p)}{b})$
$\frac{v-p+b}{\beta_H f_e} - \alpha l_r < s_o \leq \frac{v-p+b}{\beta_L f_e} - \alpha l_r$	$\lambda\left(1 - \frac{\beta_L f_e(s_o+\alpha l_r)-(v-p)}{b}\right)$
$\frac{v-p+b}{\beta_L f_e} - \alpha l_r < s_o$	0

When $(v-p)\beta_H - (v-p+b)\beta_L \geq 0$	
Conditions	$q^D(p, s_o)$
$s_o \leq \frac{v-p}{\beta_H f_e} - \alpha l_r$	1
$\frac{v-p}{\beta_H f_e} - \alpha l_r < s_o \leq \frac{v-p+b}{\beta_H f_e} - \alpha l_r$	$\lambda + (1-\lambda)(1 - \frac{\beta_H f_e(s_o+\alpha l_r)-(v-p)}{b})$
$\frac{v-p+b}{\beta_H f_e} - \alpha l_r < s_o \leq \frac{v-p}{\beta_L f_e} - \alpha l_r$	λ
$\frac{v-p}{\beta_L f_e} - \alpha l_r < s_o \leq \frac{v-p+b}{\beta_L f_e} - \alpha l_r$	$\lambda\left(1 - \frac{\beta_L f_e(s_o+\alpha l_r)-(v-p)}{b}\right)$
$\frac{v-p+b}{\beta_L f_e} - \alpha l_r < s_o$	0

When $(v-p)\beta_H - (v-p+b)\beta_L < 0$, the retailer's profit function becomes as follows:

$$
\begin{aligned}
&\Pi_r(s_o,\omega_o,p)\\
&=\begin{cases}
\Pi_r^1 = f\alpha(l_r-c_r)+f_e(s_o-\omega_o), \\
\hfill \text{if } s_o \le A_H := \frac{v-p}{\beta_H f_e} - \alpha l_r, \\
\Pi_r^2 = \Big(\lambda + (1-\lambda)(1-\frac{\beta_H f_e(s_o+\alpha l_r)-(v-p)}{b})\Big)\,(f\alpha(l_r-c_r)+f_e(s_o-\omega_o)), \\
\hfill \text{if } A_H < s_o \le A_L := \frac{v-p}{\beta_L f_e} - \alpha l_r, \\
\Pi_r^3 = \Big(\lambda(1-\frac{\beta_L f_e(s_o+\alpha l_r)-(v-p)}{b}) + (1-\lambda)(1-\frac{\beta_H f_e(s_o+\alpha l_r)-(v-p)}{b})\Big) \\
\hfill \times(f\alpha(l_r-c_r)+f_e(s_o-\omega_o)), \\
\hfill \text{if } A_L < s_o \le B_H := \frac{v-p+b}{\beta_H f_e} - \alpha l_r, \\
\Pi_r^4 = \lambda(1-\frac{\beta_L f_e(s_o+\alpha l_r)-(v-p)}{b})(f\alpha(l_r-c_r)+f_e(s_o-\omega_o)), \\
\hfill \text{if } B_H < s_o \le B_L := \frac{v-p+b}{\beta_L f_e} - \alpha l_r, \\
0, \\
\hfill \text{if } B_L < s_o.
\end{cases}
\end{aligned}
$$

Then the optimal parts retail price $s_o^*(\omega_o,p)$ becomes as in Table A.2, where $s_o^2 := -\alpha l_r + \frac{1}{2}\Big(\omega_o + \alpha c_r + \frac{b+(1-\lambda)(v-p)}{(1-\lambda)\beta_H f_e} - \frac{f_m\alpha(l_r-c_r)}{f_e}\Big)$, $s_o^3 := -\alpha l_r + \frac{1}{2}\Big(\omega_o + \alpha c_r + \frac{b+(v-p)}{(\lambda\beta_L+(1-\lambda)\beta_H)f_e} - \frac{f_m\alpha(l_r-c_r)}{f_e}\Big)$, and $s_o^4 := -\alpha l_r + \frac{1}{2}\Big(\omega_o + \alpha c_r + \frac{b+(v-p)}{\beta_L f_e} - \frac{f_m\alpha(l_r-c_r)}{f_e}\Big)$ are unconstrained maximizers of functions Π_r^2, Π_r^3, and Π_r^4, respectively. In finding the optimal $s_o^*(\omega_o,p)$, we compare the candidates in the third column of the table by evaluating $\Pi_r^2(s_o^2)$ for s_o^2, $\Pi_r^3(s_o^3)$ for s_o^3, $\Pi_r^4(s_o^4)$ for s_o^4, and zero for $s_o \ge B_L$.

In finding the optimal $s_o^*(\omega_o,p)$, we compare the candidates in the third column of the table by evaluating $\Pi_r^2(s_o^2)$ for s_o^2, $\Pi_r^4(s_o^4)$ for s_o^4, $\Pi_r^1(A_H)$ for A_H, $\Pi_r^1(A_L)$ for A_L, and zero for $s_o \ge B_L$.

Table A.2: Heterogeneity in Customer Foresight Model: $s_o^*(\omega_o, p)$ when $(v-p)\beta_H - (v-p+b)\beta_L < 0$ holds

Conditions on s_o^3 and s_o^4	Conditions on s_o^2	$s_o^*(\omega_o, p)$	Obj. Fnc. Shape
$s_o^3 < A_L$ and $s_o^4 < B_H$	$s_o^2 < A_H$	A_H	$\nearrow\searrow\searrow\searrow\rightarrow$
	$A_H \leq s_o^2 < A_L$	s_o^2	$\nearrow\bigcap\searrow\searrow\rightarrow$
	$A_L \leq s_o^2$	A_L	$\nearrow\nearrow\searrow\searrow\rightarrow$
$s_o^3 < A_L$ and $B_H \leq s_o^4 < B_L$	$s_o^2 < A_H$	A_H or s_o^4	$\nearrow\searrow\searrow\bigcap\rightarrow$
	$A_H \leq s_o^2 < A_L$	s_o^2 or s_o^4	$\nearrow\bigcap\searrow\bigcap\rightarrow$
	$A_L \leq s_o^2$	A_L or s_o^4	$\nearrow\nearrow\searrow\bigcap\rightarrow$
$s_o^3 < A_L$ and $B_L \leq s_o^4$	$s_o^2 < A_H$	A_H or $[B_L, \infty)$	$\nearrow\searrow\searrow\nearrow\rightarrow$
	$A_H \leq s_o^2 < A_L$	s_o^2 or $[B_L, \infty)$	$\nearrow\bigcap\searrow\nearrow\rightarrow$
	$A_L \leq s_o^2$	A_L or $[B_L, \infty)$	$\nearrow\nearrow\searrow\nearrow\rightarrow$
$A_L \leq s_o^3 < B_H$ and $s_o^4 < B_H$	$s_o^2 < A_H$	A_H or s_o^3	$\nearrow\searrow\bigcap\searrow\rightarrow$
	$A_H \leq s_o^2 < A_L$	s_o^2 or s_o^3	$\nearrow\bigcap\bigcap\searrow\rightarrow$
	$A_L \leq s_o^2$	s_o^3	$\nearrow\nearrow\bigcap\searrow\rightarrow$
$A_L \leq s_o^3 < B_H \leq s_o^4 < B_L$	$s_o^2 < A_H$	A_H or s_o^3 or s_o^4	$\nearrow\searrow\bigcap\bigcap\rightarrow$
	$A_H \leq s_o^2 < A_L$	s_o^2 or s_o^3 or s_o^4	$\nearrow\bigcap\bigcap\bigcap\rightarrow$
	$A_L \leq s_o^2$	s_o^3 or s_o^4	$\nearrow\nearrow\bigcap\bigcap\rightarrow$
$A_L \leq s_o^3 < B_H$ and $B_L \leq s_o^4$	$s_o^2 < A_H$	A_H or s_o^3 or $[B_L, \infty)$	$\nearrow\searrow\bigcap\nearrow\rightarrow$
	$A_H \leq s_o^2 < A_L$	s_o^2 or s_o^3 or $[B_L, \infty)$	$\nearrow\bigcap\bigcap\nearrow\rightarrow$
	$A_L \leq s_o^2$	s_o^3 or $[B_L, \infty)$	$\nearrow\nearrow\bigcap\nearrow\rightarrow$
$B_H \leq s_o^3$ and $s_o^4 < B_H$	$s_o^2 < A_H$	A_H or B_H	$\nearrow\searrow\nearrow\searrow\rightarrow$
	$A_H \leq s_o^2 < A_L$	s_o^2 or B_H	$\nearrow\bigcap\nearrow\searrow\rightarrow$
	$A_L \leq s_o^2$	B_H	$\nearrow\nearrow\nearrow\searrow\rightarrow$
$B_H \leq s_o^3$ and $B_H \leq s_o^4 < B_L$	$s_o^2 < A_H$	A_H or s_o^4	$\nearrow\searrow\nearrow\bigcap\rightarrow$
	$A_H \leq s_o^2 < A_L$	s_o^2 or s_o^4	$\nearrow\bigcap\nearrow\bigcap\rightarrow$
	$A_L \leq s_o^2$	s_o^4	$\nearrow\nearrow\nearrow\bigcap\rightarrow$
$B_H \leq s_o^3$ and $B_L \leq s_o^4$	$s_o^2 < A_H$	A_H or $[B_L, \infty)$	$\nearrow\searrow\nearrow\nearrow\rightarrow$
	$A_H \leq s_o^2 < A_L$	s_o^2 or $[B_L, \infty)$	$\nearrow\bigcap\nearrow\nearrow\rightarrow$
	$A_L \leq s_o^2$	$[B_L, \infty)$	$\nearrow\nearrow\nearrow\nearrow\rightarrow$

Table A.3: Heterogeneity in Customer Foresight Model: $s_o^*(\omega_o, p)$ when $(v-p)\beta_H - (v-p+b)\beta_L \geq 0$ holds

Conditions on s_o^4	Conditions on s_o^2	$s_o^*(\omega_o, p)$	Obj. Fnc. Shape
$s_o^4 < A_L$	$s_o^2 < A_H$	A_H or A_L	$\nearrow\searrow\nearrow\searrow\rightarrow$
	$A_H \leq s_o^2 < B_H$	s_o^2 or A_L	$\nearrow\bigcap\nearrow\searrow\rightarrow$
	$B_H \leq s_o^2$	A_L	$\nearrow\nearrow\nearrow\searrow\rightarrow$
$A_L \leq s_o^4 < B_L$	$s_o^2 < A_H$	A_H or s_o^4	$\nearrow\searrow\bigcap\searrow\rightarrow$
	$A_H \leq s_o^2 < B_H$	s_o^2 or s_o^4	$\nearrow\bigcap\bigcap\searrow\rightarrow$
	$B_H \leq s_o^2$	s_o^4	$\nearrow\nearrow\bigcap\searrow\rightarrow$
$B_L \leq s_o^4$	$s_o^2 < A_H$	A_H or $[B_L, \infty)$	$\nearrow\searrow\nearrow\searrow\rightarrow$
	$A_H \leq s_o^2 < B_H$	s_o^2 or $[B_L, \infty)$	$\nearrow\bigcap\nearrow\searrow\rightarrow$
	$B_H \leq s_o^2$	$[B_L, \infty)$	$\nearrow\nearrow\nearrow\searrow\rightarrow$

Table A.4: Equilibrium Comparison: Heterogeneity in Customer Foresight Model ($\beta_L = 0.4, \beta_H = 0.6, \lambda = 0.5$) vs. Decentralized Model ($\beta = 0.5$)

		Heterogeneous Beta Model									**Decentralized Model**						
v	b	p^*	ω_o^*	s_o^*	q_L^*	q_H^*	$\hat{q}^*$	Π_r^*	Π_m^*	CS^*	p^*	ω_o^*	s_o^*	q^*	Π_r^*	Π_m^*	CS^*
1	1	0	4.55	6.03	74%	12%	0.43	0.37	0.55	-0.63	0	5.00	6.25	0.38	0.28	0.56	-0.54
1	2	0	6.83	9.16	56%	9%	0.32	0.42	0.78	-0.71	0	7.00	9.25	0.31	0.39	0.78	-0.64
1	3	0	9.00	12.25	50%	8%	0.29	0.51	1.02	-0.85	0	9.00	12.25	0.29	0.51	1.02	-0.78
1	4	0	11.00	15.25	47%	9%	0.28	0.63	1.27	-1.01	0	11.00	15.25	0.28	0.63	1.27	-0.93
2	1	0.32	6.11	8.18	100%	16%	0.58	0.67	1.37	-1.14	0	7.00	9.25	0.63	0.78	1.56	-1.29
2	2	0	9.00	12.25	75%	12%	0.44	0.77	1.53	-1.27	0	9.00	12.25	0.44	0.77	1.53	-1.18
2	3	0	11.00	15.25	63%	12%	0.37	0.84	1.69	-1.34	0	11.00	15.25	0.38	0.84	1.69	-1.24
2	4	0	13.00	18.25	58%	11%	0.34	0.95	1.89	-1.46	0	13.00	18.25	0.34	0.95	1.89	-1.35
3	1	0	11.00	15.25	90%	0%	0.45	1.01	2.03	-1.89	0	9.00	12.25	0.88	1.53	3.06	-2.35
3	2	0	11.00	15.25	95%	17%	0.56	1.27	2.53	-2.01	0	11.00	15.25	0.56	1.27	2.53	-1.86
3	3	0	13.00	18.25	77%	15%	0.46	1.26	2.52	-1.95	0	13.00	18.25	0.46	1.26	2.52	-1.80
3	4	0	15.00	21.25	67%	14%	0.41	1.32	2.64	-2.00	0	15.00	21.25	0.41	1.32	2.64	-1.85
4	1	0	15.00	19.75	100%	0%	0.50	1.25	3.25	-2.75	0	12.13	15.75	1.00	1.94	5.06	0.00
4	2	0.63	12.22	16.60	100%	16%	0.58	1.34	3.32	-2.28	0	13.00	18.25	0.69	1.89	3.78	-2.71
4	3	0	15.00	21.25	90%	18%	0.54	1.76	3.52	-2.66	0	15.00	21.25	0.54	1.76	3.52	-2.47
4	4	0	16.99	24.24	78%	16%	0.47	1.76	3.52	-2.62	0	17.00	24.25	0.47	1.76	3.52	-2.43

Table A.5: Instances Where Heterogeneity in Customer Foresight Model Equilibrium has a Positive Product Price ($p > 0$)

Parameters						**Heterogeneous Beta Model Equilibrium**										
λ	β_L	β_H	$\hat{\beta}$	v	b	p	ω_o	s_o	q_L	q_H	$\hat{q}$	Π_r	Π_m	$\Pi_m^*/\Pi_{total}^*\%$	CS
0.5	0.40	0.60	0.500	2	1	0.32	6.11	8.18	100%	16%	0.58	0.67	1.37	67%	-1.14
0.5	0.40	0.60	0.500	4	2	0.63	12.22	16.60	100%	16%	0.58	1.34	3.32	71%	-2.28
0.5	0.40	0.75	0.575	3	3	0.26	7.40	13.43	100%	20%	0.60	1.89	1.78	49%	-1.44
0.5	0.40	0.75	0.575	4	3	1.27	7.40	13.42	100%	20%	0.60	1.89	2.38	56%	-1.44
0.5	0.40	0.75	0.575	4	4	0.35	9.87	17.99	100%	20%	0.60	2.52	2.58	51%	-1.93
0.1	0.25	0.75	0.700	1	1	0.01	3.15	4.13	100%	35%	0.42	0.26	0.24	49%	-0.19
0.1	0.25	0.95	0.880	1	2	0.73	2.53	3.60	90%	22%	0.29	0.19	0.29	60%	-0.02
0.1	0.25	0.95	0.880	1	3	0.58	3.80	5.53	90%	22%	0.29	0.29	0.43	60%	-0.04
0.1	0.25	0.95	0.880	1	4	0.49	5.01	7.38	89%	22%	0.29	0.38	0.58	60%	-0.05
0.1	0.25	0.95	0.880	2	1	1.05	2.02	2.93	100%	44%	0.50	0.29	0.53	65%	0.04
0.1	0.25	0.95	0.880	2	2	1.50	2.39	3.79	100%	29%	0.36	0.30	0.61	67%	0.00
0.1	0.25	0.95	0.880	2	3	1.20	4.19	6.16	100%	25%	0.33	0.36	0.75	67%	-0.04
0.1	0.25	0.95	0.880	2	4	0.93	5.63	8.33	100%	25%	0.32	0.48	0.89	65%	-0.06
0.1	0.25	0.95	0.880	3	1	1.02	3.05	4.53	100%	71%	0.74	0.64	1.14	64%	0.16
0.1	0.25	0.95	0.880	3	2	1.11	4.03	6.10	100%	44%	0.50	0.58	1.05	65%	0.08
0.1	0.25	0.95	0.880	3	3	1.05	5.25	7.94	100%	35%	0.42	0.61	1.12	65%	0.04
0.1	0.25	0.95	0.880	3	4	1.99	4.82	7.85	100%	29%	0.36	0.59	1.23	67%	0.00
0.1	0.25	0.95	0.880	4	1	0.45	5.15	7.23	100%	100%	1.00	1.16	2.02	63%	0.31
0.1	0.25	0.95	0.880	4	2	1.21	4.83	7.45	100%	57%	0.61	0.88	1.61	65%	0.19
0.1	0.25	0.95	0.880	4	3	1.15	6.05	9.28	100%	44%	0.50	0.87	1.58	65%	0.13
0.1	0.25	0.95	0.880	4	4	1.11	7.25	11.10	100%	37%	0.44	0.89	1.63	65%	0.08
0.1	0.40	0.95	0.895	1	1	0.41	2.38	2.72	100%	18%	0.26	0.08	0.16	67%	-0.04
0.1	0.40	0.95	0.895	2	3	0.38	5.92	7.88	100%	26%	0.33	0.36	0.77	68%	-0.05
0.1	0.40	0.95	0.895	3	4	0.97	6.86	9.92	100%	30%	0.37	0.61	1.26	67%	-0.01

Appendix B

ADDITIONAL MATERIAL FOR DOWNSTREAM COMPETITION MODEL

B.1 Analytical Solution of the Decentralized Model for a Special Case

This case is characterized by the ordering of threshold values related to ω_o as $A < B < C < D$. The corresponding assumption is given below, which is derived from $B < C$ (as $A < B$ and $C < D$ by definition).

$$\upsilon - p + \delta b - \beta f_e(s_n + \alpha l_i) - \beta f_m \alpha(l_r - c_r) > 0$$

Lemma 17 *The following statements are true regarding the manufacturer's profit function given the retailer's best response,* $\Pi_m^{CM1}(p, \omega_o, s_o^*(p, \omega_o))$*, when the conditions* $\frac{\upsilon-p}{\beta f_e} < s_n + \alpha l_i < \frac{b\delta+\upsilon-p}{\beta f_e}$ *and* $\upsilon - p + \delta b - \beta f_e(s_n + \alpha l_i) - \beta f_m \alpha(l_r - c_r) > 0$ *hold:*

$$\Pi_m^{CM1}(p, \omega_o, s_o^*(p, \omega_o))$$

$$= \begin{cases} \Pi_{m1}^{CM1}(p, \omega_o) = p - c + f_e(\omega_o - c_o) - f_m(c_o + \alpha l_r), \\ \hfill \textit{if}\ \omega_o + \alpha c_r < \frac{\upsilon-p-b+\beta f_m \alpha(l_r-c_r)}{\beta f_e}, \\ \Pi_{m2}^{CM1}(p, \omega_o) = \frac{(p-c+f_e(\omega_o-c_o)-f_m(c_o+\alpha l_r))(\upsilon-p+b-\beta(f_e(\omega_o+\alpha c_r)-f_m\alpha(l_r-c_r)))}{2b}, \\ \hfill \textit{if}\ \omega_o + \alpha c_r > \frac{\upsilon-p-b+\beta f_m \alpha(l_r-c_r)}{\beta f_e} \\ \hfill \textit{and}\ \omega_o + \alpha c_r < \frac{2\beta f_e(s_n+\alpha l_i)+\delta\beta f_m\alpha(l_r-c_r)-(2-\delta)(\upsilon-p)-\delta b}{\delta\beta f_e}, \\ \Pi_{m3}^{CM1}(p, \omega_o) = \frac{(\upsilon-p+\delta b-\beta f_e(s_n+\alpha l_i))(p-c+f_e(\omega_o-c_o)-f_m(c_o+\alpha l_r))}{\delta b}, \\ \hfill \textit{if}\ \omega_o + \alpha c_r > \frac{2\beta f_e(s_n+\alpha l_i)+\delta\beta f_m\alpha(l_r-c_r)-(2-\delta)(\upsilon-p)-\delta b}{\delta\beta f_e} \\ \hfill \textit{and}\ \omega_o + \alpha c_r < \frac{\beta f_e(2-\delta)(s_n+\alpha l_i)-(1-\delta)(\delta b+2(\upsilon-p))}{\delta\beta f_e}, \end{cases}$$

$$\Pi_m^{CM1}(p,\omega_o,s_o^*(p,\omega_o))$$

$$= \begin{cases} \Pi_{m4}^{CM1}(p,\omega_o) = \frac{(\upsilon-p+\delta b-\beta f_e(s_n+\alpha l_i))(p-c-f_m(c_o+\alpha l_r))}{\delta b} \\ \qquad + \frac{f_e(\omega_o-c_o)(b(1-\delta)-\beta f_e(\omega_o+\alpha c_r-s_n-\alpha l_i))}{2b(1-\delta)}, \\ \qquad \textit{if}\ \omega_o+\alpha c_r > \frac{\beta f_e(2-\delta)(s_n+\alpha l_i)-(1-\delta)(\delta b+2(\upsilon-p))}{\delta\beta f_e} \\ \qquad \textit{and}\ \omega_o+\alpha c_r < \frac{\beta f_e(s_n+\alpha l_i)+(1-\delta)b}{\beta f_e}, \\ \Pi_{m5}^{CM1}(p,\omega_o) = \frac{(\upsilon-p+\delta b-\beta f_e(s_n+\alpha l_i))(p-c-f_m(c_o+\alpha l_r))}{\delta b}, \\ \qquad \textit{if}\ \omega_o+\alpha c_r > \frac{\beta f_e(s_n+\alpha l_i)+(1-\delta)b}{\beta f_e}: \end{cases}$$

1. *$\Pi_m^{CM1}(p,\omega_o,s_o^*(p,\omega_o))$ is continuous in ω_o.*

2. *$\Pi_{m1}^{CM1}(p,\omega_o,s_o^*(p,\omega_o))$ is increasing in ω_o.*

3. *$\Pi_{m2}^{CM1}(p,\omega_o,s_o^*(p,\omega_o))$ is concave in ω_o.*

4. *$\Pi_{m3}^{CM1}(p,\omega_o,s_o^*(p,\omega_o))$ is increasing in ω_o.*

5. *$\Pi_{m4}^{CM1}(p,\omega_o,s_o^*(p,\omega_o))$ is concave in ω_o.*

B.2 Downstream Competition Model with Changed Sequence of Events

We have considered a change in the sequence of events, mainly in order to see if this helps in studying the manufacturer's best response analytically in the medium TCO_i case. The revised sequence of events is as follows, where we still consider a game in which the manufacturer (the leader) is followed by the retailer. All other definitions and parameters stay unchanged.

1. Manufacturer sets the wholesale price of original spare parts to the retailer, ω_o

2. Manufacturer sets the product retail price to the end customer, p

3. Retailer sets the retail price of original spare parts to the end customer, s_o

We again study the equilibrium with backward induction. The retailer's best response for the spare parts retail price, s_o, is the same as those in Section 4.1.1, only changing the representation of the validity intervals. These changes in representations are trivial for the low and high s_n regions, but somewhat more complicated for the medium s_n

region (which is actually why we attempt this at all). An overview of the retailer's best response in this region can be found in Tables B.1-B.3. Unfortunately, although being simpler than the original model, this representation still results in 11+6 different configurations, 14 of which require analytical comparison of profit function values under several constraints in order to determine the best solution. Therefore, we did not further study the next steps of the backward induction, namely the manufacturer's best response on p, ω_o and the overall equilibrium.

Table B.1: Valid intervals for p and the corresponding best responses for s_o where $\omega_o < D$ (Part 1)

Alt.	s_o^*	Shape of Π_r^{CM}	Conditions on p	Cases (Part 1)		
				$C' < A' < B'$	$A' < C' < B'$	$A' < B' < C'$
2	$s_{o2}^* = \frac{\delta\beta f_e(s_n+\alpha l_i)-(1-\delta)(\upsilon-p)}{\delta\beta f_e} - \alpha l_r$	$\nearrow\nearrow\searrow\longrightarrow$	$p < B', p > A'$ $p > C'$	$A' < p < B'$	$C' < p < B'$	infeasible
4	$s_{o4}^* = \frac{\upsilon-p}{\beta f_e} - \alpha l_r$	$\nearrow\searrow\searrow\longrightarrow$	$p < A', p > B'$ $p > C'$	infeasible	infeasible	infeasible
5	$s_{o5}^* = \frac{\upsilon-p+b+\beta f_e(\omega_o+\alpha c_r)-\beta f_m\alpha(l_r-c_r)}{2\beta f_e} - \alpha l_r$	$\nearrow\cap\searrow\longrightarrow$	$p > A', p > B'$ $p > C'$	$p > B'$	$p > B'$	$p > C'$
3	$s_{o3}^* = \frac{\beta f_e(\omega_o+\alpha c_r+s_n+\alpha l_i)+(1-\delta)b}{2\beta f_e} - \alpha l_r$	$\nearrow\nearrow\cap\longrightarrow$	$p < B', p > A'$ $p < C'$	infeasible	$A' < p < C'$	$A' < p < B'$
7	s_{o4}^* or s_{o3}^*	$\nearrow\searrow\cap\longrightarrow$	$p < A', p > B'$ $p < C'$	infeasible	infeasible	infeasible
9	s_{o5}^* or s_{o3}^*	$\nearrow\cap\cap\longrightarrow$	$p > A', p > B'$ $p < C'$	infeasible	infeasible	$B' < p < C'$

Table B.2: Valid intervals for p and the corresponding best responses for s_o where $\omega_o < D$ (Part 2)

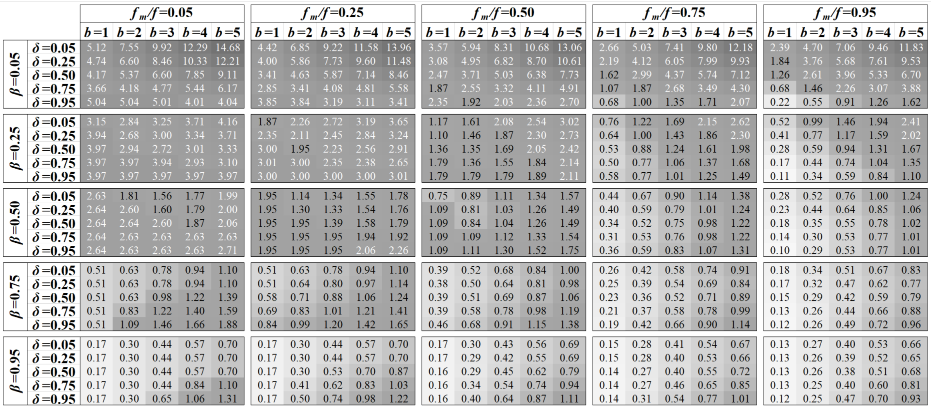
Alt.	s_o^*	Shape of Π_r^{CM}	Conditions on p	Cases (Part 2)		
				$C' < B' < A'$	$B' < C' < A'$	$B' < A' < C'$
2	$s_{o2}^* = \frac{\delta\beta f_e(s_n+\alpha l_i)-(1-\delta)(\upsilon-p)}{\delta\beta f_e} - \alpha l_r$	$\nearrow\nearrow\searrow\longrightarrow$	$p < B', p > A'$ $p > C'$	infeasible	infeasible	infeasible
4	$s_{o4}^* = \frac{\upsilon-p}{\beta f_e} - \alpha l_r$	$\nearrow\searrow\searrow\longrightarrow$	$p < A', p > B'$ $p > C'$	$B' < p < A'$	$C' < p < A'$	infeasible
5	$s_{o5}^* = \frac{\upsilon-p+b+\beta f_e(\omega_o+\alpha c_r)-\beta f_m\alpha(l_r-c_r)}{2\beta f_e} - \alpha l_r$	$\nearrow\cap\searrow\longrightarrow$	$p > A', p > B'$ $p > C'$	$p > A'$	$p > A'$	$p > C'$
3	$s_{o3}^* = \frac{\beta f_e(\omega_o+\alpha c_r+s_n+\alpha l_i)+(1-\delta)b}{2\beta f_e} - \alpha l_r$	$\nearrow\nearrow\cap\longrightarrow$	$p < B', p > A'$ $p < C'$	infeasible	infeasible	infeasible
7	s_{o4}^* or s_{o3}^*	$\nearrow\searrow\cap\longrightarrow$	$p < A', p > B'$ $p < C'$	infeasible	$B' < p < C'$	$B' < p < A'$
9	s_{o5}^* or s_{o3}^*	$\nearrow\cap\cap\longrightarrow$	$p > A', p > B'$ $p < C'$	infeasible	infeasible	$A' < p < C'$

Table B.3: Valid intervals for p and the corresponding best responses for s_o where $\omega_o > D$ (Part 3)

Alt.	s_o^*	Shape of Π_r^{CM}	Conditions on p	Cases (Part 2)	
				$A' < B'$	$B' < A'$
1	$s_{o1}^* = \frac{\beta f_e(s_n+\alpha l_i)+(1-\delta)b}{\beta f_e} - \alpha l_r$	$\nearrow\nearrow\nearrow\longrightarrow$	$p < B'$ $p > A'$	$A' < p < B'$	infeasible
6	s_{o4}^* or s_{o1}^*	$\nearrow\searrow\nearrow\longrightarrow$	$p < A'$ $p > B'$	infeasible	$B' < p < A'$
8	s_{o5}^* or s_{o1}^*	$\nearrow\cap\nearrow\longrightarrow$	$p > A'$ $p > B'$	$p > B'$	$p > A'$

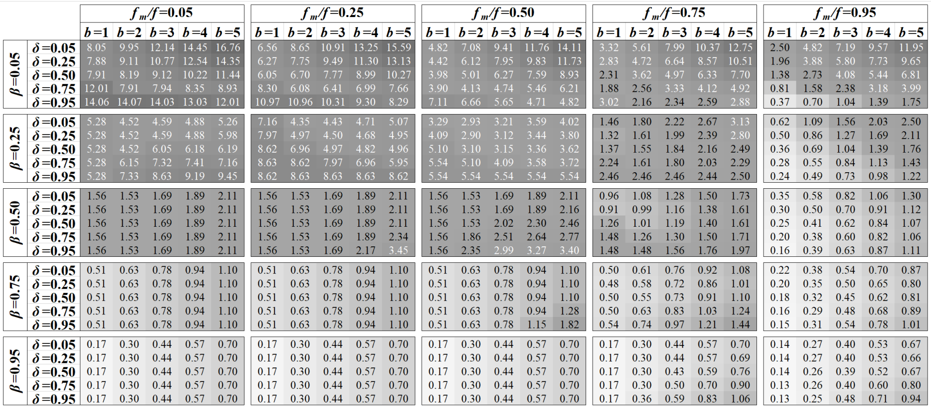
Table B.4: Distribution of equilibrium demand patterns across regions, where $p < v - \beta f_e(s_n + \alpha l_i)$ is denoted as *low*, $p > v + \delta b - \beta f_e(s_n + \alpha l_i)$ is denoted as *high*, and the values in between are denoted as *medium*

Region	q_i	q_r	p	# of instances	% of instances
-3	=0	=1	low	18,903	10.08%
-2	=0	=1	medium	11	0.01%
-1	=0	=1	high	536	0.29%
0	=0	> 0	medium	23,891	12.74%
1	=0	> 0	high	46,011	24.54%
2	=0	> 0	medium	14,430	7.70%
3	=0	=0	high	8,700	4.64%
4	> 0	=0	medium	0	0%
5	=1	=0	low	0	0%
6	> 0	> 0	low	2,188	1.17%
7	> 0	> 0	medium	72,830	38.84%
				187,500	100%



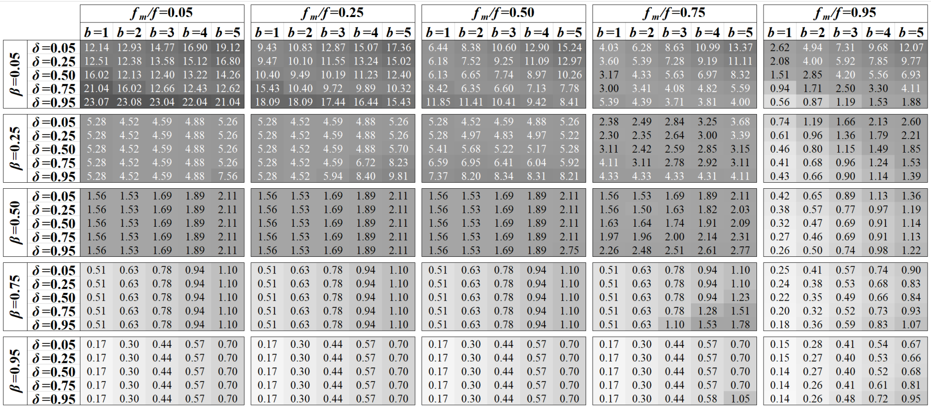
		f_m/f=0.05					f_m/f=0.25					f_m/f=0.50					f_m/f=0.75					f_m/f=0.95				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	5.12	7.55	9.92	12.29	14.68	4.42	6.85	9.22	11.58	13.96	3.57	5.94	8.31	10.68	13.06	2.66	5.03	7.41	9.80	12.18	2.39	4.70	7.06	9.46	11.83
	δ=0.25	4.74	6.60	8.46	10.33	12.21	4.00	5.86	7.73	9.60	11.48	3.08	4.95	6.82	8.70	10.61	2.19	4.12	6.05	7.99	9.93	1.84	3.76	5.68	7.61	9.53
	δ=0.50	4.17	5.37	6.60	7.85	9.11	3.41	4.63	5.87	7.14	8.46	2.47	3.71	5.03	6.38	7.73	1.62	2.99	4.37	5.74	7.12	1.26	2.61	3.96	5.33	6.70
	δ=0.75	3.66	4.18	4.77	5.44	6.17	2.85	3.41	4.08	4.81	5.58	1.87	2.55	3.32	4.11	4.91	1.07	1.87	2.68	3.49	4.30	0.68	1.46	2.26	3.07	3.88
	δ=0.95	5.04	5.04	5.01	4.01	4.04	3.85	3.84	3.19	3.11	3.41	2.35	1.92	2.03	2.36	2.70	0.68	1.00	1.35	1.71	2.07	0.22	0.55	0.91	1.26	1.62
β=0.25	δ=0.05	3.15	2.84	3.25	3.71	4.16	1.87	2.26	2.72	3.19	3.65	1.17	1.61	2.08	2.54	3.02	0.76	1.22	1.69	2.15	2.62	0.52	0.99	1.46	1.94	2.41
	δ=0.25	3.94	2.68	3.00	3.34	3.71	2.35	2.11	2.45	2.84	3.24	1.10	1.46	1.87	2.30	2.73	0.64	1.00	1.43	1.86	2.30	0.41	0.77	1.17	1.59	2.02
	δ=0.50	3.97	2.94	2.72	3.01	3.33	3.00	1.95	2.23	2.56	2.91	1.36	1.35	1.69	2.05	2.42	0.53	0.88	1.24	1.61	1.98	0.28	0.59	0.94	1.31	1.67
	δ=0.75	3.97	3.97	3.94	2.93	3.10	3.01	3.00	2.35	2.38	2.65	1.79	1.36	1.55	1.84	2.14	0.50	0.77	1.06	1.37	1.68	0.17	0.44	0.74	1.04	1.35
	δ=0.95	3.97	3.97	3.97	3.97	3.97	3.00	3.00	3.00	3.00	3.01	1.79	1.79	1.79	1.89	2.11	0.58	0.77	1.01	1.25	1.49	0.11	0.34	0.59	0.84	1.10
β=0.50	δ=0.05	2.63	1.81	1.56	1.77	1.99	1.95	1.14	1.34	1.55	1.78	0.75	0.89	1.11	1.34	1.57	0.44	0.67	0.90	1.14	1.38	0.28	0.52	0.76	1.00	1.24
	δ=0.25	2.64	2.60	1.60	1.79	2.00	1.95	1.30	1.33	1.54	1.76	1.09	0.81	1.03	1.26	1.49	0.40	0.59	0.79	1.01	1.24	0.23	0.44	0.64	0.85	1.06
	δ=0.50	2.64	2.64	2.60	1.87	2.06	1.95	1.95	1.39	1.58	1.79	1.09	0.84	1.04	1.26	1.49	0.34	0.52	0.75	0.98	1.22	0.18	0.35	0.55	0.78	1.02
	δ=0.75	2.64	2.63	2.63	2.63	2.63	1.95	1.95	1.95	1.94	1.92	1.09	1.09	1.12	1.33	1.54	0.31	0.53	0.76	0.98	1.22	0.14	0.30	0.53	0.77	1.01
	δ=0.95	2.64	2.63	2.63	2.63	2.71	1.95	1.95	1.95	2.06	2.26	1.09	1.11	1.30	1.52	1.75	0.36	0.59	0.83	1.07	1.31	0.10	0.29	0.53	0.77	1.01
β=0.75	δ=0.05	0.51	0.63	0.78	0.94	1.10	0.51	0.63	0.78	0.94	1.10	0.39	0.52	0.68	0.84	1.00	0.26	0.42	0.58	0.74	0.91	0.18	0.34	0.51	0.67	0.83
	δ=0.25	0.51	0.63	0.78	0.94	1.10	0.51	0.64	0.80	0.97	1.14	0.38	0.50	0.64	0.81	0.98	0.25	0.39	0.54	0.69	0.84	0.17	0.32	0.47	0.62	0.77
	δ=0.50	0.51	0.63	0.98	1.22	1.39	0.58	0.71	0.88	1.06	1.24	0.39	0.51	0.69	0.87	1.06	0.23	0.36	0.52	0.71	0.89	0.15	0.29	0.42	0.59	0.79
	δ=0.75	0.51	0.83	1.22	1.40	1.59	0.69	0.83	1.01	1.21	1.41	0.39	0.58	0.78	0.98	1.19	0.21	0.37	0.58	0.78	0.99	0.13	0.26	0.44	0.66	0.88
	δ=0.95	0.51	1.09	1.46	1.66	1.88	0.84	0.99	1.20	1.42	1.65	0.46	0.68	0.91	1.15	1.38	0.19	0.42	0.66	0.90	1.14	0.12	0.26	0.49	0.72	0.96
β=0.95	δ=0.05	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.43	0.56	0.69	0.15	0.28	0.41	0.54	0.67	0.13	0.27	0.40	0.53	0.66
	δ=0.25	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.44	0.57	0.70	0.17	0.29	0.42	0.55	0.69	0.15	0.28	0.40	0.53	0.66	0.13	0.26	0.39	0.52	0.65
	δ=0.50	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.53	0.70	0.87	0.16	0.29	0.45	0.62	0.79	0.14	0.27	0.40	0.55	0.72	0.13	0.26	0.38	0.51	0.68
	δ=0.75	0.17	0.30	0.44	0.84	1.10	0.17	0.41	0.62	0.83	1.03	0.16	0.34	0.54	0.74	0.94	0.14	0.27	0.46	0.65	0.85	0.13	0.25	0.40	0.60	0.81
	δ=0.95	0.17	0.30	0.65	1.06	1.31	0.17	0.50	0.74	0.98	1.22	0.16	0.40	0.64	0.87	1.11	0.14	0.31	0.54	0.77	1.01	0.12	0.25	0.47	0.70	0.93

(a) $s_n = 1$



		f_m/f=0.05					f_m/f=0.25					f_m/f=0.50					f_m/f=0.75					f_m/f=0.95				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	8.05	9.95	12.14	14.45	16.76	6.56	8.65	10.91	13.25	15.59	4.82	7.08	9.41	11.76	14.11	3.32	5.61	7.99	10.37	12.75	2.50	4.82	7.19	9.57	11.95
	δ=0.25	7.88	9.11	10.77	12.54	14.35	6.27	7.75	9.49	11.30	13.13	4.42	6.12	7.95	9.83	11.73	2.83	4.72	6.64	8.57	10.51	1.96	3.88	5.80	7.73	9.65
	δ=0.50	7.91	8.19	9.12	10.22	11.44	6.05	6.70	7.77	8.99	10.27	3.98	5.01	6.27	7.59	8.93	2.31	3.62	4.97	6.33	7.70	1.38	2.73	4.08	5.44	6.81
	δ=0.75	12.01	7.91	7.94	8.35	8.93	8.30	6.08	6.41	6.99	7.66	3.90	4.13	4.74	5.46	6.21	1.88	2.56	3.33	4.12	4.92	0.81	1.58	2.38	3.18	3.99
	δ=0.95	14.06	14.07	14.03	13.03	12.01	10.97	10.96	10.31	9.30	8.29	7.11	6.66	5.65	4.71	4.82	3.02	2.16	2.34	2.59	2.88	0.37	0.70	1.04	1.39	1.75
β=0.25	δ=0.05	5.28	4.52	4.59	4.88	5.26	7.16	4.35	4.43	4.71	5.07	3.29	2.93	3.21	3.59	4.02	1.46	1.80	2.22	2.67	3.13	0.62	1.09	1.56	2.03	2.50
	δ=0.25	5.28	4.52	4.59	4.88	5.98	7.97	4.97	4.50	4.68	4.95	4.09	2.90	3.12	3.44	3.80	1.32	1.61	1.99	2.39	2.80	0.50	0.86	1.27	1.69	2.11
	δ=0.50	5.28	4.52	6.05	6.18	6.19	8.62	6.96	4.97	4.82	4.96	5.10	3.10	3.15	3.36	3.62	1.37	1.55	1.84	2.16	2.49	0.36	0.69	1.04	1.39	1.76
	δ=0.75	5.28	6.15	7.32	7.41	7.16	8.63	8.62	7.97	6.96	5.95	5.54	5.10	4.09	3.58	3.72	2.24	1.61	1.80	2.03	2.29	0.28	0.55	0.84	1.13	1.43
	δ=0.95	5.28	7.33	8.63	9.19	9.45	8.62	8.63	8.63	8.63	8.62	5.54	5.54	5.54	5.54	5.54	2.46	2.46	2.46	2.44	2.50	0.24	0.49	0.73	0.98	1.22
β=0.50	δ=0.05	1.56	1.53	1.69	1.89	2.11	1.56	1.53	1.69	1.89	2.11	1.56	1.53	1.69	1.89	2.11	0.96	1.08	1.28	1.50	1.73	0.35	0.58	0.82	1.06	1.30
	δ=0.25	1.56	1.53	1.69	1.89	2.11	1.56	1.53	1.69	1.89	2.11	1.56	1.53	1.69	1.89	2.16	0.91	0.99	1.16	1.38	1.61	0.30	0.50	0.70	0.91	1.12
	δ=0.50	1.56	1.53	1.69	1.89	2.11	1.56	1.53	1.69	1.89	2.11	1.56	1.53	2.02	2.30	2.46	1.26	1.01	1.19	1.40	1.61	0.25	0.41	0.62	0.84	1.07
	δ=0.75	1.56	1.53	1.69	1.89	2.11	1.56	1.53	1.69	1.89	2.34	1.56	1.86	2.51	2.64	2.77	1.48	1.26	1.30	1.50	1.71	0.20	0.38	0.60	0.82	1.06
	δ=0.95	1.56	1.53	1.69	1.89	2.11	1.56	1.53	1.69	2.17	3.45	1.56	2.35	2.99	3.27	3.40	1.48	1.48	1.56	1.76	1.97	0.16	0.39	0.63	0.87	1.11
β=0.75	δ=0.05	0.51	0.63	0.78	0.94	1.10	0.51	0.63	0.78	0.94	1.10	0.51	0.63	0.78	0.94	1.10	0.50	0.61	0.76	0.92	1.08	0.22	0.38	0.54	0.70	0.87
	δ=0.25	0.51	0.63	0.78	0.94	1.10	0.51	0.63	0.78	0.94	1.10	0.51	0.63	0.78	0.94	1.10	0.48	0.58	0.72	0.86	1.01	0.20	0.35	0.50	0.65	0.80
	δ=0.50	0.51	0.63	0.78	0.94	1.10	0.51	0.63	0.78	0.94	1.10	0.51	0.63	0.78	0.94	1.10	0.50	0.55	0.73	0.91	1.10	0.18	0.32	0.45	0.62	0.81
	δ=0.75	0.51	0.63	0.78	0.94	1.10	0.51	0.63	0.78	0.94	1.10	0.51	0.63	0.78	0.94	1.28	0.50	0.63	0.83	1.03	1.24	0.16	0.29	0.48	0.68	0.89
	δ=0.95	0.51	0.63	0.78	0.94	1.10	0.51	0.63	0.78	0.94	1.10	0.51	0.63	0.78	1.15	1.82	0.54	0.74	0.97	1.21	1.44	0.15	0.31	0.54	0.78	1.01
β=0.95	δ=0.05	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.44	0.57	0.70	0.14	0.27	0.40	0.53	0.67
	δ=0.25	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.44	0.57	0.69	0.14	0.27	0.40	0.53	0.66
	δ=0.50	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.43	0.59	0.76	0.14	0.26	0.39	0.52	0.67
	δ=0.75	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.50	0.70	0.90	0.13	0.26	0.40	0.60	0.80
	δ=0.95	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.44	0.57	0.70	0.17	0.36	0.59	0.83	1.06	0.13	0.25	0.48	0.71	0.94

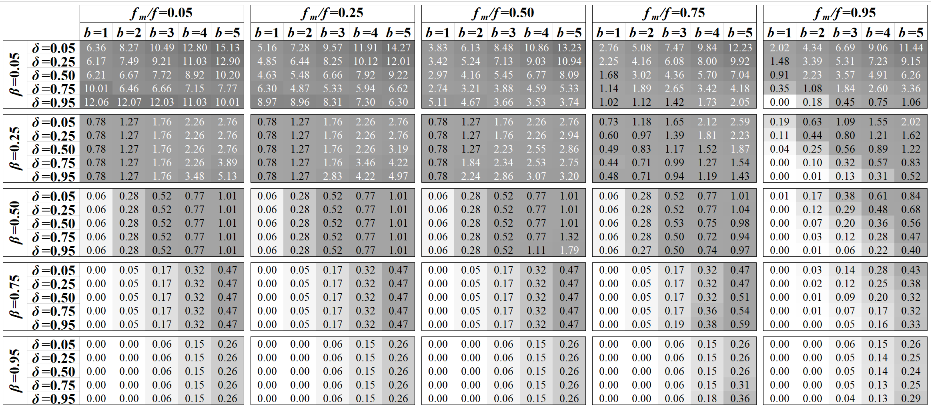
(b) $s_n = 3$



		f_m/f=0.05					f_m/f=0.25					f_m/f=0.50					f_m/f=0.75					f_m/f=0.95				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	12.14	12.93	14.77	16.90	19.12	9.43	10.83	12.87	15.07	17.36	6.44	8.38	10.60	12.90	15.24	4.03	6.28	8.63	10.99	13.37	2.62	4.94	7.31	9.68	12.07
	δ=0.25	12.51	12.38	13.58	15.12	16.80	9.47	10.10	11.55	13.24	15.02	6.18	7.52	9.25	11.09	12.97	3.60	5.39	7.28	9.19	11.11	2.08	4.00	5.92	7.85	9.77
	δ=0.50	16.02	12.13	12.40	13.22	14.26	10.40	9.49	10.19	11.23	12.40	6.13	6.65	7.74	8.97	10.26	3.17	4.33	5.63	6.97	8.32	1.51	2.85	4.20	5.56	6.93
	δ=0.75	21.04	16.02	12.66	12.43	12.62	15.43	10.40	9.72	9.89	10.32	8.42	6.35	6.60	7.13	7.78	3.00	3.41	4.08	4.82	5.59	0.94	1.71	2.50	3.30	4.11
	δ=0.95	23.07	23.08	23.04	22.04	21.04	18.09	18.09	17.44	16.44	15.43	11.85	11.41	10.41	9.42	8.41	5.39	4.39	3.71	3.81	4.00	0.56	0.87	1.19	1.53	1.88
β=0.25	δ=0.05	5.28	4.52	4.59	4.88	5.26	5.28	4.52	4.59	4.88	5.26	5.28	4.52	4.59	4.88	5.26	2.38	2.49	2.84	3.25	3.68	0.74	1.19	1.66	2.13	2.60
	δ=0.25	5.28	4.52	4.59	4.88	5.26	5.28	4.52	4.59	4.88	5.26	5.28	4.97	4.83	4.97	5.22	2.30	2.35	2.64	3.00	3.39	0.61	0.96	1.36	1.79	2.21
	δ=0.50	5.28	4.52	4.59	4.88	5.26	5.28	4.52	4.59	4.88	5.70	5.41	5.68	5.22	5.17	5.28	3.11	2.42	2.59	2.85	3.15	0.46	0.80	1.15	1.49	1.85
	δ=0.75	5.28	4.52	4.59	4.88	5.26	5.28	4.52	4.59	6.72	8.23	6.59	6.95	6.41	6.04	5.92	4.11	3.11	2.78	2.92	3.11	0.41	0.68	0.96	1.24	1.53
	δ=0.95	5.28	4.52	4.59	4.88	7.56	5.28	4.52	5.94	8.40	9.81	7.37	8.20	8.34	8.31	8.21	4.33	4.33	4.33	4.31	4.11	0.43	0.66	0.90	1.14	1.39
β=0.50	δ=0.05	1.56	1.53	1.69	1.89	2.11	1.56	1.53	1.69	1.89	2.11	1.56	1.53	1.69	1.89	2.11	1.56	1.53	1.69	1.89	2.11	0.42	0.65	0.89	1.13	1.36
	δ=0.25	1.56	1.53	1.69	1.89	2.11	1.56	1.53	1.69	1.89	2.11	1.56	1.53	1.69	1.89	2.11	1.56	1.50	1.63	1.82	2.03	0.38	0.57	0.77	0.97	1.19
	δ=0.50	1.56	1.53	1.69	1.89	2.11	1.56	1.53	1.69	1.89	2.11	1.56	1.53	1.69	1.89	2.11	1.63	1.64	1.74	1.91	2.09	0.32	0.47	0.69	0.91	1.14
	δ=0.75	1.56	1.53	1.69	1.89	2.11	1.56	1.53	1.69	1.89	2.11	1.56	1.53	1.69	1.89	2.11	1.97	1.96	2.00	2.14	2.31	0.27	0.46	0.69	0.91	1.13
	δ=0.95	1.56	1.53	1.69	1.89	2.11	1.56	1.53	1.69	1.89	2.11	1.56	1.53	1.69	1.89	2.75	2.26	2.48	2.51	2.61	2.77	0.26	0.50	0.74	0.98	1.22
β=0.75	δ=0.05	0.51	0.63	0.78	0.94	1.10	0.51	0.63	0.78	0.94	1.10	0.51	0.63	0.78	0.94	1.10	0.51	0.63	0.78	0.94	1.10	0.25	0.41	0.57	0.74	0.90
	δ=0.25	0.51	0.63	0.78	0.94	1.10	0.51	0.63	0.78	0.94	1.10	0.51	0.63	0.78	0.94	1.10	0.51	0.63	0.78	0.94	1.10	0.24	0.38	0.53	0.68	0.83
	δ=0.50	0.51	0.63	0.78	0.94	1.10	0.51	0.63	0.78	0.94	1.10	0.51	0.63	0.78	0.94	1.10	0.51	0.63	0.78	0.94	1.23	0.22	0.35	0.49	0.66	0.84
	δ=0.75	0.51	0.63	0.78	0.94	1.10	0.51	0.63	0.78	0.94	1.10	0.51	0.63	0.78	0.94	1.10	0.51	0.63	0.78	1.28	1.51	0.20	0.32	0.52	0.73	0.93
	δ=0.95	0.51	0.63	0.78	0.94	1.10	0.51	0.63	0.78	0.94	1.10	0.51	0.63	0.78	0.94	1.10	0.51	0.63	1.10	1.53	1.78	0.18	0.36	0.59	0.83	1.07
β=0.95	δ=0.05	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.44	0.57	0.70	0.15	0.28	0.41	0.54	0.67
	δ=0.25	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.44	0.57	0.70	0.15	0.27	0.40	0.53	0.66
	δ=0.50	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.44	0.57	0.70	0.14	0.27	0.40	0.52	0.68
	δ=0.75	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.44	0.57	0.70	0.14	0.26	0.41	0.61	0.81
	δ=0.95	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.44	0.58	1.05	0.14	0.26	0.48	0.72	0.95

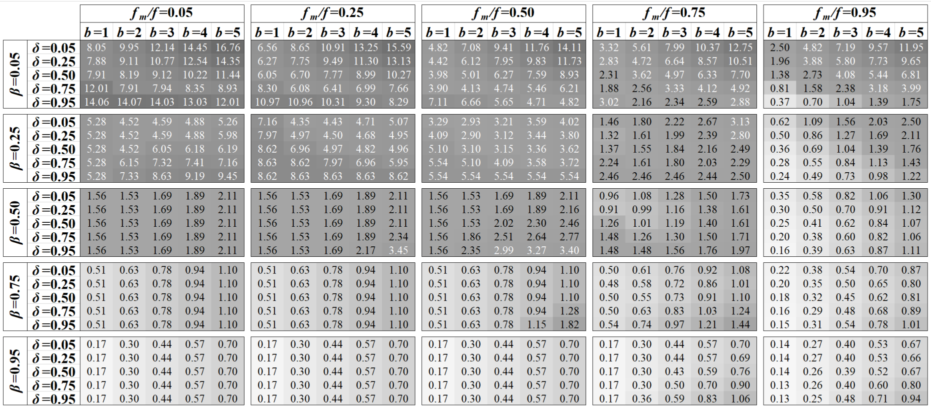
(c) $s_n = 5$

Figure B.1: Impact of s_n on the manufacturer's profit function, where the parameters are set to $f = 5, v = 3$, and values from 0 to 45.68 are highlighted as a heatmap from white to black background



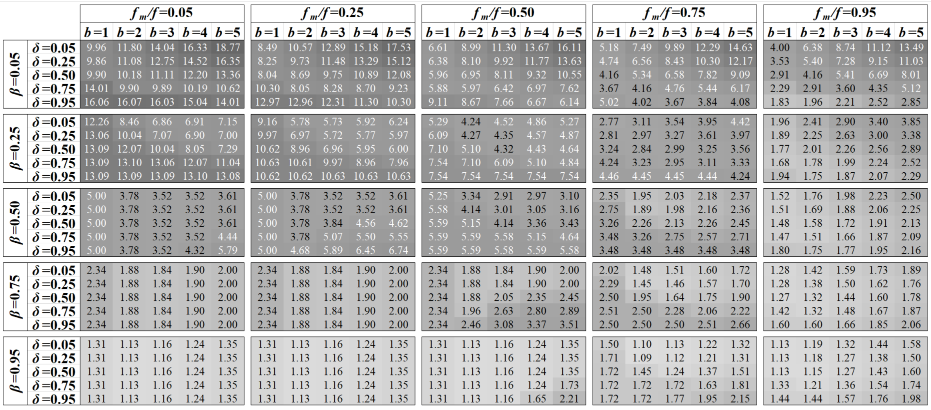
		f_m/f=0.05					f_m/f=0.25					f_m/f=0.50					f_m/f=0.75					f_m/f=0.95				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	6.36	8.27	10.49	12.80	15.13	5.16	7.28	9.57	11.91	14.27	3.83	6.13	8.48	10.86	13.23	2.76	5.08	7.47	9.84	12.23	2.02	4.34	6.69	9.06	11.44
	δ=0.25	6.17	7.49	9.21	11.03	12.90	4.85	6.44	8.25	10.12	12.01	3.42	5.24	7.13	9.03	10.94	2.25	4.16	6.08	8.00	9.92	1.48	3.39	5.31	7.23	9.15
	δ=0.50	6.21	6.67	7.72	8.92	10.20	4.63	5.48	6.66	7.92	9.22	2.97	4.16	5.45	6.77	8.09	1.68	3.02	4.36	5.70	7.04	0.91	2.23	3.57	4.91	6.26
	δ=0.75	10.01	6.46	6.66	7.15	7.77	6.30	4.87	5.33	5.94	6.62	2.74	3.21	3.88	4.59	5.33	1.14	1.89	2.65	3.42	4.18	0.35	1.08	1.84	2.60	3.36
	δ=0.95	12.06	12.07	12.03	11.03	10.01	8.97	8.96	8.31	7.30	6.30	5.11	4.67	3.66	3.53	3.74	1.02	1.12	1.42	1.73	2.05	0.00	0.18	0.45	0.75	1.06
β=0.25	δ=0.05	0.78	1.27	1.76	2.26	2.76	0.78	1.27	1.76	2.26	2.76	0.78	1.27	1.76	2.26	2.76	0.73	1.18	1.65	2.12	2.59	0.19	0.63	1.09	1.55	2.02
	δ=0.25	0.78	1.27	1.76	2.26	2.76	0.78	1.27	1.76	2.26	2.76	0.78	1.27	1.76	2.26	2.94	0.60	0.97	1.39	1.81	2.23	0.11	0.44	0.80	1.21	1.62
	δ=0.50	0.78	1.27	1.76	2.26	2.76	0.78	1.27	1.76	2.26	3.19	0.78	1.27	2.23	2.55	2.86	0.49	0.83	1.17	1.52	1.87	0.04	0.25	0.56	0.89	1.22
	δ=0.75	0.78	1.27	1.76	2.26	3.89	0.78	1.27	1.76	3.46	4.22	0.78	1.84	2.34	2.53	2.75	0.44	0.71	0.99	1.27	1.54	0.00	0.10	0.32	0.57	0.83
	δ=0.95	0.78	1.27	1.76	3.48	5.13	0.78	1.27	2.83	4.22	4.97	0.78	2.24	2.86	3.07	3.20	0.48	0.71	0.94	1.19	1.43	0.00	0.01	0.13	0.31	0.52
β=0.50	δ=0.05	0.06	0.28	0.52	0.77	1.01	0.06	0.28	0.52	0.77	1.01	0.06	0.28	0.52	0.77	1.01	0.06	0.28	0.52	0.77	1.01	0.01	0.17	0.38	0.61	0.84
	δ=0.25	0.06	0.28	0.52	0.77	1.01	0.06	0.28	0.52	0.77	1.01	0.06	0.28	0.52	0.77	1.01	0.06	0.28	0.52	0.77	1.04	0.00	0.12	0.29	0.48	0.68
	δ=0.50	0.06	0.28	0.52	0.77	1.01	0.06	0.28	0.52	0.77	1.01	0.06	0.28	0.52	0.77	1.01	0.06	0.28	0.53	0.75	0.98	0.00	0.07	0.20	0.36	0.56
	δ=0.75	0.06	0.28	0.52	0.77	1.01	0.06	0.28	0.52	0.77	1.01	0.06	0.28	0.52	0.77	1.32	0.06	0.28	0.50	0.72	0.94	0.00	0.03	0.12	0.28	0.47
	δ=0.95	0.06	0.28	0.52	0.77	1.01	0.06	0.28	0.52	0.77	1.01	0.06	0.28	0.52	1.11	1.79	0.06	0.27	0.50	0.74	0.97	0.00	0.01	0.06	0.22	0.40
β=0.75	δ=0.05	0.00	0.05	0.17	0.32	0.47	0.00	0.05	0.17	0.32	0.47	0.00	0.05	0.17	0.32	0.47	0.00	0.05	0.17	0.32	0.47	0.00	0.03	0.14	0.28	0.43
	δ=0.25	0.00	0.05	0.17	0.32	0.47	0.00	0.05	0.17	0.32	0.47	0.00	0.05	0.17	0.32	0.47	0.00	0.05	0.17	0.32	0.47	0.00	0.02	0.12	0.25	0.38
	δ=0.50	0.00	0.05	0.17	0.32	0.47	0.00	0.05	0.17	0.32	0.47	0.00	0.05	0.17	0.32	0.47	0.00	0.05	0.17	0.32	0.51	0.00	0.01	0.09	0.20	0.32
	δ=0.75	0.00	0.05	0.17	0.32	0.47	0.00	0.05	0.17	0.32	0.47	0.00	0.05	0.17	0.32	0.47	0.00	0.05	0.17	0.36	0.54	0.00	0.01	0.07	0.17	0.32
	δ=0.95	0.00	0.05	0.17	0.32	0.47	0.00	0.05	0.17	0.32	0.47	0.00	0.05	0.17	0.32	0.47	0.00	0.05	0.19	0.38	0.59	0.00	0.00	0.05	0.16	0.33
β=0.95	δ=0.05	0.00	0.00	0.06	0.15	0.26	0.00	0.00	0.06	0.15	0.26	0.00	0.00	0.06	0.15	0.26	0.00	0.00	0.06	0.15	0.26	0.00	0.00	0.06	0.15	0.26
	δ=0.25	0.00	0.00	0.06	0.15	0.26	0.00	0.00	0.06	0.15	0.26	0.00	0.00	0.06	0.15	0.26	0.00	0.00	0.06	0.15	0.26	0.00	0.00	0.05	0.14	0.25
	δ=0.50	0.00	0.00	0.06	0.15	0.26	0.00	0.00	0.06	0.15	0.26	0.00	0.00	0.06	0.15	0.26	0.00	0.00	0.06	0.15	0.26	0.00	0.00	0.05	0.14	0.24
	δ=0.75	0.00	0.00	0.06	0.15	0.26	0.00	0.00	0.06	0.15	0.26	0.00	0.00	0.06	0.15	0.26	0.00	0.00	0.06	0.15	0.31	0.00	0.00	0.05	0.13	0.25
	δ=0.95	0.00	0.00	0.06	0.15	0.26	0.00	0.00	0.06	0.15	0.26	0.00	0.00	0.06	0.15	0.26	0.00	0.00	0.06	0.18	0.36	0.00	0.00	0.04	0.13	0.29

(a) $\upsilon = 1$



		f_m/f=0.05					f_m/f=0.25					f_m/f=0.50					f_m/f=0.75					f_m/f=0.95				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	8.05	9.95	12.14	14.45	16.76	6.56	8.65	10.91	13.25	15.59	4.82	7.08	9.41	11.76	14.11	3.32	5.61	7.99	10.37	12.75	2.50	4.82	7.19	9.57	11.95
	δ=0.25	7.88	9.11	10.77	12.54	14.35	6.27	7.75	9.49	11.30	13.13	4.42	6.12	7.95	9.83	11.73	2.83	4.72	6.64	8.57	10.51	1.96	3.88	5.80	7.73	9.65
	δ=0.50	7.91	8.19	9.12	10.22	11.44	6.05	6.70	7.77	8.99	10.27	3.98	5.01	6.27	7.59	8.93	2.31	3.62	4.97	6.33	7.70	1.38	2.73	4.08	5.44	6.81
	δ=0.75	12.01	7.91	7.94	8.35	8.93	8.30	6.08	6.41	6.99	7.66	3.90	4.13	4.74	5.46	6.21	1.88	2.56	3.33	4.12	4.92	0.81	1.58	2.38	3.18	3.99
	δ=0.95	14.06	14.07	14.03	13.03	12.01	10.97	10.96	10.31	9.30	8.29	7.11	6.66	5.65	4.71	4.82	3.02	2.16	2.34	2.59	2.88	0.37	0.70	1.04	1.39	1.75
β=0.25	δ=0.05	5.28	4.52	4.59	4.88	5.26	7.16	4.35	4.43	4.71	5.07	3.29	2.93	3.21	3.59	4.02	1.46	1.80	2.22	2.67	3.13	0.62	1.09	1.56	2.03	2.50
	δ=0.25	5.28	4.52	4.59	4.88	5.98	7.97	4.97	4.50	4.68	4.95	4.09	2.90	3.12	3.44	3.80	1.32	1.61	1.99	2.39	2.80	0.50	0.86	1.27	1.69	2.11
	δ=0.50	5.28	4.52	6.05	6.18	6.19	8.62	6.96	4.97	4.82	4.96	5.10	3.10	3.15	3.36	3.62	1.37	1.55	1.84	2.16	2.49	0.36	0.69	1.04	1.39	1.76
	δ=0.75	5.28	6.15	7.32	7.41	7.16	8.63	8.62	7.97	6.96	5.95	5.54	5.10	4.09	3.58	3.72	2.24	1.61	1.80	2.03	2.29	0.28	0.55	0.84	1.13	1.43
	δ=0.95	5.28	7.33	8.63	9.19	9.45	8.62	8.63	8.63	8.63	8.62	5.54	5.54	5.54	5.54	5.54	2.46	2.46	2.46	2.44	2.50	0.24	0.49	0.73	0.98	1.22
β=0.50	δ=0.05	1.56	1.53	1.69	1.89	2.11	1.56	1.53	1.69	1.89	2.11	1.56	1.53	1.69	1.89	2.11	0.96	1.08	1.28	1.50	1.73	0.35	0.58	0.82	1.06	1.30
	δ=0.25	1.56	1.53	1.69	1.89	2.11	1.56	1.53	1.69	1.89	2.11	1.56	1.53	1.69	1.89	2.16	0.91	0.99	1.16	1.38	1.61	0.30	0.50	0.70	0.91	1.12
	δ=0.50	1.56	1.53	1.69	1.89	2.11	1.56	1.53	1.69	1.89	2.11	1.56	1.53	2.02	2.30	2.46	1.26	1.01	1.19	1.40	1.61	0.25	0.41	0.62	0.84	1.07
	δ=0.75	1.56	1.53	1.69	1.89	2.11	1.56	1.53	1.69	1.89	2.34	1.56	1.86	2.51	2.64	2.77	1.48	1.26	1.30	1.50	1.71	0.20	0.38	0.60	0.82	1.06
	δ=0.95	1.56	1.53	1.69	1.89	2.11	1.56	1.53	1.69	2.17	3.45	1.56	2.35	2.99	3.27	3.40	1.48	1.48	1.56	1.76	1.97	0.16	0.39	0.63	0.87	1.11
β=0.75	δ=0.05	0.51	0.63	0.78	0.94	1.10	0.51	0.63	0.78	0.94	1.10	0.51	0.63	0.78	0.94	1.10	0.50	0.61	0.76	0.92	1.08	0.22	0.38	0.54	0.70	0.87
	δ=0.25	0.51	0.63	0.78	0.94	1.10	0.51	0.63	0.78	0.94	1.10	0.51	0.63	0.78	0.94	1.10	0.48	0.58	0.72	0.86	1.01	0.20	0.35	0.50	0.65	0.80
	δ=0.50	0.51	0.63	0.78	0.94	1.10	0.51	0.63	0.78	0.94	1.10	0.51	0.63	0.78	0.94	1.10	0.50	0.55	0.73	0.91	1.10	0.18	0.32	0.45	0.62	0.81
	δ=0.75	0.51	0.63	0.78	0.94	1.10	0.51	0.63	0.78	0.94	1.10	0.51	0.63	0.78	0.94	1.28	0.50	0.63	0.83	1.03	1.24	0.16	0.29	0.48	0.68	0.89
	δ=0.95	0.51	0.63	0.78	0.94	1.10	0.51	0.63	0.78	0.94	1.10	0.51	0.63	0.78	1.15	1.82	0.54	0.74	0.97	1.21	1.44	0.15	0.31	0.54	0.78	1.01
β=0.95	δ=0.05	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.44	0.57	0.70	0.14	0.27	0.40	0.53	0.67
	δ=0.25	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.44	0.57	0.69	0.14	0.27	0.40	0.53	0.66
	δ=0.50	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.43	0.59	0.76	0.14	0.26	0.39	0.52	0.67
	δ=0.75	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.50	0.70	0.90	0.13	0.26	0.40	0.60	0.80
	δ=0.95	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.44	0.57	0.70	0.17	0.30	0.44	0.57	0.70	0.17	0.36	0.59	0.83	1.06	0.13	0.25	0.48	0.71	0.94

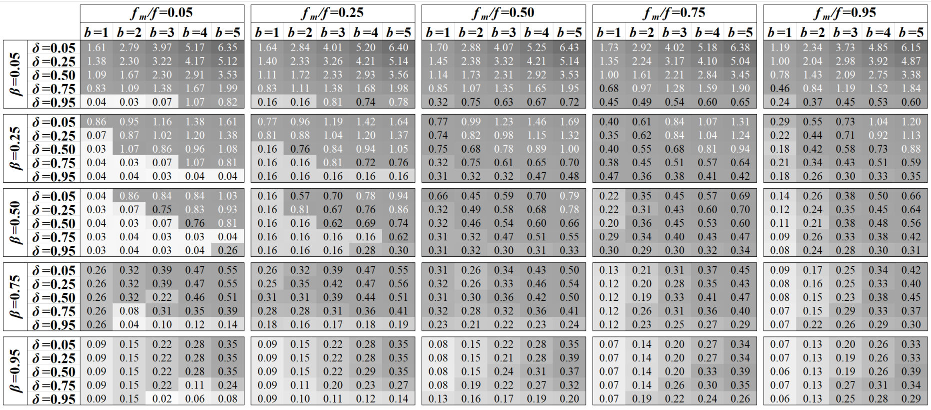
(b) $\upsilon = 3$



		f_m/f=0.05					f_m/f=0.25					f_m/f=0.50					f_m/f=0.75					f_m/f=0.95				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	9.96	11.80	14.04	16.33	18.77	8.49	10.57	12.89	15.18	17.53	6.61	8.99	11.30	13.67	16.11	5.18	7.49	9.89	12.29	14.63	4.00	6.38	8.74	11.12	13.49
	δ=0.25	9.86	11.08	12.75	14.52	16.35	8.25	9.73	11.48	13.29	15.12	6.38	8.10	9.92	11.77	13.63	4.74	6.56	8.43	10.30	12.17	3.53	5.40	7.28	9.15	11.03
	δ=0.50	9.90	10.18	11.11	12.20	13.36	8.04	8.69	9.75	10.89	12.08	5.96	6.95	8.11	9.32	10.55	4.16	5.34	6.58	7.82	9.09	2.91	4.16	5.41	6.69	8.01
	δ=0.75	14.01	9.90	9.89	10.19	10.62	10.30	8.05	8.28	8.70	9.23	5.88	5.97	6.42	6.97	7.62	3.67	4.16	4.76	5.44	6.17	2.29	2.91	3.60	4.35	5.12
	δ=0.95	16.06	16.07	16.03	15.04	14.01	12.97	12.96	12.31	11.30	10.30	9.11	8.67	7.66	6.67	6.14	5.02	4.02	3.67	3.84	4.08	1.83	1.96	2.21	2.52	2.85
β=0.25	δ=0.05	12.26	8.46	6.86	6.91	7.15	9.16	5.78	5.73	5.92	6.24	5.29	4.24	4.52	4.86	5.27	2.77	3.11	3.54	3.95	4.42	1.96	2.41	2.90	3.40	3.85
	δ=0.25	13.06	10.04	7.07	6.90	7.00	9.97	6.97	5.72	5.77	5.97	6.09	4.27	4.35	4.57	4.87	2.81	2.97	3.27	3.61	3.97	1.89	2.25	2.63	3.00	3.38
	δ=0.50	13.09	12.07	10.04	8.05	7.29	10.62	8.96	6.96	5.95	6.00	7.10	5.10	4.32	4.43	4.64	3.24	2.84	2.99	3.25	3.56	1.77	2.01	2.26	2.56	2.89
	δ=0.75	13.09	13.10	13.06	12.07	11.04	10.63	10.61	9.97	8.96	7.96	7.54	7.10	6.09	5.10	4.84	4.24	3.23	2.95	3.11	3.33	1.68	1.78	1.99	2.24	2.52
	δ=0.95	13.09	13.09	13.09	13.10	13.08	10.62	10.62	10.63	10.63	10.63	7.54	7.54	7.54	7.54	7.54	4.46	4.45	4.45	4.44	4.24	1.94	1.75	1.87	2.07	2.29
β=0.50	δ=0.05	5.00	3.78	3.52	3.52	3.61	5.00	3.78	3.52	3.52	3.61	5.25	3.34	2.91	2.97	3.10	2.35	1.95	2.03	2.18	2.37	1.52	1.76	1.98	2.23	2.50
	δ=0.25	5.00	3.78	3.52	3.52	3.61	5.00	3.78	3.52	3.52	3.61	5.58	4.14	3.01	3.05	3.16	2.75	1.89	1.98	2.16	2.36	1.51	1.69	1.88	2.06	2.25
	δ=0.50	5.00	3.78	3.52	3.52	3.61	5.00	3.78	3.84	4.56	4.62	5.59	5.15	4.14	3.36	3.43	3.26	2.26	2.13	2.26	2.45	1.48	1.58	1.72	1.91	2.13
	δ=0.75	5.00	3.78	3.52	3.52	4.44	5.00	3.78	5.07	5.50	5.55	5.59	5.59	5.58	5.15	4.64	3.48	3.26	2.75	2.57	2.71	1.47	1.51	1.66	1.87	2.09
	δ=0.95	5.00	3.78	3.52	4.32	5.79	5.00	4.68	5.89	6.45	6.74	5.59	5.59	5.58	5.59	5.58	3.48	3.48	3.48	3.48	3.48	1.80	1.75	1.77	1.95	2.16
β=0.75	δ=0.05	2.34	1.88	1.84	1.90	2.00	2.34	1.88	1.84	1.90	2.00	2.34	1.88	1.84	1.90	2.00	2.02	1.48	1.51	1.60	1.72	1.28	1.42	1.59	1.73	1.89
	δ=0.25	2.34	1.88	1.84	1.90	2.00	2.34	1.88	1.84	1.90	2.00	2.34	1.88	1.84	1.90	2.00	2.29	1.45	1.46	1.57	1.70	1.28	1.38	1.50	1.62	1.76
	δ=0.50	2.34	1.88	1.84	1.90	2.00	2.34	1.88	1.84	1.90	2.00	2.34	1.88	2.05	2.35	2.45	2.50	1.95	1.64	1.75	1.90	1.27	1.32	1.44	1.60	1.78
	δ=0.75	2.34	1.88	1.84	1.90	2.00	2.34	1.88	1.84	1.90	2.00	2.34	1.96	2.63	2.80	2.89	2.51	2.50	2.28	2.06	2.22	1.42	1.32	1.48	1.67	1.87
	δ=0.95	2.34	1.88	1.84	1.90	2.00	2.34	1.88	1.84	1.90	2.00	2.34	2.46	3.08	3.37	3.51	2.50	2.50	2.50	2.51	2.66	1.60	1.60	1.66	1.85	2.06
β=0.95	δ=0.05	1.31	1.13	1.16	1.24	1.35	1.31	1.13	1.16	1.24	1.35	1.31	1.13	1.16	1.24	1.35	1.50	1.10	1.13	1.22	1.32	1.13	1.19	1.32	1.44	1.58
	δ=0.25	1.31	1.13	1.16	1.24	1.35	1.31	1.13	1.16	1.24	1.35	1.31	1.13	1.16	1.24	1.35	1.71	1.09	1.12	1.21	1.31	1.13	1.18	1.27	1.38	1.50
	δ=0.50	1.31	1.13	1.16	1.24	1.35	1.31	1.13	1.16	1.24	1.35	1.31	1.13	1.16	1.24	1.35	1.72	1.45	1.24	1.37	1.51	1.13	1.15	1.27	1.43	1.60
	δ=0.75	1.31	1.13	1.16	1.24	1.35	1.31	1.13	1.16	1.24	1.35	1.31	1.13	1.16	1.24	1.73	1.72	1.72	1.72	1.63	1.81	1.33	1.21	1.36	1.54	1.74
	δ=0.95	1.31	1.13	1.16	1.24	1.35	1.31	1.13	1.16	1.24	1.35	1.31	1.13	1.16	1.65	2.21	1.72	1.72	1.77	1.95	2.15	1.44	1.44	1.57	1.76	1.98

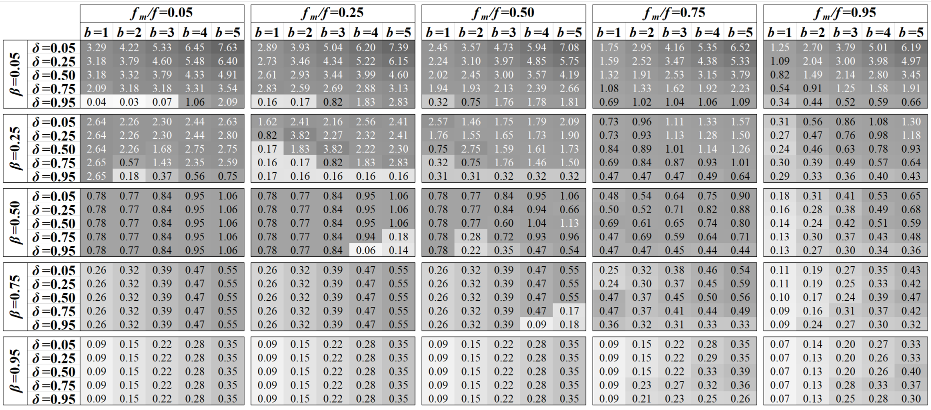
(c) $\upsilon = 5$

Figure B.2: Impact of υ on the manufacturer's profit function, where the parameters are set to $f = 5, s_n = 3$, and values from 0 to 45.68 are highlighted as a heatmap from white to black background



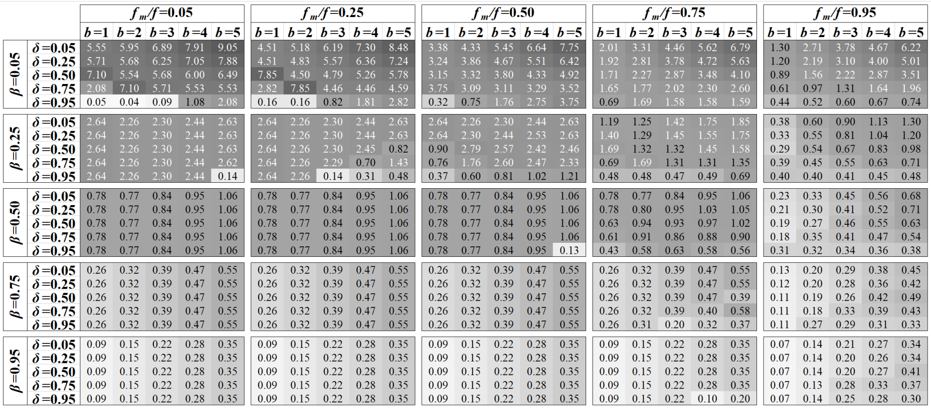
		$f_m/f=0.05$					$f_m/f=0.25$					$f_m/f=0.50$					$f_m/f=0.75$					$f_m/f=0.95$				
		$b=1$	$b=2$	$b=3$	$b=4$	$b=5$	$b=1$	$b=2$	$b=3$	$b=4$	$b=5$	$b=1$	$b=2$	$b=3$	$b=4$	$b=5$	$b=1$	$b=2$	$b=3$	$b=4$	$b=5$	$b=1$	$b=2$	$b=3$	$b=4$	$b=5$
$\beta=0.05$	$\delta=0.05$	1.61	2.79	3.97	5.17	6.35	1.64	2.84	4.01	5.20	6.40	1.70	2.88	4.07	5.25	6.43	1.73	2.92	4.02	5.18	6.38	1.19	2.34	3.73	4.85	6.15
	$\delta=0.25$	1.38	2.30	3.22	4.17	5.12	1.40	2.33	3.26	4.21	5.14	1.45	2.38	3.32	4.21	5.14	1.35	2.24	3.17	4.10	5.04	1.00	2.04	2.98	3.92	4.87
	$\delta=0.50$	1.09	1.67	2.30	2.91	3.53	1.11	1.72	2.33	2.93	3.56	1.14	1.73	2.31	2.92	3.53	1.00	1.61	2.21	2.84	3.45	0.78	1.43	2.09	2.75	3.38
	$\delta=0.75$	0.83	1.09	1.38	1.67	1.99	0.83	1.11	1.38	1.68	1.98	0.85	1.07	1.35	1.65	1.95	0.68	0.97	1.28	1.59	1.90	0.46	0.84	1.19	1.52	1.84
	$\delta=0.95$	0.04	0.03	0.07	1.07	0.82	0.16	0.16	0.81	0.74	0.78	0.32	0.75	0.63	0.67	0.72	0.45	0.49	0.54	0.60	0.65	0.24	0.37	0.45	0.53	0.60
$\beta=0.25$	$\delta=0.05$	0.86	0.95	1.16	1.38	1.61	0.77	0.96	1.19	1.42	1.64	0.77	0.99	1.23	1.46	1.69	0.40	0.61	0.84	1.07	1.31	0.29	0.55	0.73	1.04	1.20
	$\delta=0.25$	0.07	0.87	1.02	1.20	1.38	0.81	0.88	1.04	1.20	1.37	0.74	0.82	0.98	1.15	1.32	0.35	0.62	0.84	1.04	1.24	0.22	0.44	0.71	0.92	1.13
	$\delta=0.50$	0.03	1.07	0.86	0.96	1.08	0.16	0.76	0.84	0.94	1.05	0.75	0.68	0.78	0.89	1.00	0.40	0.55	0.68	0.81	0.94	0.18	0.42	0.58	0.73	0.88
	$\delta=0.75$	0.04	0.03	0.07	1.07	0.81	0.16	0.16	0.81	0.72	0.76	0.32	0.75	0.61	0.65	0.70	0.38	0.45	0.51	0.57	0.64	0.21	0.34	0.43	0.51	0.59
	$\delta=0.95$	0.04	0.04	0.03	0.04	0.04	0.16	0.16	0.16	0.16	0.16	0.31	0.32	0.32	0.47	0.48	0.47	0.36	0.38	0.41	0.42	0.18	0.26	0.30	0.33	0.35
$\beta=0.50$	$\delta=0.05$	0.04	0.86	0.84	0.84	1.03	0.16	0.57	0.70	0.78	0.94	0.66	0.45	0.59	0.70	0.79	0.22	0.35	0.45	0.57	0.69	0.14	0.26	0.38	0.50	0.66
	$\delta=0.25$	0.03	0.07	0.75	0.83	0.93	0.16	0.81	0.67	0.76	0.86	0.32	0.49	0.58	0.68	0.78	0.22	0.31	0.43	0.60	0.70	0.12	0.24	0.35	0.45	0.64
	$\delta=0.50$	0.04	0.03	0.07	0.76	0.81	0.16	0.16	0.62	0.69	0.74	0.32	0.46	0.54	0.60	0.66	0.20	0.36	0.45	0.53	0.60	0.11	0.21	0.38	0.48	0.56
	$\delta=0.75$	0.03	0.04	0.03	0.03	0.04	0.16	0.16	0.16	0.16	0.62	0.31	0.32	0.47	0.51	0.55	0.29	0.34	0.40	0.43	0.47	0.09	0.26	0.33	0.38	0.42
	$\delta=0.95$	0.03	0.04	0.03	0.04	0.26	0.16	0.16	0.16	0.28	0.30	0.31	0.32	0.30	0.31	0.33	0.30	0.29	0.30	0.32	0.34	0.08	0.24	0.28	0.30	0.31
$\beta=0.75$	$\delta=0.05$	0.26	0.32	0.39	0.47	0.55	0.26	0.32	0.39	0.47	0.55	0.31	0.26	0.34	0.43	0.50	0.13	0.21	0.31	0.37	0.45	0.09	0.17	0.25	0.34	0.42
	$\delta=0.25$	0.26	0.32	0.39	0.47	0.55	0.25	0.35	0.42	0.47	0.56	0.32	0.26	0.33	0.46	0.54	0.12	0.20	0.28	0.35	0.43	0.08	0.16	0.25	0.33	0.40
	$\delta=0.50$	0.26	0.32	0.22	0.46	0.51	0.31	0.31	0.39	0.44	0.51	0.31	0.30	0.36	0.42	0.50	0.12	0.19	0.33	0.41	0.47	0.08	0.15	0.23	0.38	0.45
	$\delta=0.75$	0.26	0.08	0.31	0.35	0.39	0.28	0.28	0.31	0.36	0.41	0.32	0.28	0.32	0.36	0.41	0.12	0.26	0.31	0.36	0.40	0.07	0.15	0.29	0.33	0.37
	$\delta=0.95$	0.26	0.04	0.10	0.12	0.14	0.18	0.16	0.17	0.18	0.19	0.23	0.21	0.22	0.23	0.24	0.12	0.23	0.25	0.27	0.29	0.07	0.22	0.26	0.29	0.30
$\beta=0.95$	$\delta=0.05$	0.09	0.15	0.22	0.28	0.35	0.09	0.15	0.22	0.28	0.35	0.08	0.15	0.22	0.28	0.35	0.07	0.14	0.20	0.27	0.34	0.07	0.13	0.20	0.26	0.33
	$\delta=0.25$	0.09	0.15	0.22	0.28	0.35	0.09	0.15	0.22	0.28	0.35	0.08	0.15	0.21	0.28	0.39	0.07	0.14	0.20	0.27	0.34	0.07	0.13	0.19	0.26	0.33
	$\delta=0.50$	0.09	0.15	0.22	0.28	0.35	0.09	0.15	0.22	0.29	0.35	0.08	0.15	0.24	0.31	0.37	0.07	0.14	0.20	0.33	0.39	0.06	0.13	0.19	0.26	0.39
	$\delta=0.75$	0.09	0.15	0.22	0.11	0.24	0.09	0.11	0.20	0.23	0.27	0.08	0.19	0.22	0.27	0.32	0.07	0.14	0.26	0.30	0.35	0.07	0.13	0.27	0.31	0.34
	$\delta=0.95$	0.09	0.15	0.02	0.06	0.08	0.09	0.10	0.11	0.12	0.14	0.13	0.16	0.17	0.19	0.20	0.07	0.19	0.22	0.24	0.26	0.06	0.13	0.25	0.28	0.29

(a) $s_n = 1$



		$f_m/f=0.05$					$f_m/f=0.25$					$f_m/f=0.50$					$f_m/f=0.75$					$f_m/f=0.95$				
		$b=1$	$b=2$	$b=3$	$b=4$	$b=5$	$b=1$	$b=2$	$b=3$	$b=4$	$b=5$	$b=1$	$b=2$	$b=3$	$b=4$	$b=5$	$b=1$	$b=2$	$b=3$	$b=4$	$b=5$	$b=1$	$b=2$	$b=3$	$b=4$	$b=5$
$\beta=0.05$	$\delta=0.05$	3.29	4.22	5.33	6.45	7.63	2.89	3.93	5.04	6.20	7.39	2.45	3.57	4.73	5.94	7.08	1.75	2.95	4.16	5.35	6.52	1.25	2.70	3.79	5.01	6.19
	$\delta=0.25$	3.18	3.79	4.60	5.48	6.40	2.73	3.46	4.34	5.22	6.15	2.24	3.10	3.97	4.85	5.75	1.59	2.52	3.47	4.38	5.33	1.09	2.04	3.00	3.98	4.97
	$\delta=0.50$	3.18	3.32	3.79	4.33	4.91	2.61	2.93	3.44	3.99	4.60	2.02	2.45	3.00	3.57	4.19	1.32	1.91	2.53	3.15	3.79	0.82	1.49	2.14	2.80	3.45
	$\delta=0.75$	2.09	3.18	3.18	3.31	3.54	2.83	2.59	2.69	2.88	3.13	1.94	1.93	2.13	2.39	2.66	1.08	1.33	1.62	1.92	2.23	0.54	0.91	1.25	1.58	1.91
	$\delta=0.95$	0.04	0.03	0.07	1.06	2.09	0.16	0.17	0.82	1.83	2.83	0.32	0.75	1.76	1.78	1.81	0.69	1.02	1.04	1.06	1.09	0.34	0.44	0.52	0.59	0.66
$\beta=0.25$	$\delta=0.05$	2.64	2.26	2.30	2.44	2.63	1.62	2.41	2.16	2.56	2.41	2.57	1.46	1.75	1.79	2.09	0.73	0.96	1.11	1.33	1.57	0.31	0.56	0.86	1.08	1.30
	$\delta=0.25$	2.64	2.26	2.30	2.44	2.80	0.82	3.82	2.27	2.32	2.41	1.76	1.55	1.65	1.73	1.90	0.73	0.93	1.13	1.28	1.50	0.27	0.47	0.76	0.98	1.18
	$\delta=0.50$	2.64	2.26	1.68	2.75	2.75	0.17	1.83	3.82	2.22	2.30	0.75	2.75	1.59	1.61	1.73	0.84	0.89	1.01	1.14	1.26	0.24	0.46	0.63	0.78	0.93
	$\delta=0.75$	2.65	0.57	1.43	2.35	2.59	0.16	0.17	0.82	1.83	2.83	0.32	0.75	1.76	1.46	1.50	0.69	0.84	0.87	0.93	1.01	0.30	0.39	0.49	0.57	0.64
	$\delta=0.95$	2.65	0.18	0.37	0.56	0.75	0.17	0.16	0.16	0.16	0.16	0.31	0.31	0.32	0.32	0.32	0.47	0.47	0.47	0.49	0.64	0.29	0.33	0.36	0.40	0.43
$\beta=0.50$	$\delta=0.05$	0.78	0.77	0.84	0.95	1.06	0.78	0.77	0.84	0.95	1.06	0.78	0.77	0.84	0.95	1.06	0.48	0.54	0.64	0.75	0.90	0.18	0.31	0.41	0.53	0.65
	$\delta=0.25$	0.78	0.77	0.84	0.95	1.06	0.78	0.77	0.84	0.95	1.06	0.78	0.77	0.84	0.94	0.66	0.50	0.52	0.71	0.82	0.88	0.16	0.28	0.38	0.49	0.68
	$\delta=0.50$	0.78	0.77	0.84	0.95	1.06	0.78	0.77	0.84	0.95	1.06	0.78	0.77	0.60	1.04	1.13	0.69	0.61	0.65	0.74	0.80	0.14	0.24	0.42	0.51	0.59
	$\delta=0.75$	0.78	0.77	0.84	0.95	1.06	0.78	0.77	0.84	0.94	0.18	0.78	0.28	0.72	0.93	0.96	0.47	0.69	0.59	0.64	0.71	0.13	0.30	0.37	0.43	0.48
	$\delta=0.95$	0.78	0.77	0.84	0.95	1.06	0.78	0.77	0.84	0.06	0.14	0.78	0.22	0.35	0.47	0.54	0.47	0.47	0.45	0.44	0.44	0.13	0.27	0.30	0.34	0.36
$\beta=0.75$	$\delta=0.05$	0.26	0.32	0.39	0.47	0.55	0.26	0.32	0.39	0.47	0.55	0.26	0.32	0.39	0.47	0.55	0.25	0.32	0.38	0.46	0.54	0.11	0.19	0.27	0.35	0.43
	$\delta=0.25$	0.26	0.32	0.39	0.47	0.55	0.26	0.32	0.39	0.47	0.55	0.26	0.32	0.39	0.47	0.55	0.24	0.30	0.37	0.45	0.59	0.11	0.19	0.25	0.33	0.42
	$\delta=0.50$	0.26	0.32	0.39	0.47	0.55	0.26	0.32	0.39	0.47	0.55	0.26	0.32	0.39	0.47	0.55	0.47	0.37	0.45	0.50	0.56	0.10	0.17	0.24	0.39	0.47
	$\delta=0.75$	0.26	0.32	0.39	0.47	0.55	0.26	0.32	0.39	0.47	0.55	0.26	0.32	0.39	0.47	0.17	0.47	0.37	0.41	0.44	0.49	0.09	0.16	0.31	0.37	0.42
	$\delta=0.95$	0.26	0.32	0.39	0.47	0.55	0.26	0.32	0.39	0.47	0.55	0.26	0.32	0.39	0.09	0.18	0.36	0.32	0.31	0.33	0.33	0.09	0.24	0.27	0.30	0.32
$\beta=0.95$	$\delta=0.05$	0.09	0.15	0.22	0.28	0.35	0.09	0.15	0.22	0.28	0.35	0.09	0.15	0.22	0.28	0.35	0.09	0.15	0.22	0.28	0.35	0.07	0.14	0.20	0.27	0.33
	$\delta=0.25$	0.09	0.15	0.22	0.28	0.35	0.09	0.15	0.22	0.28	0.35	0.09	0.15	0.22	0.28	0.35	0.09	0.15	0.22	0.29	0.35	0.07	0.13	0.20	0.26	0.33
	$\delta=0.50$	0.09	0.15	0.22	0.28	0.35	0.09	0.15	0.22	0.28	0.35	0.09	0.15	0.22	0.28	0.35	0.09	0.15	0.22	0.33	0.39	0.07	0.13	0.20	0.26	0.40
	$\delta=0.75$	0.09	0.15	0.22	0.28	0.35	0.09	0.15	0.22	0.28	0.35	0.09	0.15	0.22	0.28	0.35	0.09	0.23	0.27	0.32	0.36	0.07	0.13	0.28	0.33	0.37
	$\delta=0.95$	0.09	0.15	0.22	0.28	0.35	0.09	0.15	0.22	0.28	0.35	0.09	0.15	0.22	0.28	0.35	0.09	0.21	0.23	0.25	0.26	0.07	0.13	0.25	0.28	0.30

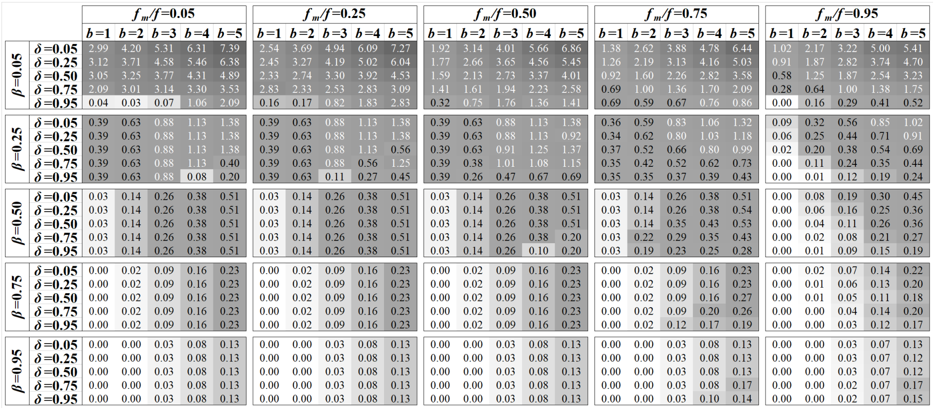
(b) $s_n = 3$



		$f_m/f=0.05$					$f_m/f=0.25$					$f_m/f=0.50$					$f_m/f=0.75$					$f_m/f=0.95$				
		$b=1$	$b=2$	$b=3$	$b=4$	$b=5$	$b=1$	$b=2$	$b=3$	$b=4$	$b=5$	$b=1$	$b=2$	$b=3$	$b=4$	$b=5$	$b=1$	$b=2$	$b=3$	$b=4$	$b=5$	$b=1$	$b=2$	$b=3$	$b=4$	$b=5$
$\beta=0.05$	$\delta=0.05$	5.55	5.95	6.89	7.91	9.05	4.51	5.18	6.19	7.30	8.48	3.38	4.33	5.45	6.64	7.75	2.01	3.31	4.46	5.62	6.79	1.30	2.71	3.78	4.67	6.22
	$\delta=0.25$	5.71	5.68	6.25	7.05	7.88	4.51	4.83	5.57	6.36	7.24	3.24	3.86	4.67	5.51	6.42	1.92	2.81	3.78	4.72	5.63	1.20	2.19	3.10	4.00	5.01
	$\delta=0.50$	7.10	5.54	5.68	6.00	6.49	7.85	4.50	4.79	5.26	5.78	3.15	3.32	3.80	4.33	4.92	1.71	2.27	2.87	3.48	4.10	0.89	1.56	2.22	2.87	3.51
	$\delta=0.75$	2.08	7.10	5.71	5.53	5.53	2.82	7.85	4.46	4.46	4.59	3.75	3.09	3.11	3.29	3.52	1.65	1.77	2.02	2.30	2.60	0.61	0.97	1.31	1.64	1.96
	$\delta=0.95$	0.05	0.04	0.09	1.08	2.08	0.16	0.16	0.82	1.81	2.82	0.32	0.75	1.76	2.75	3.75	0.69	1.69	1.58	1.58	1.59	0.44	0.52	0.60	0.67	0.74
$\beta=0.25$	$\delta=0.05$	2.64	2.26	2.30	2.44	2.63	2.64	2.26	2.30	2.44	2.63	2.64	2.26	2.30	2.44	2.63	1.19	1.25	1.42	1.75	1.85	0.38	0.60	0.90	1.13	1.30
	$\delta=0.25$	2.64	2.26	2.30	2.44	2.63	2.64	2.26	2.30	2.44	2.63	2.64	2.30	2.44	2.53	2.63	1.40	1.29	1.45	1.55	1.75	0.33	0.55	0.81	1.04	1.20
	$\delta=0.50$	2.64	2.26	2.30	2.44	2.63	2.64	2.26	2.30	2.45	0.82	0.90	2.79	2.57	2.42	2.46	1.69	1.32	1.32	1.45	1.58	0.29	0.54	0.67	0.83	0.98
	$\delta=0.75$	2.64	2.26	2.30	2.44	2.62	2.64	2.26	2.29	0.70	1.43	0.76	1.76	2.60	2.47	2.33	0.69	1.69	1.31	1.31	1.35	0.39	0.45	0.55	0.63	0.71
	$\delta=0.95$	2.64	2.26	2.30	2.44	0.14	2.64	2.26	0.14	0.31	0.48	0.37	0.60	0.81	1.02	1.21	0.48	0.48	0.47	0.49	0.69	0.40	0.40	0.41	0.45	0.48
$\beta=0.50$	$\delta=0.05$	0.78	0.77	0.84	0.95	1.06	0.78	0.77	0.84	0.95	1.06	0.78	0.77	0.84	0.95	1.06	0.78	0.77	0.84	0.95	1.06	0.23	0.33	0.45	0.56	0.68
	$\delta=0.25$	0.78	0.77	0.84	0.95	1.06	0.78	0.77	0.84	0.95	1.06	0.78	0.77	0.84	0.95	1.06	0.78	0.80	0.95	1.03	1.05	0.21	0.30	0.41	0.52	0.71
	$\delta=0.50$	0.78	0.77	0.84	0.95	1.06	0.78	0.77	0.84	0.95	1.06	0.78	0.77	0.84	0.95	1.06	0.63	0.94	0.93	0.97	1.02	0.19	0.27	0.46	0.55	0.63
	$\delta=0.75$	0.78	0.77	0.84	0.95	1.06	0.78	0.77	0.84	0.95	1.06	0.78	0.77	0.84	0.95	1.06	0.61	0.91	0.86	0.88	0.90	0.18	0.35	0.41	0.47	0.54
	$\delta=0.95$	0.78	0.77	0.84	0.95	1.06	0.78	0.77	0.84	0.95	1.06	0.78	0.77	0.84	0.95	0.13	0.43	0.58	0.63	0.58	0.56	0.31	0.32	0.34	0.36	0.38
$\beta=0.75$	$\delta=0.05$	0.26	0.32	0.39	0.47	0.55	0.26	0.32	0.39	0.47	0.55	0.26	0.32	0.39	0.47	0.55	0.26	0.32	0.39	0.47	0.55	0.13	0.20	0.29	0.38	0.45
	$\delta=0.25$	0.26	0.32	0.39	0.47	0.55	0.26	0.32	0.39	0.47	0.55	0.26	0.32	0.39	0.47	0.55	0.26	0.32	0.39	0.47	0.55	0.12	0.20	0.28	0.36	0.42
	$\delta=0.50$	0.26	0.32	0.39	0.47	0.55	0.26	0.32	0.39	0.47	0.55	0.26	0.32	0.39	0.47	0.55	0.26	0.32	0.39	0.47	0.39	0.11	0.19	0.26	0.42	0.49
	$\delta=0.75$	0.26	0.32	0.39	0.47	0.55	0.26	0.32	0.39	0.47	0.55	0.26	0.32	0.39	0.47	0.55	0.26	0.32	0.39	0.40	0.58	0.11	0.18	0.33	0.39	0.43
	$\delta=0.95$	0.26	0.32	0.39	0.47	0.55	0.26	0.32	0.39	0.47	0.55	0.26	0.32	0.39	0.47	0.55	0.26	0.31	0.20	0.32	0.37	0.11	0.27	0.29	0.31	0.33
$\beta=0.95$	$\delta=0.05$	0.09	0.15	0.22	0.28	0.35	0.09	0.15	0.22	0.28	0.35	0.09	0.15	0.22	0.28	0.35	0.09	0.15	0.22	0.28	0.35	0.07	0.14	0.21	0.27	0.34
	$\delta=0.25$	0.09	0.15	0.22	0.28	0.35	0.09	0.15	0.22	0.28	0.35	0.09	0.15	0.22	0.28	0.35	0.09	0.15	0.22	0.28	0.35	0.07	0.14	0.20	0.26	0.34
	$\delta=0.50$	0.09	0.15	0.22	0.28	0.35	0.09	0.15	0.22	0.28	0.35	0.09	0.15	0.22	0.28	0.35	0.09	0.15	0.22	0.28	0.35	0.07	0.14	0.20	0.27	0.41
	$\delta=0.75$	0.09	0.15	0.22	0.28	0.35	0.09	0.15	0.22	0.28	0.35	0.09	0.15	0.22	0.28	0.35	0.09	0.15	0.22	0.28	0.35	0.07	0.13	0.28	0.33	0.37
	$\delta=0.95$	0.09	0.15	0.22	0.28	0.35	0.09	0.15	0.22	0.28	0.35	0.09	0.15	0.22	0.28	0.35	0.09	0.15	0.22	0.10	0.20	0.07	0.14	0.25	0.28	0.30

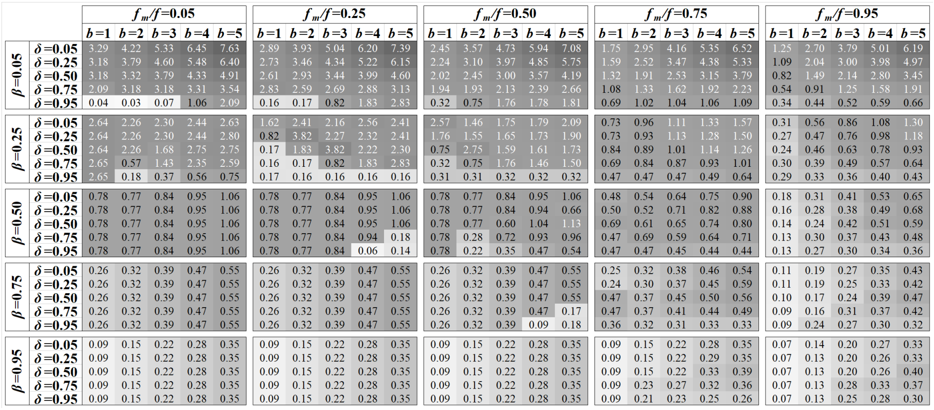
(c) $s_n = 5$

Figure B.3: Impact of s_n on the retailer's profit function, where the parameters are set to $f = 5, v = 3$, and values from 0 to 19.19 are highlighted as a heatmap from white to black background



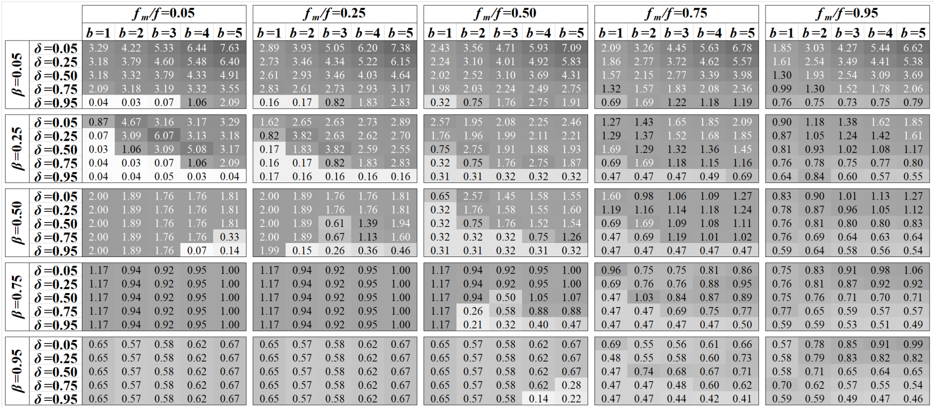
		f_m/f=0.05					f_m/f=0.25					f_m/f=0.50					f_m/f=0.75					f_m/f=0.95				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	2.99	4.20	5.31	6.31	7.39	2.54	3.69	4.94	6.09	7.27	1.92	3.14	4.01	5.66	6.86	1.38	2.62	3.88	4.78	6.44	1.02	2.17	3.22	5.00	5.41
	δ=0.25	3.12	3.71	4.58	5.46	6.38	2.45	3.27	4.19	5.02	6.04	1.77	2.66	3.65	4.56	5.45	1.26	2.19	3.13	4.16	5.03	0.91	1.87	2.82	3.74	4.70
	δ=0.50	3.05	3.25	3.77	4.31	4.89	2.33	2.74	3.30	3.92	4.53	1.59	2.13	2.73	3.37	4.01	0.92	1.60	2.26	2.82	3.58	0.58	1.25	1.87	2.54	3.23
	δ=0.75	2.09	3.01	3.14	3.30	3.53	2.83	2.33	2.53	2.83	3.09	1.41	1.61	1.94	2.23	2.58	0.69	1.00	1.36	1.70	2.09	0.28	0.64	1.00	1.38	1.75
	δ=0.95	0.04	0.03	0.07	1.06	2.09	0.16	0.17	0.82	1.83	2.83	0.32	0.75	1.76	1.36	1.41	0.69	0.59	0.67	0.76	0.86	0.00	0.16	0.29	0.41	0.52
β=0.25	δ=0.05	0.39	0.63	0.88	1.13	1.38	0.39	0.63	0.88	1.13	1.38	0.39	0.63	0.88	1.13	1.38	0.36	0.59	0.83	1.06	1.32	0.09	0.32	0.56	0.85	1.02
	δ=0.25	0.39	0.63	0.88	1.13	1.38	0.39	0.63	0.88	1.13	1.38	0.39	0.63	0.88	1.13	0.92	0.34	0.62	0.80	1.03	1.18	0.06	0.25	0.44	0.71	0.91
	δ=0.50	0.39	0.63	0.88	1.13	1.38	0.39	0.63	0.88	1.13	0.56	0.39	0.63	0.91	1.25	1.37	0.37	0.52	0.66	0.80	0.99	0.02	0.20	0.38	0.54	0.69
	δ=0.75	0.39	0.63	0.88	1.13	0.40	0.39	0.63	0.88	0.56	1.25	0.39	0.38	1.01	1.08	1.15	0.35	0.42	0.52	0.62	0.73	0.00	0.11	0.24	0.35	0.44
	δ=0.95	0.39	0.63	0.88	0.08	0.20	0.39	0.63	0.11	0.27	0.45	0.39	0.26	0.47	0.67	0.69	0.35	0.35	0.37	0.39	0.43	0.00	0.01	0.12	0.19	0.24
β=0.50	δ=0.05	0.03	0.14	0.26	0.38	0.51	0.03	0.14	0.26	0.38	0.51	0.03	0.14	0.26	0.38	0.51	0.03	0.14	0.26	0.38	0.51	0.00	0.08	0.19	0.30	0.45
	δ=0.25	0.03	0.14	0.26	0.38	0.51	0.03	0.14	0.26	0.38	0.51	0.03	0.14	0.26	0.38	0.51	0.03	0.14	0.26	0.38	0.54	0.00	0.06	0.16	0.25	0.36
	δ=0.50	0.03	0.14	0.26	0.38	0.51	0.03	0.14	0.26	0.38	0.51	0.03	0.14	0.26	0.38	0.51	0.03	0.14	0.35	0.43	0.53	0.00	0.04	0.11	0.26	0.36
	δ=0.75	0.03	0.14	0.26	0.38	0.51	0.03	0.14	0.26	0.38	0.51	0.03	0.14	0.26	0.38	0.20	0.03	0.22	0.29	0.35	0.43	0.00	0.02	0.08	0.21	0.27
	δ=0.95	0.03	0.14	0.26	0.38	0.51	0.03	0.14	0.26	0.38	0.51	0.03	0.14	0.26	0.10	0.20	0.03	0.19	0.23	0.25	0.28	0.00	0.01	0.09	0.15	0.19
β=0.75	δ=0.05	0.00	0.02	0.09	0.16	0.23	0.00	0.02	0.09	0.16	0.23	0.00	0.02	0.09	0.16	0.23	0.00	0.02	0.09	0.16	0.23	0.00	0.02	0.07	0.14	0.22
	δ=0.25	0.00	0.02	0.09	0.16	0.23	0.00	0.02	0.09	0.16	0.23	0.00	0.02	0.09	0.16	0.23	0.00	0.02	0.09	0.16	0.23	0.00	0.01	0.06	0.13	0.20
	δ=0.50	0.00	0.02	0.09	0.16	0.23	0.00	0.02	0.09	0.16	0.23	0.00	0.02	0.09	0.16	0.23	0.00	0.02	0.09	0.16	0.27	0.00	0.01	0.05	0.11	0.18
	δ=0.75	0.00	0.02	0.09	0.16	0.23	0.00	0.02	0.09	0.16	0.23	0.00	0.02	0.09	0.16	0.23	0.00	0.02	0.09	0.20	0.26	0.00	0.00	0.04	0.14	0.20
	δ=0.95	0.00	0.02	0.09	0.16	0.23	0.00	0.02	0.09	0.16	0.23	0.00	0.02	0.09	0.16	0.23	0.00	0.02	0.12	0.17	0.19	0.00	0.00	0.03	0.12	0.17
β=0.95	δ=0.05	0.00	0.00	0.03	0.08	0.13	0.00	0.00	0.03	0.08	0.13	0.00	0.00	0.03	0.08	0.13	0.00	0.00	0.03	0.08	0.13	0.00	0.00	0.03	0.07	0.13
	δ=0.25	0.00	0.00	0.03	0.08	0.13	0.00	0.00	0.03	0.08	0.13	0.00	0.00	0.03	0.08	0.13	0.00	0.00	0.03	0.08	0.13	0.00	0.00	0.03	0.07	0.12
	δ=0.50	0.00	0.00	0.03	0.08	0.13	0.00	0.00	0.03	0.08	0.13	0.00	0.00	0.03	0.08	0.13	0.00	0.00	0.03	0.08	0.13	0.00	0.00	0.03	0.07	0.12
	δ=0.75	0.00	0.00	0.03	0.08	0.13	0.00	0.00	0.03	0.08	0.13	0.00	0.00	0.03	0.08	0.13	0.00	0.00	0.03	0.08	0.17	0.00	0.00	0.02	0.07	0.17
	δ=0.95	0.00	0.00	0.03	0.08	0.13	0.00	0.00	0.03	0.08	0.13	0.00	0.00	0.03	0.08	0.13	0.00	0.00	0.03	0.10	0.14	0.00	0.00	0.02	0.07	0.15

(a) $\upsilon = 1$



		f_m/f=0.05					f_m/f=0.25					f_m/f=0.50					f_m/f=0.75					f_m/f=0.95				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	3.29	4.22	5.33	6.45	7.63	2.89	3.93	5.04	6.20	7.39	2.45	3.57	4.73	5.94	7.08	1.75	2.95	4.16	5.35	6.52	1.25	2.70	3.79	5.01	6.19
	δ=0.25	3.18	3.79	4.60	5.48	6.40	2.73	3.46	4.34	5.22	6.15	2.24	3.10	3.97	4.85	5.75	1.59	2.52	3.47	4.38	5.33	1.09	2.04	3.00	3.98	4.97
	δ=0.50	3.18	3.32	3.79	4.33	4.91	2.61	2.93	3.44	3.99	4.60	2.02	2.45	3.00	3.57	4.19	1.32	1.91	2.53	3.15	3.79	0.82	1.49	2.14	2.80	3.45
	δ=0.75	2.09	3.18	3.18	3.31	3.54	2.83	2.59	2.69	2.88	3.13	1.94	1.93	2.13	2.39	2.66	1.08	1.33	1.62	1.92	2.23	0.54	0.91	1.25	1.58	1.91
	δ=0.95	0.04	0.03	0.07	1.06	2.09	0.16	0.17	0.82	1.83	2.83	0.32	0.75	1.76	1.78	1.81	0.69	1.02	1.04	1.06	1.09	0.34	0.44	0.52	0.59	0.66
β=0.25	δ=0.05	2.64	2.26	2.30	2.44	2.63	1.62	2.41	2.16	2.56	2.41	2.57	1.46	1.75	1.79	2.09	0.73	0.96	1.11	1.33	1.57	0.31	0.56	0.86	1.08	1.30
	δ=0.25	2.64	2.26	2.30	2.44	2.80	0.82	3.82	2.27	2.32	2.41	1.76	1.55	1.65	1.73	1.90	0.73	0.93	1.13	1.28	1.50	0.27	0.47	0.76	0.98	1.18
	δ=0.50	2.64	2.26	1.68	2.75	2.75	0.17	1.83	3.82	2.22	2.30	0.75	2.75	1.59	1.61	1.73	0.84	0.89	1.01	1.14	1.26	0.24	0.46	0.63	0.78	0.93
	δ=0.75	2.65	0.57	1.43	2.35	2.59	0.16	0.17	0.82	1.83	2.83	0.32	0.75	1.76	1.46	1.50	0.69	0.84	0.87	0.93	1.01	0.30	0.39	0.49	0.57	0.64
	δ=0.95	2.65	0.18	0.37	0.56	0.75	0.17	0.16	0.16	0.16	0.16	0.31	0.31	0.32	0.32	0.32	0.47	0.47	0.47	0.49	0.64	0.29	0.33	0.36	0.40	0.43
β=0.50	δ=0.05	0.78	0.77	0.84	0.95	1.06	0.78	0.77	0.84	0.95	1.06	0.78	0.77	0.84	0.95	1.06	0.48	0.54	0.64	0.75	0.90	0.18	0.31	0.41	0.53	0.65
	δ=0.25	0.78	0.77	0.84	0.95	1.06	0.78	0.77	0.84	0.95	1.06	0.78	0.77	0.84	0.94	0.66	0.50	0.52	0.71	0.82	0.88	0.16	0.28	0.38	0.49	0.68
	δ=0.50	0.78	0.77	0.84	0.95	1.06	0.78	0.77	0.84	0.95	1.06	0.78	0.77	0.60	1.04	1.13	0.69	0.61	0.65	0.74	0.80	0.14	0.24	0.42	0.51	0.59
	δ=0.75	0.78	0.77	0.84	0.95	1.06	0.78	0.77	0.84	0.94	0.18	0.78	0.28	0.72	0.93	0.96	0.47	0.69	0.59	0.64	0.71	0.13	0.30	0.37	0.43	0.48
	δ=0.95	0.78	0.77	0.84	0.95	1.06	0.78	0.77	0.84	0.06	0.14	0.78	0.22	0.35	0.47	0.54	0.47	0.47	0.45	0.44	0.44	0.13	0.27	0.30	0.34	0.36
β=0.75	δ=0.05	0.26	0.32	0.39	0.47	0.55	0.26	0.32	0.39	0.47	0.55	0.26	0.32	0.39	0.47	0.55	0.25	0.32	0.38	0.46	0.54	0.11	0.19	0.27	0.35	0.43
	δ=0.25	0.26	0.32	0.39	0.47	0.55	0.26	0.32	0.39	0.47	0.55	0.26	0.32	0.39	0.47	0.55	0.24	0.30	0.37	0.45	0.59	0.11	0.19	0.25	0.33	0.42
	δ=0.50	0.26	0.32	0.39	0.47	0.55	0.26	0.32	0.39	0.47	0.55	0.26	0.32	0.39	0.47	0.55	0.47	0.37	0.45	0.50	0.56	0.10	0.17	0.24	0.39	0.47
	δ=0.75	0.26	0.32	0.39	0.47	0.55	0.26	0.32	0.39	0.47	0.55	0.26	0.32	0.39	0.47	0.17	0.47	0.37	0.41	0.44	0.49	0.09	0.16	0.31	0.37	0.42
	δ=0.95	0.26	0.32	0.39	0.47	0.55	0.26	0.32	0.39	0.47	0.55	0.26	0.32	0.39	0.09	0.18	0.36	0.32	0.31	0.33	0.33	0.09	0.24	0.27	0.30	0.32
β=0.95	δ=0.05	0.09	0.15	0.22	0.28	0.35	0.09	0.15	0.22	0.28	0.35	0.09	0.15	0.22	0.28	0.35	0.09	0.15	0.22	0.28	0.35	0.07	0.14	0.20	0.27	0.33
	δ=0.25	0.09	0.15	0.22	0.28	0.35	0.09	0.15	0.22	0.28	0.35	0.09	0.15	0.22	0.28	0.35	0.09	0.15	0.22	0.29	0.35	0.07	0.13	0.20	0.26	0.33
	δ=0.50	0.09	0.15	0.22	0.28	0.35	0.09	0.15	0.22	0.28	0.35	0.09	0.15	0.22	0.28	0.35	0.09	0.15	0.22	0.33	0.39	0.07	0.13	0.20	0.26	0.40
	δ=0.75	0.09	0.15	0.22	0.28	0.35	0.09	0.15	0.22	0.28	0.35	0.09	0.15	0.22	0.28	0.35	0.09	0.23	0.27	0.32	0.36	0.07	0.13	0.28	0.33	0.37
	δ=0.95	0.09	0.15	0.22	0.28	0.35	0.09	0.15	0.22	0.28	0.35	0.09	0.15	0.22	0.28	0.35	0.09	0.21	0.23	0.25	0.26	0.07	0.13	0.25	0.28	0.30

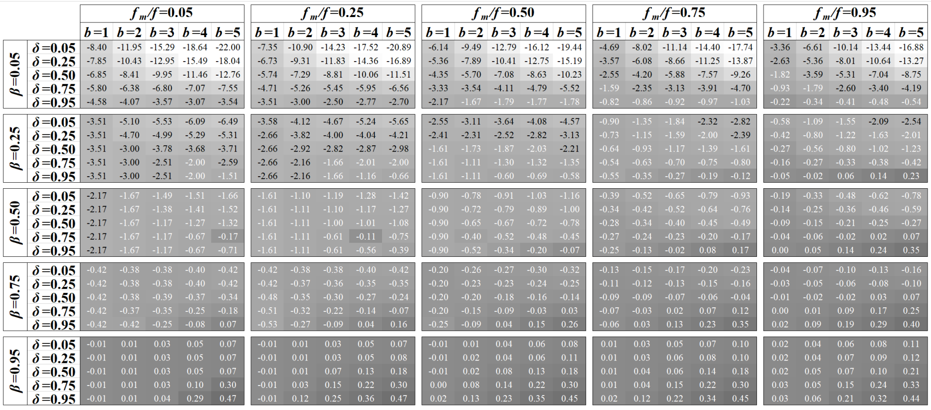
(b) $\upsilon = 3$



		f_m/f=0.05					f_m/f=0.25					f_m/f=0.50					f_m/f=0.75					f_m/f=0.95				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	3.29	4.22	5.33	6.44	7.63	2.89	3.93	5.05	6.20	7.38	2.43	3.56	4.71	5.93	7.09	2.09	3.26	4.45	5.63	6.78	1.85	3.03	4.27	5.44	6.62
	δ=0.25	3.18	3.79	4.60	5.48	6.40	2.73	3.46	4.34	5.22	6.15	2.24	3.10	4.01	4.92	5.83	1.86	2.77	3.72	4.62	5.57	1.61	2.54	3.49	4.41	5.38
	δ=0.50	3.18	3.32	3.79	4.33	4.91	2.61	2.93	3.46	4.03	4.64	2.02	2.52	3.10	3.69	4.31	1.57	2.15	2.77	3.39	3.98	1.30	1.93	2.54	3.09	3.69
	δ=0.75	2.09	3.18	3.19	3.32	3.55	2.83	2.61	2.73	2.93	3.17	1.98	2.03	2.24	2.49	2.75	1.32	1.57	1.83	2.08	2.36	0.99	1.30	1.52	1.78	2.06
	δ=0.95	0.04	0.03	0.07	1.06	2.09	0.16	0.17	0.82	1.83	2.83	0.32	0.75	1.76	2.75	1.91	0.69	1.69	1.22	1.18	1.19	0.76	0.75	0.73	0.75	0.79
β=0.25	δ=0.05	0.87	4.67	3.16	3.17	3.29	1.62	2.65	2.63	2.73	2.89	2.57	1.95	2.08	2.25	2.46	1.27	1.43	1.65	1.85	2.09	0.90	1.18	1.38	1.62	1.85
	δ=0.25	0.07	3.09	6.07	3.13	3.18	0.82	3.82	2.63	2.62	2.70	1.76	1.96	1.99	2.11	2.21	1.29	1.37	1.52	1.68	1.85	0.87	1.05	1.24	1.42	1.61
	δ=0.50	0.03	1.06	3.09	5.08	3.17	0.17	1.83	3.82	2.59	2.55	0.75	2.75	1.91	1.88	1.93	1.69	1.29	1.32	1.36	1.45	0.81	0.93	1.02	1.08	1.17
	δ=0.75	0.04	0.03	0.07	1.06	2.09	0.16	0.17	0.82	1.83	2.83	0.32	0.75	1.76	2.75	1.87	0.69	1.69	1.18	1.15	1.16	0.76	0.78	0.75	0.77	0.80
	δ=0.95	0.04	0.04	0.05	0.03	0.04	0.17	0.16	0.16	0.16	0.16	0.31	0.31	0.32	0.32	0.32	0.47	0.47	0.47	0.49	0.69	0.64	0.84	0.60	0.57	0.55
β=0.50	δ=0.05	2.00	1.89	1.76	1.76	1.81	2.00	1.89	1.76	1.76	1.81	0.65	2.57	1.45	1.58	1.55	1.60	0.98	1.06	1.09	1.27	0.83	0.90	1.01	1.13	1.27
	δ=0.25	2.00	1.89	1.76	1.76	1.81	2.00	1.89	1.76	1.76	1.81	0.32	1.76	1.58	1.55	1.60	1.19	1.16	1.14	1.18	1.24	0.78	0.87	0.96	1.05	1.12
	δ=0.50	2.00	1.89	1.76	1.76	1.81	2.00	1.89	0.61	1.39	1.94	0.32	0.75	1.76	1.52	1.54	0.69	1.69	1.09	1.08	1.11	0.76	0.81	0.80	0.80	0.83
	δ=0.75	2.00	1.89	1.76	1.76	0.33	2.00	1.89	0.67	1.13	1.60	0.32	0.32	0.32	0.75	1.26	0.47	0.69	1.19	1.01	1.02	0.76	0.69	0.64	0.63	0.64
	δ=0.95	2.00	1.89	1.76	0.07	0.14	1.99	0.15	0.26	0.36	0.46	0.31	0.31	0.32	0.31	0.32	0.47	0.47	0.47	0.47	0.47	0.59	0.64	0.58	0.56	0.54
β=0.75	δ=0.05	1.17	0.94	0.92	0.95	1.00	1.17	0.94	0.92	0.95	1.00	1.17	0.94	0.92	0.95	1.00	0.96	0.75	0.75	0.81	0.86	0.75	0.83	0.91	0.98	1.06
	δ=0.25	1.17	0.94	0.92	0.95	1.00	1.17	0.94	0.92	0.95	1.00	1.17	0.94	0.92	0.95	1.00	0.69	0.76	0.76	0.88	0.95	0.76	0.81	0.87	0.92	0.92
	δ=0.50	1.17	0.94	0.92	0.95	1.00	1.17	0.94	0.92	0.95	1.00	1.17	0.94	0.50	1.05	1.07	0.47	1.03	0.84	0.87	0.89	0.75	0.76	0.71	0.70	0.71
	δ=0.75	1.17	0.94	0.92	0.95	1.00	1.17	0.94	0.92	0.95	1.00	1.17	0.26	0.58	0.88	0.88	0.47	0.47	0.69	0.75	0.77	0.77	0.65	0.59	0.57	0.57
	δ=0.95	1.17	0.94	0.92	0.95	1.00	1.17	0.94	0.92	0.95	1.00	1.17	0.21	0.32	0.40	0.47	0.47	0.47	0.47	0.47	0.50	0.59	0.59	0.53	0.51	0.49
β=0.95	δ=0.05	0.65	0.57	0.58	0.62	0.67	0.65	0.57	0.58	0.62	0.67	0.65	0.57	0.58	0.62	0.67	0.69	0.55	0.56	0.61	0.66	0.57	0.78	0.85	0.91	0.99
	δ=0.25	0.65	0.57	0.58	0.62	0.67	0.65	0.57	0.58	0.62	0.67	0.65	0.57	0.58	0.62	0.67	0.48	0.55	0.58	0.60	0.73	0.58	0.79	0.83	0.82	0.82
	δ=0.50	0.65	0.57	0.58	0.62	0.67	0.65	0.57	0.58	0.62	0.67	0.65	0.57	0.58	0.62	0.67	0.47	0.74	0.68	0.67	0.71	0.58	0.71	0.65	0.64	0.65
	δ=0.75	0.65	0.57	0.58	0.62	0.67	0.65	0.57	0.58	0.62	0.67	0.65	0.57	0.58	0.62	0.28	0.47	0.47	0.48	0.60	0.62	0.70	0.62	0.57	0.55	0.54
	δ=0.95	0.65	0.57	0.58	0.62	0.67	0.65	0.57	0.58	0.62	0.67	0.65	0.57	0.58	0.14	0.22	0.47	0.47	0.44	0.42	0.41	0.59	0.59	0.49	0.47	0.46

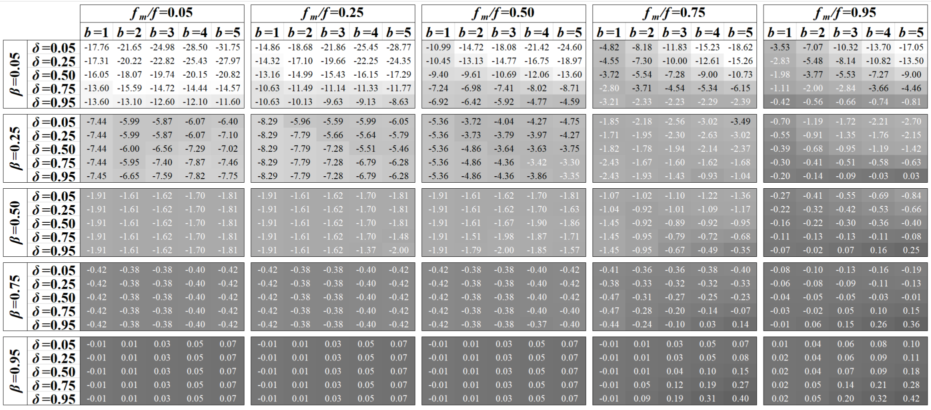
(c) $\upsilon = 5$

Figure B.4: Impact of υ on the retailer's profit function, where the parameters are set to $f = 5, s_n = 3$, and values from 0 to 19.19 are highlighted as a heatmap from white to black background



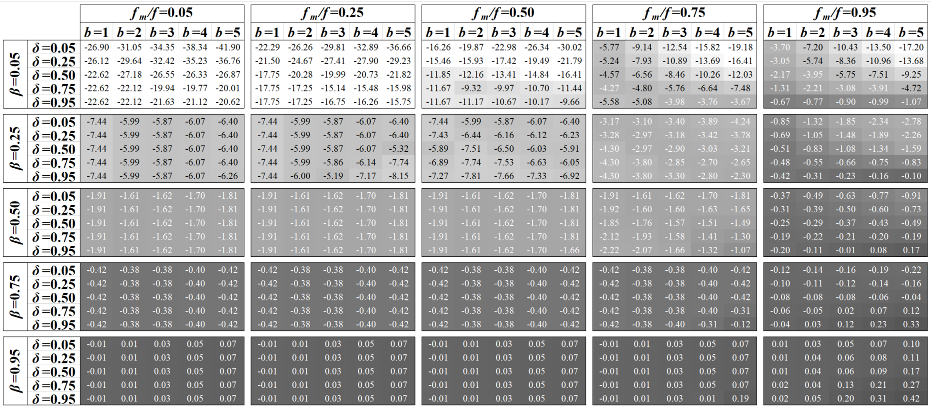
		f_m/f=0.05					f_m/f=0.25					f_m/f=0.50					f_m/f=0.75					f_m/f=0.95				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	-8.40	-11.95	-15.29	-18.64	-22.00	-7.35	-10.90	-14.23	-17.52	-20.89	-6.14	-9.49	-12.79	-16.12	-19.44	-4.69	-8.02	-11.14	-14.40	-17.74	-3.36	-6.61	-10.14	-13.44	-16.88
	δ=0.25	-7.85	-10.43	-12.95	-15.49	-18.04	-6.73	-9.31	-11.83	-14.36	-16.89	-5.36	-7.89	-10.41	-12.75	-15.19	-3.57	-6.08	-8.66	-11.25	-13.87	-2.63	-5.36	-8.01	-10.64	-13.27
	δ=0.50	-6.85	-8.41	-9.95	-11.46	-12.76	-5.74	-7.29	-8.81	-10.06	-11.51	-4.35	-5.70	-7.08	-8.63	-10.23	-2.55	-4.20	-5.88	-7.57	-9.26	-1.82	-3.59	-5.31	-7.04	-8.75
	δ=0.75	-5.80	-6.38	-6.80	-7.07	-7.55	-4.71	-5.26	-5.45	-5.95	-6.56	-3.33	-3.54	-4.11	-4.79	-5.52	-1.59	-2.35	-3.13	-3.91	-4.70	-0.93	-1.79	-2.60	-3.40	-4.19
	δ=0.95	-4.58	-4.07	-3.57	-3.07	-3.54	-3.51	-3.00	-2.50	-2.77	-2.70	-2.17	-1.67	-1.79	-1.77	-1.78	-0.82	-0.86	-0.92	-0.97	-1.03	-0.22	-0.34	-0.41	-0.48	-0.54
β=0.25	δ=0.05	-3.51	-5.10	-5.53	-6.09	-6.49	-3.58	-4.12	-4.67	-5.24	-5.65	-2.55	-3.11	-3.64	-4.08	-4.57	-0.90	-1.35	-1.84	-2.32	-2.82	-0.58	-1.09	-1.55	-2.09	-2.54
	δ=0.25	-3.51	-4.70	-4.99	-5.29	-5.31	-2.66	-3.82	-4.00	-4.04	-4.21	-2.41	-2.31	-2.52	-2.82	-3.13	-0.73	-1.15	-1.59	-2.00	-2.39	-0.42	-0.80	-1.22	-1.63	-2.01
	δ=0.50	-3.51	-3.00	-3.78	-3.68	-3.71	-2.66	-2.92	-2.82	-2.87	-2.98	-1.61	-1.73	-1.87	-2.03	-2.21	-0.64	-0.93	-1.17	-1.39	-1.61	-0.27	-0.56	-0.80	-1.02	-1.23
	δ=0.75	-3.51	-3.00	-2.51	-2.00	-2.59	-2.66	-2.16	-1.66	-2.01	-2.00	-1.61	-1.11	-1.30	-1.32	-1.35	-0.54	-0.63	-0.70	-0.75	-0.80	-0.16	-0.27	-0.33	-0.38	-0.42
	δ=0.95	-3.51	-3.00	-2.51	-2.00	-1.51	-2.66	-2.16	-1.66	-1.16	-0.66	-1.61	-1.11	-0.60	-0.69	-0.58	-0.55	-0.35	-0.27	-0.19	-0.12	-0.05	-0.02	0.06	0.14	0.23
β=0.50	δ=0.05	-2.17	-1.67	-1.49	-1.51	-1.66	-1.61	-1.10	-1.19	-1.28	-1.42	-0.90	-0.78	-0.91	-1.03	-1.16	-0.39	-0.52	-0.65	-0.79	-0.93	-0.19	-0.33	-0.48	-0.62	-0.78
	δ=0.25	-2.17	-1.67	-1.38	-1.41	-1.52	-1.61	-1.11	-1.10	-1.17	-1.27	-0.90	-0.72	-0.79	-0.89	-1.00	-0.34	-0.42	-0.52	-0.64	-0.76	-0.14	-0.25	-0.36	-0.46	-0.59
	δ=0.50	-2.17	-1.67	-1.17	-1.27	-1.32	-1.61	-1.11	-1.00	-1.01	-1.08	-0.90	-0.65	-0.67	-0.72	-0.78	-0.28	-0.34	-0.40	-0.45	-0.49	-0.09	-0.15	-0.21	-0.25	-0.27
	δ=0.75	-2.17	-1.67	-1.17	-0.67	-0.17	-1.61	-1.11	-0.61	-0.11	-0.75	-0.90	-0.40	-0.52	-0.48	-0.45	-0.27	-0.24	-0.23	-0.20	-0.17	-0.04	-0.06	-0.02	0.02	0.07
	δ=0.95	-2.17	-1.67	-1.17	-0.67	-0.71	-1.61	-1.11	-0.61	-0.56	-0.39	-0.90	-0.52	-0.34	-0.20	-0.07	-0.25	-0.13	-0.02	0.08	0.17	0.00	0.05	0.14	0.24	0.35
β=0.75	δ=0.05	-0.42	-0.38	-0.38	-0.40	-0.42	-0.42	-0.38	-0.38	-0.40	-0.42	-0.20	-0.26	-0.27	-0.30	-0.32	-0.13	-0.15	-0.17	-0.20	-0.23	-0.04	-0.07	-0.10	-0.13	-0.16
	δ=0.25	-0.42	-0.38	-0.38	-0.40	-0.42	-0.42	-0.37	-0.36	-0.35	-0.35	-0.20	-0.23	-0.23	-0.24	-0.25	-0.11	-0.12	-0.13	-0.15	-0.16	-0.03	-0.05	-0.06	-0.08	-0.10
	δ=0.50	-0.42	-0.38	-0.39	-0.37	-0.34	-0.48	-0.35	-0.30	-0.27	-0.24	-0.20	-0.20	-0.18	-0.16	-0.14	-0.09	-0.09	-0.07	-0.06	-0.04	-0.01	-0.02	-0.02	0.03	0.07
	δ=0.75	-0.42	-0.37	-0.35	-0.25	-0.18	-0.51	-0.32	-0.22	-0.14	-0.07	-0.20	-0.15	-0.09	-0.03	0.03	-0.07	-0.03	0.02	0.07	0.12	0.00	0.01	0.09	0.17	0.25
	δ=0.95	-0.42	-0.42	-0.25	-0.08	0.07	-0.53	-0.27	-0.09	0.04	0.16	-0.25	-0.09	0.04	0.15	0.26	-0.06	0.03	0.13	0.23	0.35	0.02	0.09	0.19	0.29	0.40
β=0.95	δ=0.05	-0.01	0.01	0.03	0.05	0.07	-0.01	0.01	0.03	0.05	0.07	-0.01	0.01	0.04	0.06	0.08	0.01	0.03	0.05	0.07	0.10	0.02	0.04	0.06	0.08	0.11
	δ=0.25	-0.01	0.01	0.03	0.05	0.07	-0.01	0.01	0.03	0.05	0.08	-0.01	0.02	0.04	0.06	0.11	0.01	0.03	0.06	0.08	0.10	0.02	0.04	0.07	0.09	0.12
	δ=0.50	-0.01	0.01	0.03	0.05	0.07	-0.01	0.01	0.07	0.13	0.18	-0.01	0.02	0.08	0.13	0.18	0.01	0.04	0.06	0.14	0.18	0.02	0.05	0.07	0.10	0.21
	δ=0.75	-0.01	0.01	0.03	0.10	0.30	-0.01	0.03	0.15	0.22	0.30	0.00	0.08	0.14	0.22	0.30	0.01	0.04	0.15	0.22	0.30	0.03	0.05	0.15	0.24	0.33
	δ=0.95	-0.01	0.01	0.04	0.29	0.47	-0.01	0.12	0.25	0.36	0.47	0.02	0.13	0.23	0.35	0.45	0.02	0.12	0.22	0.34	0.45	0.03	0.06	0.21	0.32	0.44

(a) $s_n = 1$



		f_m/f=0.05					f_m/f=0.25					f_m/f=0.50					f_m/f=0.75					f_m/f=0.95				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	-17.76	-21.65	-24.98	-28.50	-31.75	-14.86	-18.68	-21.86	-25.45	-28.77	-10.99	-14.72	-18.08	-21.42	-24.60	-4.82	-8.18	-11.83	-15.23	-18.62	-3.53	-7.07	-10.32	-13.70	-17.05
	δ=0.25	-17.31	-20.22	-22.82	-25.43	-27.97	-14.32	-17.10	-19.66	-22.25	-24.35	-10.45	-13.13	-14.77	-16.75	-18.97	-4.55	-7.30	-10.00	-12.61	-15.26	-2.83	-5.48	-8.14	-10.82	-13.50
	δ=0.50	-16.05	-18.07	-19.74	-20.15	-20.82	-13.16	-14.99	-15.43	-16.15	-17.29	-9.40	-9.61	-10.69	-12.06	-13.60	-3.72	-5.54	-7.28	-9.00	-10.73	-1.98	-3.77	-5.53	-7.27	-9.00
	δ=0.75	-13.60	-15.59	-14.72	-14.44	-14.57	-10.63	-11.49	-11.14	-11.33	-11.77	-7.24	-6.98	-7.41	-8.02	-8.71	-2.80	-3.71	-4.54	-5.34	-6.15	-1.11	-2.00	-2.84	-3.66	-4.46
	δ=0.95	-13.60	-13.10	-12.60	-12.10	-11.60	-10.63	-10.13	-9.63	-9.13	-8.63	-6.92	-6.42	-5.92	-4.77	-4.59	-3.21	-2.33	-2.23	-2.29	-2.39	-0.42	-0.56	-0.66	-0.74	-0.81
β=0.25	δ=0.05	-7.44	-5.99	-5.87	-6.07	-6.40	-8.29	-5.96	-5.59	-5.99	-6.05	-5.36	-3.72	-4.04	-4.27	-4.75	-1.85	-2.18	-2.56	-3.02	-3.49	-0.70	-1.19	-1.72	-2.21	-2.70
	δ=0.25	-7.44	-5.99	-5.87	-6.07	-7.10	-8.29	-7.79	-5.66	-5.64	-5.79	-5.36	-3.73	-3.79	-3.97	-4.27	-1.71	-1.95	-2.30	-2.63	-3.02	-0.55	-0.91	-1.35	-1.76	-2.15
	δ=0.50	-7.44	-6.00	-6.56	-7.29	-7.02	-8.29	-7.79	-7.28	-5.51	-5.46	-5.36	-4.86	-3.64	-3.63	-3.75	-1.82	-1.78	-1.94	-2.14	-2.37	-0.39	-0.68	-0.95	-1.19	-1.42
	δ=0.75	-7.44	-5.95	-7.40	-7.87	-7.46	-8.29	-7.79	-7.28	-6.79	-6.28	-5.36	-4.86	-4.36	-3.42	-3.30	-2.43	-1.67	-1.60	-1.62	-1.68	-0.30	-0.41	-0.51	-0.58	-0.63
	δ=0.95	-7.45	-6.65	-7.59	-7.82	-7.75	-8.29	-7.79	-7.28	-6.79	-6.28	-5.36	-4.86	-4.36	-3.86	-3.35	-2.43	-1.93	-1.43	-0.93	-1.04	-0.20	-0.14	-0.09	-0.03	0.03
β=0.50	δ=0.05	-1.91	-1.61	-1.62	-1.70	-1.81	-1.91	-1.61	-1.62	-1.70	-1.81	-1.91	-1.61	-1.62	-1.70	-1.81	-1.07	-1.02	-1.10	-1.22	-1.36	-0.27	-0.41	-0.55	-0.69	-0.84
	δ=0.25	-1.91	-1.61	-1.62	-1.70	-1.81	-1.91	-1.61	-1.62	-1.70	-1.81	-1.91	-1.61	-1.62	-1.70	-1.63	-1.04	-0.92	-1.01	-1.09	-1.17	-0.22	-0.32	-0.42	-0.53	-0.66
	δ=0.50	-1.91	-1.61	-1.62	-1.70	-1.81	-1.91	-1.61	-1.62	-1.70	-1.81	-1.91	-1.61	-1.67	-1.90	-1.86	-1.45	-0.92	-0.89	-0.92	-0.95	-0.16	-0.22	-0.30	-0.36	-0.40
	δ=0.75	-1.91	-1.61	-1.62	-1.70	-1.81	-1.91	-1.61	-1.62	-1.70	-1.48	-1.91	-1.51	-1.98	-1.87	-1.71	-1.45	-0.95	-0.79	-0.72	-0.68	-0.11	-0.13	-0.13	-0.11	-0.08
	δ=0.95	-1.91	-1.61	-1.62	-1.70	-1.81	-1.91	-1.61	-1.62	-1.37	-2.00	-1.91	-1.79	-2.00	-1.85	-1.57	-1.45	-0.95	-0.67	-0.49	-0.35	-0.07	-0.02	0.07	0.16	0.25
β=0.75	δ=0.05	-0.42	-0.38	-0.38	-0.40	-0.42	-0.42	-0.38	-0.38	-0.40	-0.42	-0.42	-0.38	-0.38	-0.40	-0.42	-0.41	-0.36	-0.36	-0.38	-0.40	-0.08	-0.10	-0.13	-0.16	-0.19
	δ=0.25	-0.42	-0.38	-0.38	-0.40	-0.42	-0.42	-0.38	-0.38	-0.40	-0.42	-0.42	-0.38	-0.38	-0.40	-0.42	-0.38	-0.33	-0.32	-0.32	-0.33	-0.06	-0.08	-0.09	-0.11	-0.13
	δ=0.50	-0.42	-0.38	-0.38	-0.40	-0.42	-0.42	-0.38	-0.38	-0.40	-0.42	-0.42	-0.38	-0.38	-0.40	-0.42	-0.47	-0.31	-0.27	-0.25	-0.23	-0.04	-0.05	-0.05	-0.03	-0.01
	δ=0.75	-0.42	-0.38	-0.38	-0.40	-0.42	-0.42	-0.38	-0.38	-0.40	-0.42	-0.42	-0.38	-0.38	-0.40	-0.40	-0.47	-0.28	-0.20	-0.14	-0.07	-0.03	-0.02	0.05	0.10	0.15
	δ=0.95	-0.42	-0.38	-0.38	-0.40	-0.42	-0.42	-0.38	-0.38	-0.40	-0.42	-0.42	-0.38	-0.38	-0.37	-0.40	-0.44	-0.24	-0.10	0.03	0.14	-0.01	0.06	0.15	0.26	0.36
β=0.95	δ=0.05	-0.01	0.01	0.03	0.05	0.07	-0.01	0.01	0.03	0.05	0.07	-0.01	0.01	0.03	0.05	0.07	-0.01	0.01	0.03	0.05	0.07	0.01	0.04	0.06	0.08	0.10
	δ=0.25	-0.01	0.01	0.03	0.05	0.07	-0.01	0.01	0.03	0.05	0.07	-0.01	0.01	0.03	0.05	0.07	-0.01	0.01	0.03	0.05	0.08	0.02	0.04	0.06	0.09	0.11
	δ=0.50	-0.01	0.01	0.03	0.05	0.07	-0.01	0.01	0.03	0.05	0.07	-0.01	0.01	0.03	0.05	0.07	-0.01	0.01	0.04	0.10	0.15	0.02	0.04	0.07	0.09	0.18
	δ=0.75	-0.01	0.01	0.03	0.05	0.07	-0.01	0.01	0.03	0.05	0.07	-0.01	0.01	0.03	0.05	0.07	-0.01	0.05	0.12	0.19	0.27	0.02	0.05	0.14	0.21	0.28
	δ=0.95	-0.01	0.01	0.03	0.05	0.07	-0.01	0.01	0.03	0.05	0.07	-0.01	0.01	0.03	0.05	0.07	-0.01	0.09	0.19	0.31	0.40	0.02	0.05	0.20	0.32	0.42

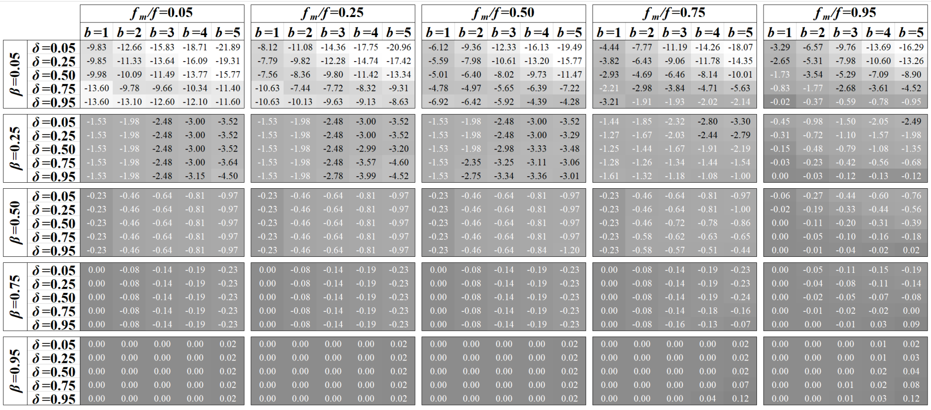
(b) $s_n = 3$



		f_m/f=0.05					f_m/f=0.25					f_m/f=0.50					f_m/f=0.75					f_m/f=0.95				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	-26.90	-31.05	-34.35	-38.34	-41.90	-22.29	-26.26	-29.81	-32.89	-36.66	-16.26	-19.87	-22.98	-26.34	-30.02	-5.77	-9.14	-12.54	-15.82	-19.18	-3.70	-7.20	-10.43	-13.50	-17.20
	δ=0.25	-26.12	-29.64	-32.42	-35.23	-36.76	-21.50	-24.67	-27.41	-27.90	-29.23	-15.46	-15.93	-17.42	-19.49	-21.79	-5.24	-7.93	-10.89	-13.69	-16.41	-3.05	-5.74	-8.36	-10.96	-13.68
	δ=0.50	-22.62	-27.18	-26.55	-26.33	-26.87	-17.75	-20.28	-19.99	-20.73	-21.82	-11.85	-12.16	-13.41	-14.84	-16.41	-4.57	-6.56	-8.46	-10.26	-12.03	-2.17	-3.95	-5.75	-7.51	-9.25
	δ=0.75	-22.62	-22.12	-19.94	-19.77	-20.01	-17.75	-17.25	-15.14	-15.48	-15.98	-11.67	-9.32	-9.97	-10.70	-11.44	-4.27	-4.80	-5.76	-6.64	-7.48	-1.31	-2.21	-3.08	-3.91	-4.72
	δ=0.95	-22.62	-22.12	-21.63	-21.12	-20.62	-17.75	-17.25	-16.75	-16.26	-15.75	-11.67	-11.17	-10.67	-10.17	-9.66	-5.58	-5.08	-3.98	-3.76	-3.67	-0.67	-0.77	-0.90	-0.99	-1.07
β=0.25	δ=0.05	-7.44	-5.99	-5.87	-6.07	-6.40	-7.44	-5.99	-5.87	-6.07	-6.40	-7.44	-5.99	-5.87	-6.07	-6.40	-3.17	-3.10	-3.40	-3.89	-4.24	-0.85	-1.32	-1.85	-2.34	-2.78
	δ=0.25	-7.44	-5.99	-5.87	-6.07	-6.40	-7.44	-5.99	-5.87	-6.07	-6.40	-7.43	-6.44	-6.16	-6.12	-6.23	-3.28	-2.97	-3.18	-3.42	-3.78	-0.69	-1.05	-1.48	-1.89	-2.26
	δ=0.50	-7.44	-5.99	-5.87	-6.07	-6.40	-7.44	-5.99	-5.87	-6.07	-5.32	-5.89	-7.51	-6.50	-6.03	-5.91	-4.30	-2.97	-2.90	-3.03	-3.21	-0.51	-0.83	-1.08	-1.34	-1.59
	δ=0.75	-7.44	-5.99	-5.87	-6.07	-6.40	-7.44	-5.99	-5.86	-6.14	-7.74	-6.89	-7.74	-7.53	-6.63	-6.05	-4.30	-3.80	-2.85	-2.70	-2.65	-0.48	-0.55	-0.66	-0.75	-0.83
	δ=0.95	-7.44	-5.99	-5.87	-6.07	-6.26	-7.44	-6.00	-5.19	-7.17	-8.15	-7.27	-7.81	-7.66	-7.33	-6.92	-4.30	-3.80	-3.30	-2.80	-2.30	-0.42	-0.31	-0.23	-0.16	-0.10
β=0.50	δ=0.05	-1.91	-1.61	-1.62	-1.70	-1.81	-1.91	-1.61	-1.62	-1.70	-1.81	-1.91	-1.61	-1.62	-1.70	-1.81	-1.91	-1.61	-1.62	-1.70	-1.81	-0.37	-0.49	-0.63	-0.77	-0.91
	δ=0.25	-1.91	-1.61	-1.62	-1.70	-1.81	-1.91	-1.61	-1.62	-1.70	-1.81	-1.91	-1.61	-1.62	-1.70	-1.81	-1.92	-1.60	-1.60	-1.63	-1.65	-0.31	-0.39	-0.50	-0.60	-0.73
	δ=0.50	-1.91	-1.61	-1.62	-1.70	-1.81	-1.91	-1.61	-1.62	-1.70	-1.81	-1.91	-1.61	-1.62	-1.70	-1.81	-1.85	-1.76	-1.57	-1.51	-1.49	-0.25	-0.29	-0.37	-0.43	-0.49
	δ=0.75	-1.91	-1.61	-1.62	-1.70	-1.81	-1.91	-1.61	-1.62	-1.70	-1.81	-1.91	-1.61	-1.62	-1.70	-1.81	-2.12	-1.93	-1.58	-1.41	-1.30	-0.19	-0.22	-0.21	-0.20	-0.19
	δ=0.95	-1.91	-1.61	-1.62	-1.70	-1.81	-1.91	-1.61	-1.62	-1.70	-1.81	-1.91	-1.61	-1.62	-1.70	-1.66	-2.22	-2.07	-1.66	-1.32	-1.07	-0.20	-0.11	-0.01	0.08	0.17
β=0.75	δ=0.05	-0.42	-0.38	-0.38	-0.40	-0.42	-0.42	-0.38	-0.38	-0.40	-0.42	-0.42	-0.38	-0.38	-0.40	-0.42	-0.42	-0.38	-0.38	-0.40	-0.42	-0.12	-0.14	-0.16	-0.19	-0.22
	δ=0.25	-0.42	-0.38	-0.38	-0.40	-0.42	-0.42	-0.38	-0.38	-0.40	-0.42	-0.42	-0.38	-0.38	-0.40	-0.42	-0.42	-0.38	-0.38	-0.40	-0.42	-0.10	-0.11	-0.12	-0.14	-0.16
	δ=0.50	-0.42	-0.38	-0.38	-0.40	-0.42	-0.42	-0.38	-0.38	-0.40	-0.42	-0.42	-0.38	-0.38	-0.40	-0.42	-0.42	-0.38	-0.38	-0.40	-0.42	-0.08	-0.08	-0.08	-0.06	-0.04
	δ=0.75	-0.42	-0.38	-0.38	-0.40	-0.42	-0.42	-0.38	-0.38	-0.40	-0.42	-0.42	-0.38	-0.38	-0.40	-0.42	-0.42	-0.38	-0.38	-0.40	-0.31	-0.06	-0.05	0.02	0.07	0.12
	δ=0.95	-0.42	-0.38	-0.38	-0.40	-0.42	-0.42	-0.38	-0.38	-0.40	-0.42	-0.42	-0.38	-0.38	-0.40	-0.42	-0.42	-0.38	-0.40	-0.31	-0.12	-0.04	0.03	0.12	0.23	0.33
β=0.95	δ=0.05	-0.01	0.01	0.03	0.05	0.07	-0.01	0.01	0.03	0.05	0.07	-0.01	0.01	0.03	0.05	0.07	-0.01	0.01	0.03	0.05	0.07	0.01	0.03	0.05	0.07	0.10
	δ=0.25	-0.01	0.01	0.03	0.05	0.07	-0.01	0.01	0.03	0.05	0.07	-0.01	0.01	0.03	0.05	0.07	-0.01	0.01	0.03	0.05	0.07	0.01	0.04	0.06	0.08	0.11
	δ=0.50	-0.01	0.01	0.03	0.05	0.07	-0.01	0.01	0.03	0.05	0.07	-0.01	0.01	0.03	0.05	0.07	-0.01	0.01	0.03	0.05	0.07	0.01	0.04	0.06	0.09	0.17
	δ=0.75	-0.01	0.01	0.03	0.05	0.07	-0.01	0.01	0.03	0.05	0.07	-0.01	0.01	0.03	0.05	0.07	-0.01	0.01	0.03	0.05	0.07	0.02	0.04	0.13	0.21	0.27
	δ=0.95	-0.01	0.01	0.03	0.05	0.07	-0.01	0.01	0.03	0.05	0.07	-0.01	0.01	0.03	0.05	0.07	-0.01	0.01	0.03	0.01	0.19	0.02	0.05	0.20	0.31	0.42

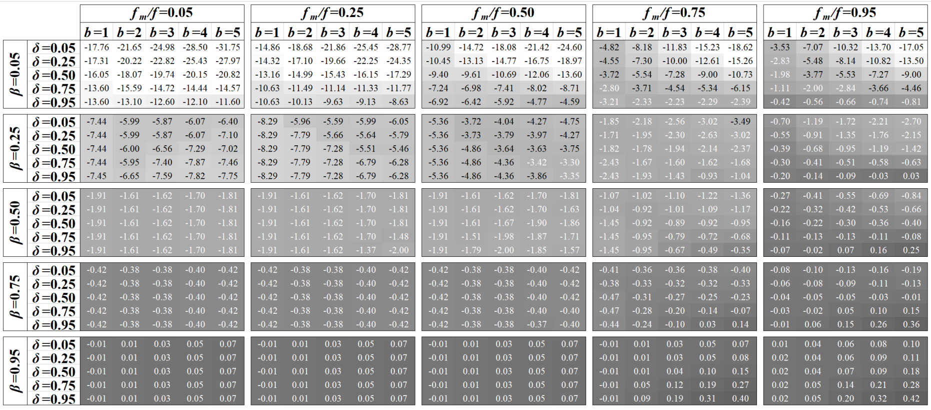
(c) $s_n = 5$

Figure B.5: Impact of s_n on consumer surplus, where the parameters are set to $f = 5, v = 3$, values from -15.00 to 2.48 are highlighted as a heatmap from white to black background, and values below -15.00 are shown with a white background



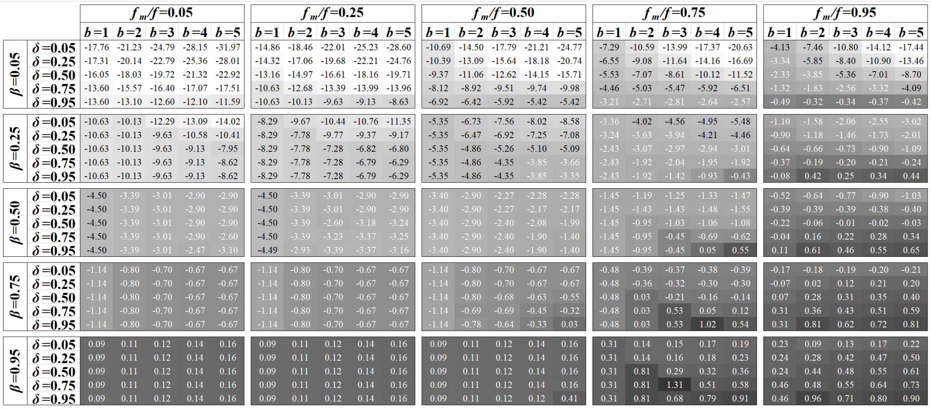
		f_m/f=0.05					f_m/f=0.25					f_m/f=0.50					f_m/f=0.75					f_m/f=0.95				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	-9.83	-12.66	-15.83	-18.71	-21.89	-8.12	-11.08	-14.36	-17.75	-20.96	-6.12	-9.36	-12.33	-16.13	-19.49	-4.44	-7.77	-11.19	-14.26	-18.07	-3.29	-6.57	-9.76	-13.69	-16.29
	δ=0.25	-9.85	-11.33	-13.64	-16.09	-19.31	-7.79	-9.82	-12.28	-14.74	-17.42	-5.59	-7.98	-10.61	-13.20	-15.77	-3.82	-6.43	-9.06	-11.78	-14.35	-2.65	-5.31	-7.98	-10.60	-13.26
	δ=0.50	-9.98	-10.09	-11.49	-13.77	-15.77	-7.56	-8.36	-9.80	-11.42	-13.34	-5.01	-6.40	-8.02	-9.73	-11.47	-2.93	-4.69	-6.46	-8.14	-10.01	-1.73	-3.54	-5.29	-7.09	-8.90
	δ=0.75	-13.60	-9.78	-9.66	-10.34	-11.40	-10.63	-7.44	-7.72	-8.32	-9.31	-4.78	-4.97	-5.65	-6.39	-7.22	-2.21	-2.98	-3.84	-4.71	-5.63	-0.83	-1.77	-2.68	-3.61	-4.52
	δ=0.95	-13.60	-13.10	-12.60	-12.10	-11.60	-10.63	-10.13	-9.63	-9.13	-8.63	-6.92	-6.42	-5.92	-4.39	-4.28	-3.21	-1.91	-1.93	-2.02	-2.14	-0.02	-0.37	-0.59	-0.78	-0.95
β=0.25	δ=0.05	-1.53	-1.98	-2.48	-3.00	-3.52	-1.53	-1.98	-2.48	-3.00	-3.52	-1.53	-1.98	-2.48	-3.00	-3.52	-1.44	-1.85	-2.32	-2.80	-3.30	-0.45	-0.98	-1.50	-2.05	-2.49
	δ=0.25	-1.53	-1.98	-2.48	-3.00	-3.52	-1.53	-1.98	-2.48	-3.00	-3.52	-1.53	-1.98	-2.48	-3.00	-3.29	-1.27	-1.67	-2.03	-2.44	-2.79	-0.31	-0.72	-1.10	-1.57	-1.98
	δ=0.50	-1.53	-1.98	-2.48	-3.00	-3.52	-1.53	-1.98	-2.48	-2.99	-3.20	-1.53	-1.98	-2.98	-3.33	-3.48	-1.25	-1.44	-1.67	-1.91	-2.19	-0.15	-0.48	-0.79	-1.08	-1.35
	δ=0.75	-1.53	-1.98	-2.48	-3.00	-3.64	-1.53	-1.98	-2.48	-3.57	-4.60	-1.53	-2.35	-3.25	-3.11	-3.06	-1.28	-1.26	-1.34	-1.44	-1.54	-0.03	-0.23	-0.42	-0.56	-0.68
	δ=0.95	-1.53	-1.98	-2.48	-3.15	-4.50	-1.53	-1.98	-2.78	-3.99	-4.52	-1.53	-2.75	-3.34	-3.36	-3.01	-1.61	-1.32	-1.18	-1.08	-1.00	0.00	-0.03	-0.12	-0.13	-0.12
β=0.50	δ=0.05	-0.23	-0.46	-0.64	-0.81	-0.97	-0.23	-0.46	-0.64	-0.81	-0.97	-0.23	-0.46	-0.64	-0.81	-0.97	-0.23	-0.46	-0.64	-0.81	-0.97	-0.06	-0.27	-0.44	-0.60	-0.76
	δ=0.25	-0.23	-0.46	-0.64	-0.81	-0.97	-0.23	-0.46	-0.64	-0.81	-0.97	-0.23	-0.46	-0.64	-0.81	-0.97	-0.23	-0.46	-0.64	-0.81	-1.00	-0.02	-0.19	-0.33	-0.44	-0.56
	δ=0.50	-0.23	-0.46	-0.64	-0.81	-0.97	-0.23	-0.46	-0.64	-0.81	-0.97	-0.23	-0.46	-0.64	-0.81	-0.97	-0.23	-0.46	-0.72	-0.78	-0.86	0.00	-0.11	-0.20	-0.31	-0.39
	δ=0.75	-0.23	-0.46	-0.64	-0.81	-0.97	-0.23	-0.46	-0.64	-0.81	-0.97	-0.23	-0.46	-0.64	-0.81	-0.97	-0.23	-0.58	-0.62	-0.63	-0.65	0.00	-0.05	-0.10	-0.16	-0.18
	δ=0.95	-0.23	-0.46	-0.64	-0.81	-0.97	-0.23	-0.46	-0.64	-0.81	-0.97	-0.23	-0.46	-0.64	-0.84	-1.20	-0.23	-0.58	-0.57	-0.51	-0.44	0.00	-0.01	-0.04	-0.02	0.02
β=0.75	δ=0.05	0.00	-0.08	-0.14	-0.19	-0.23	0.00	-0.08	-0.14	-0.19	-0.23	0.00	-0.08	-0.14	-0.19	-0.23	0.00	-0.08	-0.14	-0.19	-0.23	0.00	-0.05	-0.11	-0.15	-0.19
	δ=0.25	0.00	-0.08	-0.14	-0.19	-0.23	0.00	-0.08	-0.14	-0.19	-0.23	0.00	-0.08	-0.14	-0.19	-0.23	0.00	-0.08	-0.14	-0.19	-0.23	0.00	-0.04	-0.08	-0.11	-0.14
	δ=0.50	0.00	-0.08	-0.14	-0.19	-0.23	0.00	-0.08	-0.14	-0.19	-0.23	0.00	-0.08	-0.14	-0.19	-0.23	0.00	-0.08	-0.14	-0.19	-0.24	0.00	-0.02	-0.05	-0.07	-0.08
	δ=0.75	0.00	-0.08	-0.14	-0.19	-0.23	0.00	-0.08	-0.14	-0.19	-0.23	0.00	-0.08	-0.14	-0.19	-0.23	0.00	-0.08	-0.14	-0.18	-0.16	0.00	-0.01	-0.02	-0.02	0.00
	δ=0.95	0.00	-0.08	-0.14	-0.19	-0.23	0.00	-0.08	-0.14	-0.19	-0.23	0.00	-0.08	-0.14	-0.19	-0.23	0.00	-0.08	-0.16	-0.13	-0.07	0.00	0.00	-0.01	0.03	0.09
β=0.95	δ=0.05	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.01	0.02
	δ=0.25	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.01	0.03
	δ=0.50	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.02	0.04
	δ=0.75	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.07	0.00	0.00	0.01	0.02	0.08
	δ=0.95	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.04	0.12	0.00	0.00	0.01	0.03	0.12

(a) $\upsilon = 1$



		f_m/f=0.05					f_m/f=0.25					f_m/f=0.50					f_m/f=0.75					f_m/f=0.95				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	-17.76	-21.65	-24.98	-28.50	-31.75	-14.86	-18.68	-21.86	-25.45	-28.77	-10.99	-14.72	-18.08	-21.42	-24.60	-4.82	-8.18	-11.83	-15.23	-18.62	-3.53	-7.07	-10.32	-13.70	-17.05
	δ=0.25	-17.31	-20.22	-22.82	-25.43	-27.97	-14.32	-17.10	-19.66	-22.25	-24.35	-10.45	-13.13	-14.77	-16.75	-18.97	-4.55	-7.30	-10.00	-12.61	-15.26	-2.83	-5.48	-8.14	-10.82	-13.50
	δ=0.50	-16.05	-18.07	-19.74	-20.15	-20.82	-13.16	-14.99	-15.43	-16.15	-17.29	-9.40	-9.61	-10.69	-12.06	-13.60	-3.72	-5.54	-7.28	-9.00	-10.73	-1.98	-3.77	-5.53	-7.27	-9.00
	δ=0.75	-13.60	-15.59	-14.72	-14.44	-14.57	-10.63	-11.49	-11.14	-11.33	-11.77	-7.24	-6.98	-7.41	-8.02	-8.71	-2.80	-3.71	-4.54	-5.34	-6.15	-1.11	-2.00	-2.84	-3.66	-4.46
	δ=0.95	-13.60	-13.10	-12.60	-12.10	-11.60	-10.63	-10.13	-9.63	-9.13	-8.63	-6.92	-6.42	-5.92	-4.77	-4.59	-3.21	-2.33	-2.23	-2.29	-2.39	-0.42	-0.56	-0.66	-0.74	-0.81
β=0.25	δ=0.05	-7.44	-5.99	-5.87	-6.07	-6.40	-8.29	-5.96	-5.59	-5.99	-6.05	-5.36	-3.72	-4.04	-4.27	-4.75	-1.85	-2.18	-2.56	-3.02	-3.49	-0.70	-1.19	-1.72	-2.21	-2.70
	δ=0.25	-7.44	-5.99	-5.87	-6.07	-7.10	-8.29	-7.79	-5.66	-5.64	-5.79	-5.36	-3.73	-3.79	-3.97	-4.27	-1.71	-1.95	-2.30	-2.63	-3.02	-0.55	-0.91	-1.35	-1.76	-2.15
	δ=0.50	-7.44	-6.00	-6.56	-7.29	-7.02	-8.29	-7.79	-7.28	-5.51	-5.46	-5.36	-4.86	-3.64	-3.63	-3.75	-1.82	-1.78	-1.94	-2.14	-2.37	-0.39	-0.68	-0.95	-1.19	-1.42
	δ=0.75	-7.44	-5.95	-7.40	-7.87	-7.46	-8.29	-7.79	-7.28	-6.79	-6.28	-5.36	-4.86	-4.36	-3.42	-3.30	-2.43	-1.67	-1.60	-1.62	-1.68	-0.30	-0.41	-0.51	-0.58	-0.63
	δ=0.95	-7.45	-6.65	-7.59	-7.82	-7.75	-8.29	-7.79	-7.28	-6.79	-6.28	-5.36	-4.86	-4.36	-3.86	-3.35	-2.43	-1.93	-1.43	-0.93	-1.04	-0.20	-0.14	-0.09	-0.03	0.03
β=0.50	δ=0.05	-1.91	-1.61	-1.62	-1.70	-1.81	-1.91	-1.61	-1.62	-1.70	-1.81	-1.91	-1.61	-1.62	-1.70	-1.81	-1.07	-1.02	-1.10	-1.22	-1.36	-0.27	-0.41	-0.55	-0.69	-0.84
	δ=0.25	-1.91	-1.61	-1.62	-1.70	-1.81	-1.91	-1.61	-1.62	-1.70	-1.81	-1.91	-1.61	-1.62	-1.70	-1.63	-1.04	-0.92	-1.01	-1.09	-1.17	-0.22	-0.32	-0.42	-0.53	-0.66
	δ=0.50	-1.91	-1.61	-1.62	-1.70	-1.81	-1.91	-1.61	-1.62	-1.70	-1.81	-1.91	-1.61	-1.67	-1.90	-1.86	-1.45	-0.92	-0.89	-0.92	-0.95	-0.16	-0.22	-0.30	-0.36	-0.40
	δ=0.75	-1.91	-1.61	-1.62	-1.70	-1.81	-1.91	-1.61	-1.62	-1.70	-1.48	-1.91	-1.51	-1.98	-1.87	-1.71	-1.45	-0.95	-0.79	-0.72	-0.68	-0.11	-0.13	-0.13	-0.11	-0.08
	δ=0.95	-1.91	-1.61	-1.62	-1.70	-1.81	-1.91	-1.61	-1.62	-1.37	-2.00	-1.91	-1.79	-2.00	-1.85	-1.57	-1.45	-0.95	-0.67	-0.49	-0.35	-0.07	-0.02	0.07	0.16	0.25
β=0.75	δ=0.05	-0.42	-0.38	-0.38	-0.40	-0.42	-0.42	-0.38	-0.38	-0.40	-0.42	-0.42	-0.38	-0.38	-0.40	-0.42	-0.41	-0.36	-0.36	-0.38	-0.40	-0.08	-0.10	-0.13	-0.16	-0.19
	δ=0.25	-0.42	-0.38	-0.38	-0.40	-0.42	-0.42	-0.38	-0.38	-0.40	-0.42	-0.42	-0.38	-0.38	-0.40	-0.42	-0.38	-0.33	-0.32	-0.32	-0.33	-0.06	-0.08	-0.09	-0.11	-0.13
	δ=0.50	-0.42	-0.38	-0.38	-0.40	-0.42	-0.42	-0.38	-0.38	-0.40	-0.42	-0.42	-0.38	-0.38	-0.40	-0.42	-0.47	-0.31	-0.27	-0.25	-0.23	-0.04	-0.05	-0.05	-0.03	-0.01
	δ=0.75	-0.42	-0.38	-0.38	-0.40	-0.42	-0.42	-0.38	-0.38	-0.40	-0.42	-0.42	-0.38	-0.38	-0.40	-0.40	-0.47	-0.28	-0.20	-0.14	-0.07	-0.03	-0.02	0.05	0.10	0.15
	δ=0.95	-0.42	-0.38	-0.38	-0.40	-0.42	-0.42	-0.38	-0.38	-0.40	-0.42	-0.42	-0.38	-0.38	-0.37	-0.40	-0.44	-0.24	-0.10	0.03	0.14	-0.01	0.06	0.15	0.26	0.36
β=0.95	δ=0.05	-0.01	0.01	0.03	0.05	0.07	-0.01	0.01	0.03	0.05	0.07	-0.01	0.01	0.03	0.05	0.07	-0.01	0.01	0.03	0.05	0.07	0.01	0.04	0.06	0.08	0.10
	δ=0.25	-0.01	0.01	0.03	0.05	0.07	-0.01	0.01	0.03	0.05	0.07	-0.01	0.01	0.03	0.05	0.07	-0.01	0.01	0.03	0.05	0.08	0.02	0.04	0.06	0.09	0.11
	δ=0.50	-0.01	0.01	0.03	0.05	0.07	-0.01	0.01	0.03	0.05	0.07	-0.01	0.01	0.03	0.05	0.07	-0.01	0.01	0.04	0.10	0.15	0.02	0.04	0.07	0.09	0.18
	δ=0.75	-0.01	0.01	0.03	0.05	0.07	-0.01	0.01	0.03	0.05	0.07	-0.01	0.01	0.03	0.05	0.07	-0.01	0.05	0.12	0.19	0.27	0.02	0.05	0.14	0.21	0.28
	δ=0.95	-0.01	0.01	0.03	0.05	0.07	-0.01	0.01	0.03	0.05	0.07	-0.01	0.01	0.03	0.05	0.07	-0.01	0.09	0.19	0.31	0.40	0.02	0.05	0.20	0.32	0.42

(b) $\upsilon = 3$



		f_m/f=0.05					f_m/f=0.25					f_m/f=0.50					f_m/f=0.75					f_m/f=0.95				
		b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5	b=1	b=2	b=3	b=4	b=5
β=0.05	δ=0.05	-17.76	-21.23	-24.79	-28.15	-31.97	-14.86	-18.46	-22.01	-25.23	-28.60	-10.69	-14.50	-17.79	-21.21	-24.77	-7.29	-10.59	-13.99	-17.37	-20.63	-4.13	-7.46	-10.80	-14.12	-17.44
	δ=0.25	-17.31	-20.14	-22.79	-25.36	-28.01	-14.32	-17.06	-19.68	-22.21	-24.76	-10.39	-13.09	-15.64	-18.18	-20.74	-6.55	-9.08	-11.64	-14.16	-16.69	-3.34	-5.85	-8.40	-10.90	-13.46
	δ=0.50	-16.05	-18.03	-19.72	-21.32	-22.92	-13.16	-14.97	-16.61	-18.16	-19.71	-9.37	-11.06	-12.62	-14.15	-15.71	-5.53	-7.07	-8.61	-10.12	-11.52	-2.33	-3.85	-5.36	-7.01	-8.70
	δ=0.75	-13.60	-15.57	-16.40	-17.07	-17.51	-10.63	-12.68	-13.39	-13.99	-13.96	-8.12	-8.92	-9.51	-9.74	-9.98	-4.46	-5.03	-5.47	-5.92	-6.51	-1.32	-1.83	-2.56	-3.32	-4.09
	δ=0.95	-13.60	-13.10	-12.60	-12.10	-11.59	-10.63	-10.13	-9.63	-9.13	-8.63	-6.92	-6.42	-5.92	-5.42	-5.42	-3.21	-2.71	-2.81	-2.64	-2.57	-0.49	-0.32	-0.34	-0.37	-0.42
β=0.25	δ=0.05	-10.63	-10.13	-12.29	-13.09	-14.02	-8.29	-9.67	-10.44	-10.76	-11.35	-5.35	-6.73	-7.56	-8.02	-8.58	-3.36	-4.02	-4.56	-4.95	-5.48	-1.10	-1.58	-2.06	-2.55	-3.02
	δ=0.25	-10.63	-10.13	-9.63	-10.58	-10.41	-8.29	-7.78	-9.77	-9.37	-9.17	-5.35	-6.47	-6.92	-7.25	-7.08	-3.24	-3.63	-3.94	-4.21	-4.46	-0.90	-1.18	-1.46	-1.73	-2.01
	δ=0.50	-10.63	-10.13	-9.63	-9.13	-7.95	-8.29	-7.78	-7.28	-6.82	-6.80	-5.35	-4.86	-5.26	-5.10	-5.09	-2.43	-3.07	-2.97	-2.94	-3.01	-0.64	-0.66	-0.73	-0.90	-1.09
	δ=0.75	-10.63	-10.13	-9.63	-9.13	-8.62	-8.29	-7.78	-7.28	-6.79	-6.29	-5.35	-4.86	-4.35	-3.85	-3.66	-2.43	-1.92	-2.04	-1.95	-1.92	-0.37	-0.19	-0.20	-0.21	-0.24
	δ=0.95	-10.63	-10.13	-9.63	-9.13	-8.62	-8.29	-7.78	-7.28	-6.79	-6.29	-5.35	-4.86	-4.35	-3.85	-3.35	-2.43	-1.92	-1.42	-0.93	-0.43	-0.08	0.42	0.25	0.34	0.44
β=0.50	δ=0.05	-4.50	-3.39	-3.01	-2.90	-2.90	-4.50	-3.39	-3.01	-2.90	-2.90	-3.40	-2.90	-2.27	-2.28	-2.28	-1.45	-1.19	-1.25	-1.33	-1.47	-0.52	-0.64	-0.77	-0.90	-1.03
	δ=0.25	-4.50	-3.39	-3.01	-2.90	-2.90	-4.50	-3.39	-3.01	-2.90	-2.90	-3.40	-2.90	-2.27	-2.17	-2.17	-1.45	-1.43	-1.43	-1.48	-1.55	-0.39	-0.39	-0.39	-0.38	-0.40
	δ=0.50	-4.50	-3.39	-3.01	-2.90	-2.90	-4.50	-3.39	-2.60	-3.18	-3.24	-3.40	-2.90	-2.40	-2.08	-1.99	-1.45	-0.95	-1.03	-1.06	-1.08	-0.22	-0.06	-0.01	-0.02	-0.03
	δ=0.75	-4.50	-3.39	-3.01	-2.90	-2.60	-4.50	-3.39	-3.23	-3.37	-3.25	-3.40	-2.90	-2.40	-1.90	-1.40	-1.45	-0.95	-0.45	-0.69	-0.62	-0.04	0.16	0.22	0.28	0.34
	δ=0.95	-4.50	-3.39	-3.01	-2.47	-3.10	-4.49	-2.93	-3.39	-3.37	-3.16	-3.40	-2.90	-2.40	-1.90	-1.40	-1.45	-0.95	-0.45	0.05	0.55	0.11	0.61	0.46	0.55	0.65
β=0.75	δ=0.05	-1.14	-0.80	-0.70	-0.67	-0.67	-1.14	-0.80	-0.70	-0.67	-0.67	-1.14	-0.80	-0.70	-0.67	-0.67	-0.48	-0.39	-0.37	-0.38	-0.39	-0.17	-0.18	-0.19	-0.20	-0.21
	δ=0.25	-1.14	-0.80	-0.70	-0.67	-0.67	-1.14	-0.80	-0.70	-0.67	-0.67	-1.14	-0.80	-0.70	-0.67	-0.67	-0.48	-0.36	-0.32	-0.30	-0.30	-0.07	0.02	0.12	0.21	0.20
	δ=0.50	-1.14	-0.80	-0.70	-0.67	-0.67	-1.14	-0.80	-0.70	-0.67	-0.67	-1.14	-0.80	-0.68	-0.63	-0.55	-0.48	0.03	-0.21	-0.16	-0.14	0.07	0.28	0.31	0.35	0.40
	δ=0.75	-1.14	-0.80	-0.70	-0.67	-0.67	-1.14	-0.80	-0.70	-0.67	-0.67	-1.14	-0.69	-0.69	-0.45	-0.32	-0.48	0.03	0.53	0.05	0.12	0.31	0.36	0.43	0.51	0.59
	δ=0.95	-1.14	-0.80	-0.70	-0.67	-0.67	-1.14	-0.80	-0.70	-0.67	-0.67	-1.14	-0.78	-0.64	-0.33	0.03	-0.48	0.03	0.53	1.02	0.54	0.31	0.81	0.62	0.72	0.81
β=0.95	δ=0.05	0.09	0.11	0.12	0.14	0.16	0.09	0.11	0.12	0.14	0.16	0.09	0.11	0.12	0.14	0.16	0.31	0.14	0.15	0.17	0.19	0.23	0.09	0.13	0.17	0.22
	δ=0.25	0.09	0.11	0.12	0.14	0.16	0.09	0.11	0.12	0.14	0.16	0.09	0.11	0.12	0.14	0.16	0.31	0.14	0.16	0.18	0.23	0.24	0.28	0.42	0.47	0.50
	δ=0.50	0.09	0.11	0.12	0.14	0.16	0.09	0.11	0.12	0.14	0.16	0.09	0.11	0.12	0.14	0.16	0.31	0.81	0.29	0.32	0.36	0.24	0.44	0.48	0.55	0.61
	δ=0.75	0.09	0.11	0.12	0.14	0.16	0.09	0.11	0.12	0.14	0.16	0.09	0.11	0.12	0.14	0.16	0.31	0.81	1.31	0.51	0.58	0.46	0.48	0.55	0.64	0.73
	δ=0.95	0.09	0.11	0.12	0.14	0.16	0.09	0.11	0.12	0.14	0.16	0.09	0.11	0.12	0.12	0.41	0.31	0.81	0.68	0.79	0.91	0.46	0.96	0.71	0.80	0.90

(c) $\upsilon = 5$

Figure B.6: Impact of υ on consumer surplus, where the parameters are set to $f = 5, s_n = 3$, values from -15.00 to 2.48 are highlighted as a heatmap from white to black background, and values below -15.00 are shown with a white background

Appendix C

PROOFS FOR CHAPTER 3 & APPENDIX A

Proof of Lemma 9. $\Pi_{r0}^{D}(\omega_o, s_o)$, $\Pi_{r1}^{D}(\omega_o, s_o)$ and $\Pi_{r2}^{D}(\omega_o, s_o)$ are all continuous in s_o. To complete the proof for continuity, we only need to check the two transition points. It is left to the reader to verify that the following equations hold:

$$\begin{aligned} \Pi_{r0}^{D}\left(\omega_o, \frac{\upsilon - p}{\beta f_e} - \alpha l_r\right) &= \Pi_{r1}^{D}\left(\omega_o, \frac{\upsilon - p}{\beta f_e} - \alpha l_r\right) \\ \Pi_{r1}^{D}\left(\omega_o, \frac{\upsilon - p + b}{\beta f_e} - \alpha l_r\right) &= \Pi_{r2}^{D}\left(\omega_o, \frac{\upsilon - p + b}{\beta f_e} - \alpha l_r\right) \end{aligned}$$

Thus, $\Pi_{r}^{D}(\omega_o, s_o)$ is continuous in s_o.

Next, analyze the first and second derivatives of $\Pi_{r0}^{D}(\omega_o, s_o)$ and $\Pi_{r1}^{D}(\omega_o, s_o)$ to establish the other two points: $\frac{\partial \Pi_{r0}^{D}(\omega_o,s_o)}{\partial s_o} = f_e > 0$, $\frac{\partial^2 \Pi_{r0}^{D}(\omega_o,s_o)}{\partial s_o^2} = 0$, $\frac{\partial \Pi_{r1}^{D}(\omega_o,s_o)}{\partial s_o} = \frac{f_e}{b}\left(\upsilon - p + b - \beta(f\alpha(l_r - c_r) + f_e(2s_o + \alpha l_r - \omega_o))\right)$, $\frac{\partial^2 \Pi_{r1}^{D}(\omega_o,s_o)}{\partial s_o^2} = -\frac{2\beta f_e^2}{b} < 0$. ■

Proof of Proposition 1. First, find the unconstrained maximizer of $\Pi_{r1}^{D}(\omega_o, s_o)$, which is given by the FOC (due to concavity as proven in Lemma 9:

$$\frac{\partial \Pi_{r1}^{D}(\omega_o, s_o)}{\partial s_o} = 0 \Leftrightarrow \hat{s}_o(\omega_o) = \frac{\upsilon - p + b}{2\beta f_e} + \frac{\omega_o - \alpha l_r}{2} - \frac{f\alpha(l_r - c_r)}{2f_e}$$

Then, we have three possibilities depending on where $\hat{s}_o$ is, as compared to the boundaries of the interval where $\Pi_{r1}^{D}(\omega_o, s_o)$ is valid, i.e., $\left[\frac{\upsilon-p}{\beta f_e} - \alpha l_r, \frac{\upsilon-p+b}{\beta f_e} - \alpha l_r\right]$. In all three possibilities, we have $\Pi_{r0}^{D}(\omega_o, s_o)$ increasing on s_o on the interval $(-\infty, \frac{\upsilon-p}{\beta f_e} - \alpha l_r\Big)$ and $\Pi_{r2}^{D}(\omega_o, s_o) = 0$ constant on s_o on the interval $\Big(\frac{\upsilon-p+b}{\beta f_e} - \alpha l_r , \infty)$ (as proven in Lemma 9).

(i) When $\hat{s}_o(\omega_o) < \frac{\upsilon-p}{\beta f_e} - \alpha l_r$, $\Pi_{r1}^{D}(\omega_o, s_o)$ is decreasing on s_o. Then, the maximum is realized on $s_o(\omega_o) = \frac{\upsilon-p}{\beta f_e} - \alpha l_r$.

(ii) When $\frac{v-p}{\beta f_e} - \alpha l_r \leq \hat{s}_o(\omega_o) \leq \frac{v-p+b}{\beta f_e} - \alpha l_r$, $\Pi^D_{r1}(\omega_o, s_o)$ is concave on s_o. Then, the maximum is realized on $s_o(\omega_o) = \hat{s}_o(\omega_o) = \frac{v-p+b}{2\beta f_e} + \frac{\omega_o - \alpha l_r}{2} - \frac{f\alpha(l_r - c_r)}{2f_e}$.

(iii) When $\hat{s}_o(\omega_o) > \frac{v-p+b}{\beta f_e} - \alpha l_r$, $\Pi^D_{r1}(\omega_o, s_o)$ is increasing on s_o. Then, the maximum is realized on $s_o(\omega_o) = \left[\frac{v-p+b}{\beta f_e} - \alpha l_r \text{ , } \infty\right)$.

■

Proof of Lemma 10. In order to find $\Pi^D_m(\omega_o, s^*_o(\omega_o))$, we simply substitute s_o with $s^*_o(\omega_o)$ in $\Pi^D_m(\omega_o, s_o)$. The rest of the proof follows similar to proof of Lemma 9. ■

Proof of Corollary 1. Follows from the equilibrium results reported in Table 3.2. ■

Proof of Proposition 2. We solve for the equilibrium by using backward induction. The retailer's best response is given in Proposition 1 and the corresponding profit function of the manufacturer and relevant properties of this function are given in Lemma 10.

First, find the unconstrained optimizer of $\Pi^D_{m1}(\omega_o)$, which is given by the FOC:

$$\frac{\partial \Pi^D_{m1}(\omega_o)}{\partial \omega_o} = 0 \Leftrightarrow \hat{\omega}_o = \frac{v-p+b}{2\beta f_e} + \frac{f_m(c_o + \alpha l_r + \alpha(l_r - c_r)) - p + c}{2f_e} + \frac{c_o - \alpha c_r}{2}$$

Then, we have three possibilities depending on where $\hat{\omega}_o$ is, as compared to the boundaries of the interval where $\Pi^D_{m1}(\omega_o)$ is valid, i.e., $\left[\frac{v-p-b}{\beta f_e} + \frac{f\alpha(l_r-c_r)}{f_e} - \alpha l_r, \frac{v-p+b}{\beta f_e} + \frac{f\alpha(l_r-c_r)}{f_e} - \alpha l_r\right]$. In all three possibilities, we have $\Pi^D_{m0}(\omega_o)$ increasing on ω_o on the interval $\left(-\infty, \frac{v-p-b}{\beta f_e} + \frac{f\alpha(l_r-c_r)}{f_e} - \alpha l_r\right)$ and $\Pi^D_{m2}(\omega_o) = 0$ constant on ω_o on the interval $\left(\frac{v-p+b}{\beta f_e} + \frac{f\alpha(l_r-c_r)}{f_e} - \alpha l_r \text{ , } \infty\right)$ (as proven in Lemma 10).

(i) When $\hat{\omega}_o < \frac{v-p-b}{\beta f_e} + \frac{f\alpha(l_r-c_r)}{f_e} - \alpha l_r$, $\Pi^D_{m1}(\omega_o)$ is decreasing on ω_o. Then, the maximum is realized on $\omega_o = \frac{v-p-b}{\beta f_e} + \frac{f\alpha(l_r-c_r)}{f_e} - \alpha l_r$.

(ii) When $\frac{v-p-b}{\beta f_e} + \frac{f\alpha(l_r-c_r)}{f_e} - \alpha l_r \leq \hat{\omega}_o \leq \frac{v-p+b}{\beta f_e} + \frac{f\alpha(l_r-c_r)}{f_e} - \alpha l_r$, $\Pi^D_{m1}(\omega_o)$ is concave on ω_o. Then, the maximum is realized on $\omega_o = \hat{\omega}_o = \frac{v-p+b}{2\beta f_e} + \frac{f_m(c_o+\alpha l_r+\alpha(l_r-c_r))-p+c}{2f_e} + \frac{c_o-\alpha c_r}{2}$.

(iii) When $\hat{\omega}_o > \frac{v-p+b}{\beta f_e} + \frac{f\alpha(l_r-c_r)}{f_e} - \alpha l_r$, $\Pi^M_{m1}(\omega_o)$ is increasing on ω_o. Then, the maximum is realized on $\omega_o \in \left[\frac{v-p+b}{\beta f_e} + \frac{f\alpha(l_r-c_r)}{f_e} - \alpha l_r \text{ , } \infty\right)$.

s_o^* follows from $\omega = \omega_o^*$. ∎

Proof of Lemma 11. $\Pi_m^D(\omega_o^*, s_o^*, p)$ follows directly from Proposition 2. Since all three pieces of the function are rational, it is clear that each piece is continuous in p. To complete the proof for continuity, we only need to show continuity in the two transition points $\upsilon - 3b - \beta(f(c_o + \alpha c_r) - p + c)$ and $\upsilon + b - \beta(f(c_o + \alpha c_r) - p + c)$. It is left to the reader to verify that the following equations hold:

$$\begin{aligned} \Pi_{m0}^{D1}\left(\frac{\upsilon - 3b - \beta(f(c_o + \alpha c_r) + c)}{1 - \beta}\right) &= \Pi_{m1}^{D1}\left(\frac{\upsilon - 3b - \beta(f(c_o + \alpha c_r) + c)}{1 - \beta}\right) \\ \Pi_{m1}^{D1}\left(\frac{\upsilon + b - \beta(f(c_o + \alpha c_r) + c)}{1 - \beta}\right) &= \Pi_{m2}^{D1}\left(\frac{\upsilon + b - \beta(f(c_o + \alpha c_r) + c)}{1 - \beta}\right) \end{aligned}$$

Thus, $\Pi_m^D(\omega_o^*, s_o^*, p)$ is continuous in p.

Next, analyze the first and second derivatives of the pieces of the profit function, in order to establish the truth of the other two statements: $\frac{\partial \Pi_{m0}^{D1}(p)}{\partial p} = -\frac{1-\beta}{\beta} < 0$, $\frac{\partial^2 \Pi_{m0}^{D1}(p)}{\partial p^2} = 0$, $\frac{\partial^2 \Pi_{m1}^{D1}(p)}{\partial p^2} = \frac{(\beta-1)^2}{4b\beta} > 0$. ∎

Proof of Proposition 3. As shown in Lemma 11, the manufacturer's profit function $\Pi_m^D(\omega_o^*, s_o^*, p)$ is decreasing up until the first threshold value, then convex decreasing to zero until the second threshold value. Therefore, the maximum is realized at the minimum possible value for p, 0. ∎

Proof of Corollary 2. Analysis of the first and second derivatives of the functions in each interval show that Π_m^D and Π_r^D are convex decreasing functions on β; whereas CS^D is a concave increasing function on β. The derivatives are given below where $F = f(c_o + \alpha c_r) + c$.

$$\begin{aligned} \frac{\partial \Pi_{m0}}{\partial \beta} &= -\frac{b}{\beta^2} < 0 & \frac{\partial^2 \Pi_{m0}}{\partial \beta^2} &= \frac{2b}{\beta^3} > 0 \\ \frac{\partial \Pi_{m1}}{\partial \beta} &= -\frac{(\upsilon+b-\beta F)(\upsilon+b+\beta F)}{16b\beta^2} < 0 & \frac{\partial^2 \Pi_{m1}}{\partial \beta^2} &= \frac{(\upsilon+b)^2}{8b\beta^3} > 0 \\ \frac{\partial \Pi_{r0}}{\partial \beta} &= -\frac{\upsilon-b}{\beta^2} < 0 & \frac{\partial^2 \Pi_{r0}}{\partial \beta^2} &= \frac{2(\upsilon-b)}{\beta^3} > 0 \\ \frac{\partial \Pi_{r1}}{\partial \beta} &= -\frac{(\upsilon+b-\beta F)(\upsilon+b+\beta F)}{8b\beta^2} < 0 & \frac{\partial^2 \Pi_{r1}}{\partial \beta^2} &= \frac{(\upsilon+b)^2}{4b\beta^3} > 0 \\ \frac{\partial CS_0}{\partial \beta} &= \frac{b\upsilon}{\beta^2} > 0 & \frac{\partial^2 CS_0}{\partial \beta^2} &= -\frac{2b\upsilon}{\beta^3} < 0 \\ \frac{\partial CS_1}{\partial \beta} &= \frac{3(\upsilon+b)(\upsilon+b-\beta^2 F)+\beta^2 F^2(1-\beta)}{16\beta^2} > 0 & \frac{\partial^2 CS_1}{\partial \beta^2} &= -\frac{F^2}{16} - \frac{3(\upsilon+b)^2}{8\beta^3} < 0 \end{aligned}$$

∎

Proof of Lemma 12. The profit function follows from plugging in $q(s_o)$ from Equation 3.5 into Equation 3.6.

$\Pi_0^{MC}(s_o)$, $\Pi_1^{MC}(s_o)$ and $\Pi_2^{MC}(s_o)$ are all continuous in s_o. To complete the proof for continuity, we only need to check the two transition points. It is left to the reader to verify that the following equations hold:

$$\begin{aligned} \Pi_0^{MC}\left(\frac{\upsilon-p}{\beta f_e}-\alpha l_r\right) &= \Pi_1^{MC}\left(\frac{\upsilon-p}{\beta f_e}-\alpha l_r\right) \\ \Pi_1^{MC}\left(\frac{\upsilon-p+b}{\beta f_e}-\alpha l_r\right) &= \Pi_2^{MC}\left(\frac{\upsilon-p+b}{\beta f_e}-\alpha l_r\right) \end{aligned}$$

Thus, $\Pi^{MC}(s_o)$ is continuous in s_o.

Next, analyze the first and second derivatives of $\Pi_0^{MC}(s_o)$ and $\Pi_1^{MC}(s_o)$ to establish the other two points: $\frac{\partial \Pi_0^{MC}(s_o)}{\partial s_o} = f_e > 0$, $\frac{\partial^2 \Pi_0^{MC}(s_o)}{\partial s_o^2} = 0$, $\frac{\partial \Pi_1^{MC}(s_o)}{\partial s_o} = \frac{f_e}{b}\left(\upsilon - p + b - \beta\left(p - c - f(c_o + \alpha c_r) + 2f_e(s_o + \alpha l_r)\right)\right)$, $\frac{\partial^2 \Pi_1^{C}(s_o)}{\partial s_o^2} = -\frac{2\beta f_e^2}{b} < 0$.
■

Proof of Proposition 11. First, find the unconstrained maximizer of $\Pi_1^C(s_o)$, which is given by the FOC (due to concavity as proven in Lemma 12):

$$\frac{\partial \Pi_1^{MC}(s_o)}{\partial s_o} = 0 \Leftrightarrow \hat{s}_o = \frac{\upsilon - p + b}{2\beta f_e} + \frac{f(c_o + \alpha c_r) - p + c}{2 f_e} - \alpha l_r$$

Then, we have three possibilities depending on where $\hat{s}_o$ is, as compared to the boundaries of the interval where $\Pi_1^{MC}(s_o)$ is valid, *i.e.,* $\left[\frac{\upsilon-p}{\beta f_e} - \alpha l_r, \frac{\upsilon-p+b}{\beta f_e} - \alpha l_r\right]$. In all three possibilities, we have $\Pi_0^{MC}(s_o)$ increasing on s_o on the interval $\left(-\infty, \frac{\upsilon-p}{\beta f_e} - \alpha l_r\right)$ and $\Pi_2^{MC}(s_o) = 0$ constant on s_o on the interval $\left(\frac{\upsilon-p+b}{\beta f_e} - \alpha l_r\ ,\infty\right)$ (as proven in Lemma 12).

(i) When $\hat{s}_o < \frac{\upsilon-p}{\beta f_e} - \alpha l_r$, $\Pi_1^{MC}(s_o)$ is decreasing on s_o. Then, the maximum is realized on $s_o = \frac{\upsilon-p}{\beta f_e} - \alpha l_r$.

(ii) When $\frac{\upsilon-p}{\beta f_e} - \alpha l_r \le \hat{s}_o \le \frac{\upsilon-p+b}{\beta f_e} - \alpha l_r$, $\Pi_1^{MC}(s_o)$ is convex on s_o. Then, the maximum is realized on $s_o = \hat{s}_o = \frac{\upsilon-p+b}{2\beta f_e} + \frac{f(c_o+\alpha c_r)-p+c}{2f_e} - \alpha l_r$.

(iii) When $\hat{s}_o > \frac{\upsilon-p+b}{\beta f_e} - \alpha l_r$, $\Pi_1^{MC}(s_o)$ is increasing on s_o. Then, the maximum is realized on $s_o \in \left[\frac{\upsilon-p+b}{\beta f_e} - \alpha l_r\ ,\infty\right)$.

■

Proof of Lemma 13. $\Pi^{MC}(p, s_o^*(p))$ follows directly from Proposition 11.

Since all three pieces of the function are rational, it is clear that each piece is continuous in p. To complete the proof for continuity, we only need to show continuity in the two transition points $\upsilon - b - \beta(f(c_o + \alpha c_r) - p + c)$ and $\upsilon + b - \beta(f(c_o + \alpha c_r) - p + c)$. It is left to the reader to verify that the following equations hold:

$$\begin{aligned}
\Pi_0^{MC}\left(\frac{\upsilon - b - \beta(f(c_o + \alpha c_r) + c)}{1-\beta}\right) &= \Pi_1^{MC}\left(\frac{\upsilon - b - \beta(f(c_o + \alpha c_r) + c)}{1-\beta}\right) \\
\Pi_1^{MC}\left(\frac{\upsilon + b - \beta(f(c_o + \alpha c_r) + c)}{1-\beta}\right) &= \Pi_2^{MC}\left(\frac{\upsilon + b - \beta(f(c_o + \alpha c_r) + c)}{1-\beta}\right)
\end{aligned}$$

Thus, $\Pi^{MC}(p, s_o^*(p))$ is continuous in p.

Next, analyze the first and second derivatives of the pieces of the profit function, in order to establish the truth of the other two statements: $\frac{\partial \Pi_0^{MC}(p)}{\partial p} = -\frac{1-\beta}{\beta} < 0$, $\frac{\partial^2 \Pi_0^{MC}(p)}{\partial p^2} = 0$, $\frac{\partial^2 \Pi_1^{MC}(p)}{\partial p^2} = \frac{(\beta-1)^2}{2b\beta} > 0$. ■

Proof of Proposition 12. Since the profit function is decreasing up until the first threshold value, then convex decreasing to zero until the second threshold value, the maximum is realized at the minimum possible value for p, 0. ■

Proof of Lemma 14. $\Pi_{m0}^T(p)$, $\Pi_{m1}^T(p)$ and $\Pi_{m2}^T(p)$ are all continuous in p. To complete the proof for continuity, we only need to check the two transition points. It is left to the reader to verify that the following equations hold:

$$\begin{aligned}
\Pi_{m0}^T\left(\upsilon - \beta f(s_t + \alpha l_t)\right) &= \Pi_{m1}^T\left(\upsilon - \beta f(s_t + \alpha l_t)\right) \\
\Pi_{m1}^T\left(\upsilon + b - \beta f(s_t + \alpha l_t)\right) &= \Pi_{m2}^T\left(\upsilon + b - \beta f(s_t + \alpha l_t)\right)
\end{aligned}$$

Thus, $\Pi_m^T(p)$ is continuous in p.

Next, analyze the first and second derivatives of $\Pi_{m0}^T(p)$ and $\Pi_{m1}^T(p)$ to establish the other two points: $\frac{\partial \Pi_{m0}^T(p)}{\partial p} = 1 > 0$, $\frac{\partial^2 \Pi_{m0}^T(p)}{\partial p^2} = 0$, $\frac{\partial^2 \Pi_{m1}^T(p)}{\partial p^2} = -\frac{2}{b} < 0$. ■

Proof of Proposition 4. First, find the unconstrained maximizer of $\Pi_{m1}^T(p)$, which is given by the FOC (due to concavity as proven in Lemma 14):

$$\frac{\partial \Pi_{m1}^T(p)}{\partial p} = 0 \Leftrightarrow \hat{p} = \frac{1}{2}\left(\upsilon + b + c - \beta f(s_t + \alpha l_t)\right)$$

Then, we have three possibilities depending on where $\hat{p}$ is, as compared to the boundaries of the interval where $\Pi_{m1}^T(p)$ is valid, *i.e.,* $[\upsilon - \beta f(s_t + \alpha l_t), \upsilon + b - \beta f(s_t + \alpha l_t)]$. In all three possibilities, we have $\Pi_{m0}^T(p)$ increasing on p on the interval $(-\infty, \upsilon - \beta f(s_t + \alpha l_t))$ and $\Pi_{m2}^T(p) = 0$ constant on p on the interval $(\upsilon + b - \beta f(s_t + \alpha l_t)\ , \infty)$ (as proven in Lemma 14).

(i) When $\hat{p} < \upsilon - \beta f(s_t + \alpha l_t)$, $\Pi^T_{m1}(p)$ is decreasing on p. Then, the maximum is realized on $p = \upsilon - \beta f(s_t + \alpha l_t)$.

(ii) When $\upsilon - \beta f(s_t + \alpha l_t) \leq \hat{p} \leq \upsilon + b - \beta f(s_t + \alpha l_t)$, $\Pi^T_{m1}(p)$ is concave on p. Then, the maximum is realized on $p = \hat{p} = \frac{1}{2}(\upsilon + b + c - \beta f(s_t + \alpha l_t))$.

(iii) When $\hat{p} > \upsilon + b - \beta f(s_t + \alpha l_t)$, $\Pi^T_{m1}(p)$ is increasing on p. Then, the maximum is realized on $p \in [\upsilon + b - \beta f(s_t + \alpha l_t)\ , \infty)$.

■

Proof of Corollary 3. Skipped. ■

Proof of Lemma 15. In order to find $\Pi^D_m(\omega_o, p, s^*_o)$, we simply substitute s_o with s^*_o in $\Pi^D_m(\omega_o, p, s_o)$. The objective function is the same as the one given in Lemma 10, the only difference being the threshold values arranged with respect to p (instead of ω_o).

Since all three pieces of the function are rational, it is clear that each piece is continuous in p. To complete the proof for continuity, we only need to show continuity in the two transition points $\upsilon - b + \beta\left(f_m\alpha(l_r - c_r) - f_e(\omega_o + \alpha c_r)\right)$ and $\upsilon + b + \beta\left(f_m\alpha(l_r - c_r) - f_e(\omega_o + \alpha c_r)\right)$. It is left to the reader to verify that the following equations hold:

$$\begin{aligned}
&\Pi^D_{m0}\left(\upsilon - b + \beta\left(f_m\alpha(l_r - c_r) - f_e(\omega_o + \alpha c_r)\right)\right) \\
&\qquad\qquad = \Pi^D_{m1}\left(\upsilon - b + \beta\left(f_m\alpha(l_r - c_r) - f_e(\omega_o + \alpha c_r)\right)\right) \\
&\Pi^D_{m1}\left(\upsilon + b + \beta\left(f_m\alpha(l_r - c_r) - f_e(\omega_o + \alpha c_r)\right)\right) \\
&\qquad\qquad = \Pi^D_{m2}\left(\upsilon + b + \beta\left(f_m\alpha(l_r - c_r) - f_e(\omega_o + \alpha c_r)\right)\right)
\end{aligned}$$

Thus, $\Pi^D_m(\omega_o, s^*_o(\omega_o))$ is continuous in p.

Next, analyze the first and second derivatives of the pieces of the profit function, in order to establish the truth of the other two statements.

$$\begin{aligned}
\frac{\partial \Pi^D_{m0}(\omega_o, p)}{\partial p} &= 1 > 0 \\
\frac{\partial^2 \Pi^D_{m0}(\omega_o, p)}{\partial p^2} &= 0
\end{aligned}$$

$$\frac{\partial^2 \Pi_{m1}^D(\omega_o,p)}{\partial p^2} \;=\; -\frac{1}{b} < 0$$

■

Proof of Proposition 13. First, find the unconstrained maximizer of $\Pi_{m1}^D(\omega_o,p)$, which is given by the FOC (due to concavity as proven in Lemma 15):

$$\frac{\partial \Pi_{m1}^D(\omega_o,p)}{\partial p} = 0 \;\Leftrightarrow$$

$$\hat{p}(\omega_o) = \frac{1}{2}\left(v + b + c + f_m\left(\alpha(l_r - c_r) + c_o + \alpha c_r\right) - f_e(\omega_o - c_o) + T\right)$$

Then, we have three possibilities depending on where $\hat{p}$ is, as compared to the boundaries of the interval where $\Pi_{m1}^D(\omega_o,p)$ is valid, *i.e.,* $[v - b + \beta\left(f_m\alpha(l_r - c_r) - f_e(\omega_o + \alpha c_r)\right), v + b + \beta\left(f_m\alpha(l_r - c_r) - f_e(\omega_o + \alpha c_r)\right)]$. In all three possibilities, we have $\Pi_{m0}^D(\omega_o,p)$ increasing on p on the interval $(-\infty, v - b + \beta\left(f_m\alpha(l_r - c_r) - f_e(\omega_o + \alpha c_r)\right))$ and $\Pi_{m1}^D(\omega_o,s_o) = 0$ constant on s_o on the interval $(v + b + \beta\left(f_m\alpha(l_r - c_r) - f_e(\omega_o + \alpha c_r)\right), \infty)$ (as proven in Lemma 15).

- When $\hat{p}(\omega_o) < v - b + \beta\left(f_m\alpha(l_r - c_r) - f_e(\omega_o + \alpha c_r)\right)$, $\Pi_{m1}^D(\omega_o,p)$ is decreasing on p. Then, the maximum is realized on $p(\omega_o) = v - b + \beta\left(f_m\alpha(l_r - c_r) - f_e(\omega_o + \alpha c_r)\right)$.
- When $v - b + \beta\left(f_m\alpha(l_r - c_r) - f_e(\omega_o + \alpha c_r)\right) \le \hat{p}(\omega_o) \le v + b + \beta\left(f_m\alpha(l_r - c_r) - f_e(\omega_o + \alpha c_r)\right)$, $\Pi_{m1}^D(\omega_o,p)$ is concave on p. Then, the maximum is realized on $p(\omega_o) = \hat{p}(\omega_o) = \frac{v+b+c+f_m((1+\beta)\alpha(l_r-c_r)+c_o+\alpha c_r)-f_e((\omega_o-c_o)+\beta(\omega_o+\alpha c_r))}{2}$.
- When $\hat{p}(\omega_o) > v + b + \beta\left(f_m\alpha(l_r - c_r) - f_e(\omega_o + \alpha c_r)\right)$, $\Pi_{m1}^D(\omega_o,p)$ is increasing on p. Then, the maximum is realized on $p(\omega_o) = [v + b + \beta\left(f_m\alpha(l_r - c_r) - f_e(\omega_o + \alpha c_r)\right), \infty)$.

■

Proof of Proposition 14. In order to find $\Pi_m^D(\omega_o,p^*,s_o^*)$, we simply substitute p with p^* from Proposition 13 in $\Pi_m^D(\omega_o,p,s_o)$.

Since all three pieces of the function are rational, it is clear that each piece is continuous in ω_o. To complete the proof for continuity, we only need to show continuity in the two transition points $\frac{f_m\alpha(l_r-c_r)}{f_e} + \frac{f(c_o+\alpha c_r)+c-v+3b}{(1-\beta)f_e} - \alpha c_r$ and $\frac{f_m\alpha(l_r-c_r)}{f_e} + \frac{f(c_o+\alpha c_r)+c-v-b}{(1-\beta)f_e} - \alpha c_r$. It is left to the reader to verify that the following equations hold:

$$\Pi^D_{m0}\left(\frac{f_m\alpha(l_r-c_r)}{f_e} + \frac{f(c_o+\alpha c_r)+c-v+3b}{(1-\beta)f_e} - \alpha c_r\right)$$
$$= \Pi^D_{m1}\left(\frac{f_m\alpha(l_r-c_r)}{f_e} + \frac{f(c_o+\alpha c_r)+c-v+3b}{(1-\beta)f_e} - \alpha c_r\right)$$
$$\Pi^D_{m1}\left(\frac{f_m\alpha(l_r-c_r)}{f_e} + \frac{f(c_o+\alpha c_r)+c-v-b}{(1-\beta)f_e} - \alpha c_r\right)$$
$$= \Pi^D_{m2}\left(\frac{f_m\alpha(l_r-c_r)}{f_e} + \frac{f(c_o+\alpha c_r)+c-v-b}{(1-\beta)f_e} - \alpha c_r\right)$$

Thus, $\Pi^D_m(\omega_o, s^*_o, p^*)$ is continuous in ω_o.

Next, analyze the first and second derivatives of the pieces of the profit function, in order to establish the truth of the other two statements.

$$\begin{aligned}\frac{\partial \Pi^D_{m0}(\omega_o)}{\partial \omega_o} &= f_e(1-\beta) > 0\\ \frac{\partial^2 \Pi^D_{m0}(\omega_o)}{\partial \omega_o^2} &= 0\end{aligned}$$

$$\frac{\partial^2 \Pi^D_{m1}(\omega_o)}{\partial \omega_o^2} = \frac{f_e^2(\beta-1)^2}{4b} > 0$$

■

Proof of Lemma 16. Since all three pieces of the function are rational, it is clear that each piece is continuous in P. To complete the proof for continuity, we only need to show continuity in the two transition points $v-b+f_m\alpha(l_r-c_r)$ and $v+b+f_m\alpha(l_r-c_r)$. It is left to the reader to verify that the following equations hold:

$$\begin{aligned}\Pi^D_{m0}\left(v-b+f_m\alpha(l_r-c_r)\right) &= \Pi^D_{m1}\left(v-b+f_m\alpha(l_r-c_r)\right)\\ \Pi^D_{m1}\left(v+b+f_m\alpha(l_r-c_r)\right) &= \Pi^D_{m2}\left(v+b+f_m\alpha(l_r-c_r)\right)\end{aligned}$$

Thus, $\Pi^D_m(P)$ is continuous in P.

Next, analyze the first and second derivatives of the pieces of the profit function, in order to establish the truth of the other two statements:$\frac{\partial \Pi^D_{m0}(P)}{\partial P} = 1 > 0$, $\frac{\partial^2 \Pi^D_{m0}(P)}{\partial P^2} = 0$,

$\frac{\partial \Pi^D_{m1}(P)}{\partial P} = \frac{1}{2b}\left(\upsilon + b + c + f(c_o + \alpha c_r) + 2 f_m \alpha (l_r - c_r) - 2P\right)$, $\frac{\partial^2 \Pi^D_{m1}(P)}{\partial P^2} = -\frac{1}{b} < 0$. ■

Proof of Proposition 15. The manufacturer's profit function and its relevant properties are given in Lemma 16. First, find the unconstrained optimizer of $\Pi^D_{m1}(P)$, which is given by the FOC:

$$\frac{\partial \Pi^D_{m1}(P)}{\partial P} = 0 \Leftrightarrow \hat{P} = \frac{\upsilon + b + c + f(c_o + \alpha c_r)}{2} + f_m \alpha (l_r - c_r)$$

Then, we have three possibilities depending on where $\hat{P}$ is, as compared to the boundaries of the interval where $\Pi^D_{m1}(P)$ is valid, *i.e.,* $[\upsilon - b + f_m \alpha (l_r - c_r), \upsilon + b + f_m \alpha (l_r - c_r)]$. In all three possibilities, we have $\Pi^D_{m0}(P)$ increasing on P on the interval $(-\infty, \upsilon - b + f_m \alpha (l_r - c_r))$ and $\Pi^D_{m2}(P) = 0$ constant on P on the interval $(\upsilon + b + f_m \alpha (l_r - c_r)\ , \infty)$ (as proven in Lemma 16).

(i) When $\hat{P} < \upsilon - b + f_m \alpha (l_r - c_r)$, $\Pi^D_{m1}(P)$ is decreasing on P. Then, the maximum is realized on $P = \upsilon - b + f_m \alpha (l_r - c_r)$.

(ii) When $\upsilon - b + f_m \alpha (l_r - c_r) \leq \hat{P} \leq \upsilon + b + f_m \alpha (l_r - c_r)$, $\Pi^D_{m1}(P)$ is concave on P. Then, the maximum is realized on $P = \hat{P} = \frac{\upsilon + b + c + f(c_o + \alpha c_r)}{2} + f_m \alpha (l_r - c_r)$.

(iii) When $\hat{P} > \upsilon + b + f_m \alpha (l_r - c_r)$, $\Pi^D_{m1}(P)$ is increasing on P. Then, the maximum is realized on $P \in [\upsilon + b + f_m \alpha (l_r - c_r)\ , \infty)$.

■

Appendix D

PROOFS FOR CHAPTER 4 & APPENDIX B

Proof of Lemma 1.

$$\begin{aligned}
U_r^C(\theta_1, p, s_o) &> U_i^C(\theta_1, p, s_o) \\
(\upsilon - p) + \theta_1 - \beta f_e(\alpha l_r + s_o) &> (\upsilon - p) + \delta\theta_1 - \beta f_e(\alpha l_i + s_n) \\
\theta_1 &> \frac{\beta f_e\left(s_o - s_n + \alpha(l_r - l_i)\right)}{1-\delta} \\
\theta_2 &> \theta_1 > \frac{\beta f_e\left(s_o - s_n + \alpha(l_r - l_i)\right)}{1-\delta}
\end{aligned}$$

$$\begin{aligned}
& U_r^C(\theta_2, p, s_o) - U_i^C(\theta_2, p, s_o) \\
&= (\upsilon - p) + \theta_2 - \beta f_e(\alpha l_r + s_o) - (\upsilon - p) - \delta\theta_2 + \beta f_e(\alpha l_i + s_n) \\
&= (1-\delta)\theta_2 - \beta f_e\left(s_o - s_n + \alpha(l_r - l_i)\right) \\
&> (1-\delta)\frac{\beta f_e\left(s_o - s_n + \alpha(l_r - l_i)\right)}{1-\delta} - \beta f_e\left(s_o - s_n + \alpha(l_r - l_i)\right) \\
&> 0
\end{aligned}$$

■

Proof of Lemma 2.

First, find the demand for the retailer and the original products, as this is the first choice of the customers (as given by Lemma 1). A customer purchases the product and uses the retailer for after-sales services if $U_r^C(\theta, p, s_o) \geq 0$ and $U_r^C(\theta, p, s_o) \geq U_i^C(\theta, p, s_o)$. Compare the last customer that is indifferent between not buying and buying and using the retailer (where $U_r^C(\theta_{r0}, p, s_o) = 0$), with the customer that is indifferent between buying and using the retailer and buying and using the independent workshop (where $U_r^C(\theta_{ri}, p, s_o) = U_i^C(\theta_{ri}, p, s_o)$).

$$\begin{aligned}
\theta_{r0} &= \beta f_e(\alpha l_r + s_o) - (\upsilon - p) = \beta f_e(s_o + \alpha l_r) - (\upsilon - p) \\
\theta_{ri} &= \frac{\beta f_e\left(s_o - s_n - \alpha(l_r - l_i)\right)}{1-\delta} = \frac{\beta f_e((s_o + \alpha l_r) - (s_n + \alpha l_i))}{1-\delta}
\end{aligned}$$

Case 1: $\theta_{r0} > \theta_{ri}$

In this case, there is no demand for the independent workshop, because the condition of the case implies that the utility for the retailer is always higher than that of the independent workshop:

$$\begin{aligned}
\theta_{r0} &> \theta_{ri} \\
\beta f_e(\alpha l_r + s_o) - (\upsilon - p) &> \frac{\beta f_e\left(s_o - s_n - \alpha(l_r - l_i)\right)}{1-\delta} \\
(\upsilon - p) - \beta f_e(\alpha l_i + s_n) &< \delta\left((\upsilon - p) - \beta f_e(\alpha l_r + s_o)\right)
\end{aligned}$$

The sub-cases are then defined only on the comparison between θ_{r0}, b and 0 and the demand is derived as follows, the same as in the Monopoly Model:

$$\begin{aligned}
q^C(\omega_o, s_o) = q_r^C(\omega_o, s_o) &= \int_{\beta f_e(\alpha l_r + s_o) - (\upsilon - p)}^{b} \frac{1}{b} d\theta \\
&= \begin{cases} 1, & \text{if } s_o < \frac{(\upsilon-p)}{\beta f_e} - \alpha l_r \\ 1 - \frac{\beta f_e(\alpha l_r + s_o) - (\upsilon - p)}{b}, & \text{if } \frac{(\upsilon-p)}{\beta f_e} - \alpha l_r \le s_o \le \frac{b + (\upsilon - p)}{\beta f_e} - \alpha l_r \\ 0, & \text{if } s_o > \frac{b+(\upsilon-p)}{\beta f_e} - \alpha l_r. \end{cases}
\end{aligned}$$

Case 2: $\theta_{ri} > \theta_{r0}$

In this case, there is demand for the independent workshop. Compare the last customer that is indifferent between not buying and buying and using the independent workshop (where $U_i^C(\theta_{i0}, p, s_o) = 0$), with the customer that is indifferent between buying and using the retailer and buying and using the independent workshop (where $U_r^C(\theta_{ri}, p, s_o) = U_i^C(\theta_{ri}, p, s_o)$). First, verify that $\theta_{ri} > \theta_{i0}$, due to the condition derived on $\theta_{ri} > \theta_{r0}$:

$$\begin{aligned}
\theta_{ri} - \theta_{i0} &= \frac{\beta f_e(s_o - s_n + \alpha(l_r - l_i))}{1-\delta} - \frac{\beta f_e(s_n + \alpha l_i) - (\upsilon - p)}{\delta} \\
&= \beta f_e(\delta s_o - s_n + \alpha(\delta l_r - l_i)) + (1-\delta)(\upsilon - p) \\
&> 0
\end{aligned}$$

The sub-cases are then defined on the comparison between θ_{ri}, θ_{i0}, b and 0:

- If $\theta_{ri} > \theta_{i0} > b > 0$, then $q_r^D(p, s_o) = q_i^D(p, s_o) = 0$. That is, $q_r^D(p, s_o) = q_i^D(p, s_o) = 0$, if $(s_n + \alpha l_i) > \frac{b\delta + (\upsilon - p)}{\beta f_e}$ and $(s_n + \alpha l_i) - \delta(s_o + \alpha l_r) < \frac{(1-\delta)(\upsilon - p)}{\beta f_e}$.

- If $\theta_{ri} > b > \theta_{i0} > 0$, then $q_r^D(p, s_o) = 0$ and $q_i^D(p, s_o) = \int_{\theta_{i|0}}^{b} \frac{1}{b} d\theta$. That is, $q_r^D(p, s_o) = 0$ and $q_i^D(p, s_o) = 1 - \frac{\beta f_e(s_n + \alpha l_i) - (\upsilon - p)}{b\delta}$, if $\frac{b\delta + (\upsilon - p)}{\beta f_e} > (s_n + \alpha l_i) > \frac{(\upsilon - p)}{\beta f_e}$ and $(s_o + \alpha l_r) - (s_n + \alpha l_i) > \frac{b(1-\delta)}{\beta f_e}$.

- If $\theta_{ri} > b > 0 > \theta_{i0}$, then $q_r^D(p, s_o) = 0$ and $q_i^D(p, s_o) = 1$. That is, $q_r^D(p, s_o) = 0$ and $q_i^D(p, s_o) = 1$, if $(s_n + \alpha l_i) < \frac{(\upsilon - p)}{\beta f_e}$ and $(s_o + \alpha l_r) - (s_n + \alpha l_i) > \frac{b(1-\delta)}{\beta f_e}$.

- If $b > \theta_{ri} > \theta_{i0} > 0$, then $q_r^D(p, s_o) = \int_{\theta_{r|i}}^{b} \frac{1}{b} d\theta$ and $q_i^D(p, s_o) = \int_{\theta_{i0}}^{\theta_{ri}} \frac{1}{b} d\theta$. That is, $q_r^D(p, s_o) = 1 - \frac{\beta f_e((s_o + \alpha l_r) - (s_n + \alpha l_i))}{b(1-\delta)}$ and $q_i^D(p, s_o) = \frac{\beta f_e((s_o + \alpha l_r) - (s_n + \alpha l_i))}{b(1-\delta)} - \frac{\beta f_e(s_n + \alpha l_i) - (\upsilon - p)}{b\delta}$, if $(s_n + \alpha l_i) > \frac{(\upsilon - p)}{\beta f_e}$, $(s_o + \alpha l_r) - (s_n + \alpha l_i) < \frac{b(1-\delta)}{\beta f_e}$, and $(s_n + \alpha l_i) - \delta(s_o + \alpha l_r) < \frac{(1-\delta)(\upsilon - p)}{\beta f_e}$.

- If $b > \theta_{ri} > 0 > \theta_{i0}$, then $q_r^D(p, s_o) = \int_{\theta_{ri}}^{b} \frac{1}{b} d\theta$ and $q_i^D(p, s_o) = \int_{0}^{\theta_{ri}} \frac{1}{b} d\theta$. That is, $q_r^D(p, s_o) = 1 - \frac{\beta f_e((s_o + \alpha l_r) - (s_n + \alpha l_i)))}{b(1-\delta)}$ and $q_i^D(p, s_o) = \frac{\beta f_e((s_o + \alpha l_r) - (s_n + \alpha l_i))}{b(1-\delta)}$, if $(s_n + \alpha l_i) < \frac{(\upsilon - p)}{\beta f_e}$ and $\frac{(\upsilon - p)}{\beta f_e} > (s_o + \alpha l_r) - (s_n + \alpha l_i) > 0$.

- If $b > 0 > \theta_{ri} > \theta_{i0}$, then $q_r^D(p, s_o) = 1$ and $q_i^D(p, s_o) = 0$. That is, $q_r^D(p, s_o) = 1$ and $q_i^D(p, s_o) = 0$, if $(s_n + \alpha l_i) - \delta(s_o + \alpha l_r) < \frac{(1-\delta)(\upsilon - p)}{\beta f_e}$ and $(s_o + \alpha l_r) - (s_n + \alpha l_i) < 0$.

The detailed graphical depiction can be found in Figure D.1. ■

Proof of Proposition 5. Please refer to the proof of Proposition xx in Appendix yy. ■

Proof of Lemma 3. The profit function follows from plugging in $q_i^C(p, s_o)$ and $q_r^C(p, s_o)$ in Equation 4.3. $\Pi_{r(-3)}^{CL}(p, \omega_o, s_o)$, $\Pi_{r(6)}^{CL}(p, \omega_o, s_o)$ and $\Pi_{r(5)}^{CL}(p, \omega_o, s_o)$ are all continuous in s_o. To complete the proof for continuity, we only need to check the two transition points. It is left to the reader to verify that the following equations hold, where $s_o^1 = s_n + \alpha l_i - \alpha l_r$ and $s_o^2 = s_n + \alpha l_i + \frac{b(1-\delta)}{\beta f_e} - \alpha l_r$:

$$\begin{aligned} \Pi_{r(-3)}^{CL}\left(p, \omega_o, s_o^1\right) &= \Pi_{r(6)}^{CL}\left(p, \omega_o, s_o^1\right) \\ \Pi_{r(6)}^{CL}\left(p, \omega_o, s_o^2\right) &= \Pi_{r(5)}^{CL}\left(p, \omega_o, s_o^2\right) \end{aligned}$$

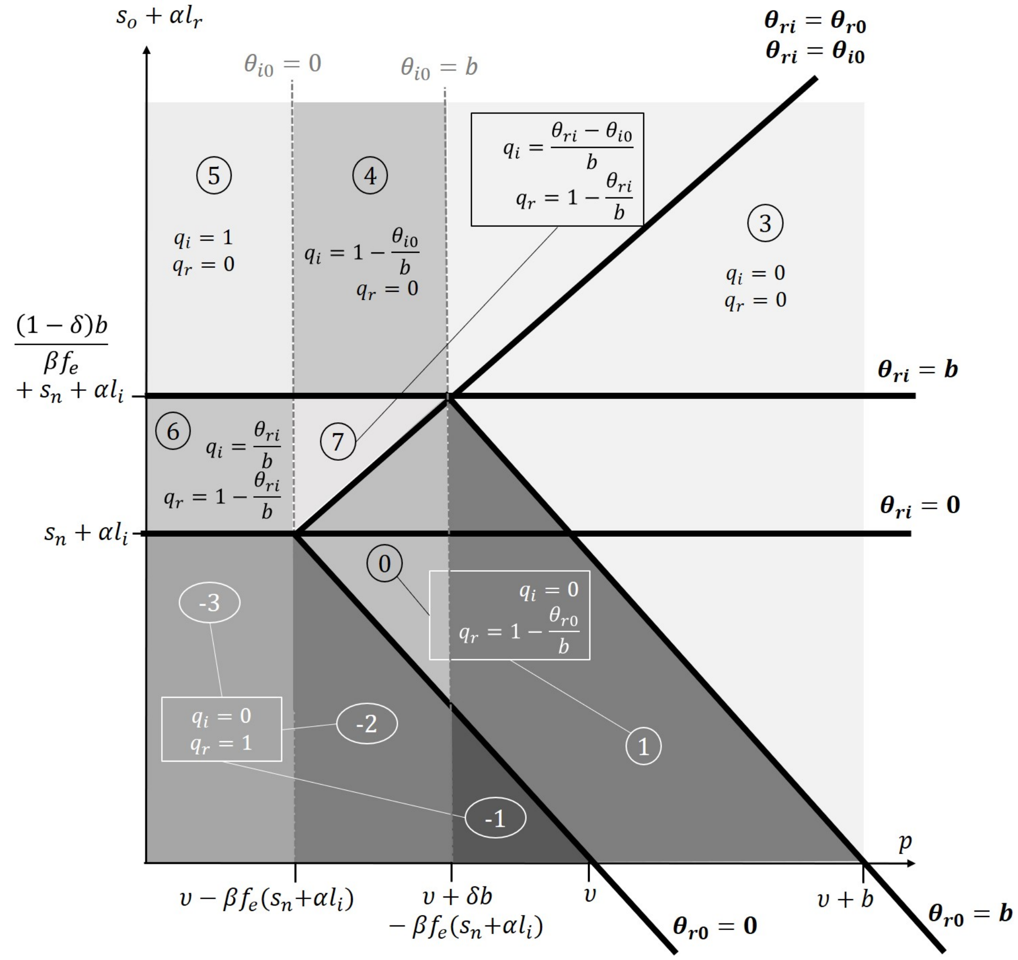


Figure D.1: Demand function for different s_o and s_n combinations where $\theta_{ri} = \frac{\beta f_e((s_o+\alpha l_r)-(s_n+\alpha l_i))}{1-\delta}$, $\theta_{r0} = \beta f_e(s_o + \alpha l_r) - (\upsilon - p)$, and $\theta_{i0} = \frac{\beta f_e(s_n+\alpha l_i)-(\upsilon-p)}{\delta}$

Thus, $\Pi_r^{CL}(p, \omega_o, s_o)$ is continuous in s_o. Next, analyze the first and second derivatives of $\Pi_{r(-3)}^{CL}(p, \omega_o, s_o)$ and $\Pi_{r(6)}^{CL}(p, \omega_o, s_o)$ to establish points 2 and 3:

$$\begin{aligned}\frac{\partial \Pi_{r(-3)}^{CL}(p, \omega_o, s_o)}{\partial s_o} &= f_e > 0 \\ \frac{\partial^2 \Pi_{r(6)}^{CL}(p, \omega_o, s_o)}{\partial s_o^2} &= -\frac{2\beta f_e^2}{b(1-\delta)} < 0\end{aligned}$$

■

Proof of Proposition 6. First, find the unconstrained maximizer of $\Pi_{r(6)}^{CL}(p, \omega_o, s_o)$, which is given by the FOC (due to concavity as proven in Lemma 3):

$$\frac{\partial \Pi_{r(6)}^{CL}(p, \omega_o, s_o)}{\partial s_o} = 0 \Leftrightarrow \hat{s}_o(p, \omega_o) = \frac{b(1-\delta) + \beta f_e(\omega_o + \alpha c_r + s_n + \alpha l_i)}{2\beta f_e} - \alpha l_r$$

Then, we have three possibilities depending on where $\hat{s}_o$ is, as compared to the boundaries of the interval where $\Pi_{r(6)}^{CL}(p, \omega_o, s_o)$ is valid, *i.e.,* $\left[s_n + \alpha l_i - \alpha l_r, \frac{b(1-\delta)}{\beta f_e} + s_n + \alpha l_i - \alpha l_r\right]$. In all three possibilities, we have $\Pi_{r(-3)}^{CL}(p, \omega_o, s_o)$ increasing on s_o on the interval $(-\infty, s_n + \alpha l_i - \alpha l_r)$ and $\Pi_{r(5)}^{CL}(p, \omega_o, s_o) = f_m \alpha(l_r - c_r)$ constant on s_o on the interval $\left(\frac{b(1-\delta)}{\beta f_e} + s_n + \alpha l_i - \alpha l_r\ , \infty\right)$ (as proven in Lemma 3).

- When $\hat{s}_o(\omega_o) < s_n + \alpha l_i - \alpha l_r$, $\Pi_{r(6)}^{CL}(p, \omega_o, s_o)$ is decreasing on s_o. Then, the maximum is realized on $s_o(\omega_o) = s_n + \alpha l_i - \alpha l_r$.
- When $s_n + \alpha l_i - \alpha l_r \leq \hat{s}_o(\omega_o) \leq \frac{b(1-\delta)}{\beta f_e} + s_n + \alpha l_i - \alpha l_r$, $\Pi_{r(6)}^{CL}(p, \omega_o, s_o)$ is convex on s_o. Then, the maximum is realized on $s_o(\omega_o) = \hat{s}_o(\omega_o) = \frac{b(1-\delta)+\beta f_e(\omega_o+\alpha c_r+s_n+\alpha l_r)}{2\beta f_e} - \alpha l_r$.
- When $\hat{s}_o(\omega_o) > \frac{b(1-\delta)}{\beta f_e} + s_n + \alpha l_i - \alpha l_r$, $\Pi_{r(6)}^{CL}(p, \omega_o, s_o)$ is increasing on s_o. Then, the maximum is realized on $s_o(\omega_o) = \left[\frac{b(1-\delta)}{\beta f_e} + s_n + \alpha l_i - \alpha l_r\ , \infty\right)$.

■

Proof of Lemma 4. The profit function follows from plugging in $q_i^D(p, s_o)$ and $q_r^D(p, s_o)$ in Equation 4.3. $\Pi_{r(-2)}^{CM}(p, \omega_o, s_o)$, $\Pi_{r(0)}^{CM}(p, \omega_o, s_o)$, $\Pi_{r(7)}^{CM}(p, \omega_o, s_o)$, and $\Pi_{r(4)}^{CM}(p, \omega_o, s_o)$ are all continuous in s_o. To complete the proof for continuity, we only need to check the three transition points. It is left to the reader to verify that the following equations hold, where $s_o^1 = \frac{\upsilon - p}{\beta f_e} - \alpha l_r$, $s_o^2 = \frac{s_n + \alpha l_i}{\delta} - \frac{(1-\delta)(\upsilon - p)}{\delta \beta f_e} - \alpha l_r$, and $s_o^3 = \frac{b(1-\delta)}{\beta f_e} + s_n + \alpha l_i - \alpha l_r$:

$$\begin{aligned}
\Pi_{r(-2)}^{CM}\left(p, \omega_o, s_o^1\right) &= \Pi_{r(0)}^{CM}\left(p, \omega_o, s_o^1\right) \\
\Pi_{r(0)}^{CM}\left(p, \omega_o, s_o^2\right) &= \Pi_{r(7)}^{CM}\left(p, \omega_o, s_o^2\right) \\
\Pi_{r(7)}^{CM}\left(p, \omega_o, s_o^3\right) &= \Pi_{r(4)}^{CM}\left(p, \omega_o, s_o^3\right)
\end{aligned}$$

Thus, $\Pi_r^{CM}(p, \omega_o, s_o)$ is continuous in s_o. Next, analyze the first and second derivatives of $\Pi_{r(-2)}^{CM}(p, \omega_o, s_o)$, $\Pi_{r(0)}^{CM}(p, \omega_o, s_o)$ and $\Pi_{r(7)}^{CM}(p, \omega_o, s_o)$ to establish points 2-4:

$$\begin{aligned}
\frac{\partial \Pi_{r(-2)}^{CM}(p, \omega_o, s_o)}{\partial s_o} &= f_e > 0 \\
\frac{\partial^2 \Pi_{r(-2)}^{CM}(p, \omega_o, s_o)}{\partial s_o^2} &= 0 \\
\frac{\partial \Pi_{r(0)}^{CM}(p, \omega_o, s_o)}{\partial s_o} &= \frac{f_e}{b}\left(\upsilon - p + b - \beta(f\alpha(l_r - c_r) + f_e(2s_o + \alpha l_r - \omega_o))\right) \\
\frac{\partial^2 \Pi_{r(0)}^{CM}(p, \omega_o, s_o)}{\partial s_o^2} &= -\frac{2\beta f_e^2}{b} < 0 \\
\frac{\partial^2 \Pi_{r(7)}^{CM}(p, \omega_o, s_o)}{\partial s_o^2} &= -\frac{2\beta f_e^2}{b(1-\delta)} < 0
\end{aligned}$$

■

Proof of Proposition 7. First, find the unconstrained maximizers of $\Pi_{r(0)}^{CM}(p, \omega_o, s_o)$ and $\Pi_{r(7)}^{CM}(p, \omega_o, s_o)$, which are given by their respective FOCs (due to concavity as proven in Lemma 4):

$$\begin{aligned}
\frac{\partial \Pi_{r(0)}^{CM}(p, \omega_o, s_o)}{\partial s_o} &= 0 \Leftrightarrow \\
\hat{s}_{o0}(p, \omega_o) &= \frac{\upsilon - p + b + \beta\left(f_e(\omega_o + \alpha c_r) - f_m \alpha (l_r - c_r)\right)}{2\beta f_e} - \alpha l_r
\end{aligned}$$

$$\frac{\partial \Pi_{r(7)}^{CM}(p, \omega_o, s_o)}{\partial s_o} = 0 \Leftrightarrow \hat{s}_{o7}(p, \omega_o) = \frac{b(1-\delta) + \beta f_e(\omega_o + \alpha c_r + s_n + \alpha l_i)}{2\beta f_e} - \alpha l_r$$

Then, we have nine possibilities depending on where $\hat{s}_{o0}$ and $\hat{s}_{o7}$ are, as compared to the boundaries of the interval where they are respectively valid, *i.e.*, $\left[\frac{\upsilon - p}{\beta f_e} - \alpha l_r, \frac{\beta f_e(s_n + \alpha l_i) - (1-\delta)(\upsilon - p)}{\delta \beta f_e} - \alpha l_r\right]$ for $\Pi_{r(2)}^{CM}(p, \omega_o, s_o)$ and $\left[\frac{\beta f_e(s_n + \alpha l_i) - (1-\delta)(\upsilon - p)}{\delta \beta f_e} - \alpha l_r, \frac{\beta f_e(s_n + \alpha l_i) + (1-\delta) b}{\beta f_e} - \alpha l_r\right]$ for $\Pi_{r(7)}^{CM}(p, \omega_o, s_o)$. In all nine possibilities, we have $\Pi^{CM}_{r(-2)}(p, \omega_o, s_o)$ increasing on s_o on the interval $(-\infty, \frac{v-p}{\beta f_e} - \alpha l_r\Big)$ and $\Pi^{CM}_{r(4)}(p, \omega_o, s_o) = f_m\alpha(l_r - c_r)\left(1 - \frac{\beta f_e(s_n+\alpha l_i)-v+p}{b\delta}\right)$ constant on s_o on the interval $\Big(\frac{\beta f_e(s_n+\alpha l_i)+(1-\delta)b}{\beta f_e} - \alpha l_r\ , \infty)$ (as proven in Lemma 4).

- $\nearrow \nearrow \nearrow \longrightarrow$ When $\hat{s}_{o0}(\omega_o) > \frac{\beta f_e(s_n+\alpha l_i)-(1-\delta)(v-p)}{\delta\beta f_e} - \alpha l_r$, $\Pi^{CM}_{r(2)}(p, \omega_o, s_o)$ is increasing on s_o. The corresponding condition on ω_o is $\omega_o > \frac{2\beta f_e(s_n+\alpha l_i)+\delta\beta f_m\alpha(l_r-c_r)-(2-\delta)(v-p)-\delta b}{\delta\beta f_e} - \alpha c_r$. Additionally, when $\hat{s}_{o7}(\omega_o) > \frac{\beta f_e(s_n+\alpha l_i)+(1-\delta)b}{\beta f_e} - \alpha l_r$, $\Pi^{CM}_{r(7)}(p, \omega_o, s_o)$ is also increasing on s_o. The corresponding second condition on ω_o is $\omega_o > \frac{\beta f_e(s_n+\alpha l_i)+(1-\delta)b}{\beta f_e} - \alpha c_r$. Then, the maximum is realized on $s_o(p, \omega_o) = \frac{\beta f_e(s_n+\alpha l_i)+b(1-\delta)}{\beta f_e} - \alpha l_r$.

- $\nearrow \nearrow \searrow \longrightarrow$ When $\hat{s}_{o0}(\omega_o) > \frac{\beta f_e(s_n+\alpha l_i)-(1-\delta)(v-p)}{\delta\beta f_e} - \alpha l_r$, $\Pi^{CM}_{r(2)}(p, \omega_o, s_o)$ is increasing on s_o. The corresponding condition on ω_o is $\omega_o > \frac{2\beta f_e(s_n+\alpha l_i)+\delta\beta f_m\alpha(l_r-c_r)-(2-\delta)(v-p)-\delta b}{\delta\beta f_e} - \alpha c_r$. Additionally, when $\hat{s}_{o7}(\omega_o) < \frac{\beta f_e(s_n+\alpha l_i)-(1-\delta)(v-p)}{\delta\beta f_e} - \alpha l_r$, $\Pi^{CM}_{r(7)}(p, \omega_o, s_o)$ is decreasing on s_o. The corresponding second condition on ω_o is $\omega_o < \frac{\beta f_e(2-\delta)(s_n+\alpha l_i)-(1-\delta)(\delta b+2(v-p))}{\delta\beta f_e} - \alpha c_r$. Then, the maximum is realized on $s_o(p, \omega_o) = \frac{\beta f_e(s_n+\alpha l_i)-(1-\delta)(v-p)}{\delta\beta f_e} - \alpha l_r$.

- $\nearrow \nearrow \quad \cap \quad \longrightarrow$ When $\hat{s}_{o0}(\omega_o) > \frac{\beta f_e(s_n+\alpha l_i)-(1-\delta)(v-p)}{\delta\beta f_e} - \alpha l_r$, $\Pi^{CM}_{r(2)}(\omega_o, s_o)$ is increasing on s_o. The corresponding condition on ω_o is $\omega_o > \frac{2\beta f_e(s_n+\alpha l_i)+\delta\beta f_m\alpha(l_r-c_r)-(2-\delta)(v-p)-\delta b}{\delta\beta f_e} - \alpha c_r$. Additionally, when $\frac{\beta f_e(s_n+\alpha l_i)-(1-\delta)(v-p)}{\delta\beta f_e} - \alpha l_r < \hat{s}_{o7}(\omega_o) < \frac{\beta f_e(s_n+\alpha l_i)+(1-\delta)b}{\beta f_e} - \alpha l_r$, $\Pi^{CM}_{r7}(\omega_o, s_o)$ is concave on s_o. The corresponding second condition on ω_o is $\frac{\beta f_e(s_n+\alpha l_i)+(1-\delta)b}{\beta f_e} - \alpha c_r > \omega_o > \frac{\beta f_e(2-\delta)(s_n+\alpha l_i)-(1-\delta)(\delta b+2(v-p))}{\delta\beta f_e} - \alpha c_r$. Then, the maximum is realized on $s_o(p, \omega_o) = \hat{s}_{o7}(p, \omega_o) = \frac{b(1-\delta)+\beta f_e(\omega_o+\alpha c_r+s_n+\alpha l_i)}{2\beta f_e} - \alpha l_r$.

- $\nearrow \searrow \searrow \longrightarrow$ When $\hat{s}_{o0}(\omega_o) < \frac{v-p}{\beta f_e} - \alpha l_r$, $\Pi^{CM}_{r(2)}(\omega_o, s_o)$ is decreasing on s_o. The corresponding condition on ω_o is $\omega_o < \frac{v-p-b+\beta f_m\alpha(l_r-c_r)}{\beta f_e} - \alpha c_r$. Additionally, when $\hat{s}_{o7}(\omega_o) < \frac{\beta f_e(s_n+\alpha l_i)-(1-\delta)(v-p)}{\delta\beta f_e} - \alpha l_r$, $\Pi^{CM}_{r(7)}(p, \omega_o, s_o)$ is also decreasing on s_o. The corresponding second condition on ω_o is $\omega_o < \frac{\beta f_e(2-\delta)(s_n+\alpha l_i)-(1-\delta)(\delta b+2(v-p))}{\delta\beta f_e} - \alpha c_r$. Then, the maximum is realized on $s_o(p, \omega_o) = \frac{v-p}{\beta f_e} - \alpha l_r$.

- $\nearrow \cap \searrow \longrightarrow$ When $\frac{\beta f_e(s_n+\alpha l_i)-(1-\delta)(v-p)}{\delta\beta f_e} - \alpha l_r > \hat{s}_{o0}(\omega_o) > \frac{v-p}{\beta f_e} - \alpha l_r$, $\Pi^{CM}_{r(2)}(p, \omega_o, s_o)$ is concave on s_o. The corresponding condition on ω_o is

$\frac{2\beta f_e(s_n+\alpha l_i)+\delta\beta f_m\alpha(l_r-c_r)-(2-\delta)(\upsilon-p)-\delta b}{\delta\beta f_e} - \alpha c_r > \omega_o > \frac{\upsilon-p-b+\beta f_m\alpha(l_r-c_r)}{\beta f_e} - \alpha c_r$. Additionally, when $\hat{s}_{o7}(\omega_o) < \frac{\beta f_e(s_n+\alpha l_i)-(1-\delta)(\upsilon-p)}{\delta\beta f_e} - \alpha l_r$, $\Pi^{CM}_{r(7)}(p, \omega_o, s_o)$ is decreasing on s_o. The corresponding second condition on ω_o is $\omega_o < \frac{\beta f_e(2-\delta)(s_n+\alpha l_i)-(1-\delta)(\delta b+2(\upsilon-p))}{\delta\beta f_e} - \alpha c_r$. Then, the maximum is realized on $s_o(p, \omega_o) = \hat{s}_{o0}(p, \omega_o) = \frac{\upsilon-p+b+\beta f_e(\omega_o+\alpha c_r)-\beta f_m\alpha(l_r-c_r)}{2\beta f_e} - \alpha l_r$.

- $\nearrow\searrow\nearrow \longrightarrow$ When $\hat{s}_{o0}(\omega_o) < \frac{\upsilon-p}{\beta f_e} - \alpha l_r$, $\Pi^{CM}_{r(2)}(p, \omega_o, s_o)$ is decreasing on s_o. The corresponding condition on ω_o is $\omega_o < \frac{\upsilon-p-b+\beta f_m\alpha(l_r-c_r)}{\beta f_e} - \alpha c_r$. Additionally, when $\hat{s}_{o7}(\omega_o) > \frac{\beta f_e(s_n+\alpha l_i)+(1-\delta)b}{\beta f_e} - \alpha l_r$, $\Pi^{CM}_{r(7)}(p, \omega_o, s_o)$ is increasing on s_o. The corresponding second condition on ω_o is $\omega_o > \frac{\beta f_e(s_n+\alpha l_i)+(1-\delta)b}{\beta f_e} - \alpha c_r$. Then, there are two possible alternatives based on how the objective function value compares at the two candidate points:

  - The maximum is realized on $s_o(p, \omega_o) = \frac{\upsilon-p}{\beta f_e} - \alpha l_r$ if $\Pi^{CM}_{r(2)}(p, \omega_o, \frac{\upsilon-p}{\beta f_e} - \alpha l_r) > \Pi^{CM}_{r(7)}(p, \omega_o, \frac{\beta f_e(s_n+\alpha l_i)+b(1-\delta)}{\beta f_e} - \alpha l_r)$, *i.e.*, $\frac{\upsilon-p+\beta(f_m\alpha(l_r-c_r)-f_e(\omega_o+\alpha c_r))}{\beta} > \frac{f_m\alpha(l_r-c_r)\beta(\upsilon-p+b\delta-\beta f_e(s_n+\alpha l_i))}{b\delta}$.

  - The maximum is realized on $s_o(p, \omega_o) = \frac{\beta f_e(s_n+\alpha l_i)+b(1-\delta)}{\beta f_e} - \alpha l_r$ if $\Pi^{CM}_{r(2)}(p, \omega_o, \frac{\upsilon-p}{\beta f_e} - \alpha l_r) < \Pi^{CM}_{r(7)}(p, \omega_o, \frac{\beta f_e(s_n+\alpha l_i)+b(1-\delta)}{\beta f_e} - \alpha l_r)$, *i.e.*, $\frac{\upsilon-p+\beta(f_m\alpha(l_r-c_r)-f_e(\omega_o+\alpha c_r))}{\beta} < \frac{f_m\alpha(l_r-c_r)\beta(\upsilon-p+b\delta-\beta f_e(s_n+\alpha l_i))}{b\delta}$.

- $\nearrow\searrow\ \cap\ \longrightarrow$ When $\hat{s}_{o0}(\omega_o) < \frac{\upsilon-p}{\beta f_e} - \alpha l_r$, $\Pi^{CM}_{r(2)}(p, \omega_o, s_o)$ is decreasing on s_o. The corresponding condition on ω_o is $\omega_o < \frac{\upsilon-p-b+\beta f_m\alpha(l_r-c_r)}{\beta f_e} - \alpha c_r$. Additionally, when $\frac{\beta f_e(s_n+\alpha l_i)-(1-\delta)(\upsilon-p)}{\delta\beta f_e} - \alpha l_r < \hat{s}_{o7}(\omega_o) < \frac{\beta f_e(s_n+\alpha l_i)+(1-\delta)b}{\beta f_e} - \alpha l_r$, $\Pi^{CM}_{r(7)}(p, \omega_o, s_o)$ is concave on s_o. The corresponding second condition on ω_o is $\frac{\beta f_e(s_n+\alpha l_i)+(1-\delta)b}{\beta f_e} - \alpha c_r > \omega_o > \frac{\beta f_e(2-\delta)(s_n+\alpha l_i)-(1-\delta)(\delta b+2(\upsilon-p))}{\delta\beta f_e} - \alpha c_r$. Then, there are two possible alternatives based on how the objective function value compares at the two candidate points:

  - The maximum is realized on $s_o(p, \omega_o) = \frac{\upsilon-p}{\beta f_e} - \alpha l_r$ if $\Pi^{CM}_{r(2)}(p, \omega_o, \frac{\upsilon-p}{\beta f_e} - \alpha l_r) > \Pi^{CM}_{r(7)}(p, \omega_o, \frac{b(1-\delta)+\beta f_e(c_o+\alpha c_r+s_n+\alpha l_i)}{2\beta f_e} - \alpha l_r)$, *i.e.*, $\frac{\upsilon-p+\beta(f_m\alpha(l_r-c_r)-f_e(\omega_o+\alpha c_r))}{\beta} > \frac{f_m\alpha(l_r-c_r)(\upsilon-p-\beta f_e(s_n+\alpha l_i)+b\delta)}{\delta b} + \frac{(b(1-\delta)+\beta f_e(s_n+\alpha l_i-\omega_o-\alpha c_r))^2}{4\beta b(1-\delta)}$.

  - The maximum is realized on $s_o(p, \omega_o) = \frac{b(1-\delta)+\beta f_e(c_o+\alpha c_r+s_n+\alpha l_i)}{2\beta f_e} - \alpha l_r$ if $\Pi^{CM}_{r(2)}(p, \omega_o, \frac{\upsilon-p}{\beta f_e} - \alpha l_r) < \Pi^{CM}_{r(7)}(p, \omega_o, \frac{b(1-\delta)+\beta f_e(c_o+\alpha c_r+s_n+\alpha l_i)}{2\beta f_e} -$

$\alpha l_r)$, *i.e.*, $\frac{\upsilon-p+\beta(f_m\alpha(l_r-c_r)-f_e(\omega_o+\alpha c_r))}{\beta} < \frac{f_m\alpha(l_r-c_r)(\upsilon-p-\beta f_e(s_n+\alpha l_i)+b\delta)}{\delta b} + \frac{\alpha c_r\beta f_e-b(1-\delta)-\beta f_e(s_n+\alpha l_i-\omega_o)^2}{4\beta b(1-\delta)}$.

- $\nearrow \ \cap \ \nearrow \longrightarrow$ When $\frac{\beta f_e(s_n+\alpha l_i)-(1-\delta)(\upsilon-p)}{\delta\beta f_e} - \alpha l_r > \hat{s}_{o0}(\omega_o) > \frac{\upsilon-p}{\beta f_e} - \alpha l_r$, $\Pi^{CM}_{r(2)}(p,\omega_o,s_o)$ is concave on s_o. The corresponding condition on ω_o is $\frac{2\beta f_e(s_n+\alpha l_i)+\delta\beta f_m\alpha(l_r-c_r)-(2-\delta)(\upsilon-p)-\delta b}{\delta\beta f_e} - \alpha c_r > \omega_o > \frac{\upsilon-p-b+\beta f_m\alpha(l_r-c_r)}{\beta f_e} - \alpha c_r$. Additionally, when $\hat{s}_{o7}(\omega_o) > \frac{\beta f_e(s_n+\alpha l_i)+(1-\delta)b}{\beta f_e} - \alpha l_r$, $\Pi^{CM}_{r(7)}(p,\omega_o,s_o)$ is increasing on s_o. The corresponding second condition on ω_o is $\omega_o > \frac{\beta f_e(s_n+\alpha l_i)+(1-\delta)b}{\beta f_e} - \alpha c_r$. Then, there are two possible alternatives based on how the objective function value compares at the two candidate points:
    - The maximum is realized on $s_o(p,\omega_o) = \frac{\upsilon-p+b+\beta f_e(\omega_o+\alpha c_r)-\beta f_m\alpha(l_r-c_r)}{2\beta f_e} - \alpha l_r$ if $\Pi^{CM}_{r(2)}(p,\omega_o,\frac{\upsilon-p+b+\beta f_e(\omega_o+\alpha c_r)-\beta f_m\alpha(l_r-c_r)}{2\beta f_e} - \alpha l_r) > \Pi^{CM}_{r(7)}(p,\omega_o,\frac{\beta f_e(s_n+\alpha l_i)+b(1-\delta)}{\beta f_e} - \alpha l_r)$, *i.e.*, $\frac{(\upsilon-p+b-\beta(f_e(\omega_o+\alpha c_r)-f_m\alpha(l_r-c_r)))^2}{4\beta b} > \frac{f_m\alpha(l_r-c_r)\beta(\upsilon-p+b\delta-\beta f_e(s_n+\alpha l_i))}{b\delta}$.
    - The maximum is realized on $s_o(p,\omega_o) = \frac{\beta f_e(s_n+\alpha l_i)+b(1-\delta)}{\beta f_e} - \alpha l_r$ if $\Pi^{CM}_{r(2)}(p,\omega_o,\frac{\upsilon-p+b+\beta f_e(\omega_o+\alpha c_r)-\beta f_m\alpha(l_r-c_r)}{2\beta f_e} - \alpha l_r) < \Pi^{CM}_{r(7)}(p,\omega_o,\frac{\beta f_e(s_n+\alpha l_i)+b(1-\delta)}{\beta f_e} - \alpha l_r)$, *i.e.*, $\frac{(\upsilon-p+b-\beta(f_e(\omega_o+\alpha c_r)-f_m\alpha(l_r-c_r)))^2}{4\beta b} < \frac{f_m\alpha(l_r-c_r)\beta(\upsilon-p+b\delta-\beta f_e(s_n+\alpha l_i))}{b\delta}$.
- $\nearrow \ \cap\cap \ \longrightarrow$ When $\frac{\beta f_e(s_n+\alpha l_i)-(1-\delta)(\upsilon-p)}{\delta\beta f_e} - \alpha l_r > \hat{s}_{o0}(\omega_o) > \frac{\upsilon-p}{\beta f_e} - \alpha l_r$, $\Pi^{CM}_{r(2)}(p,\omega_o,s_o)$ is concave on s_o. The corresponding condition on ω_o is $\frac{2\beta f_e(s_n+\alpha l_i)+\delta\beta f_m\alpha(l_r-c_r)-(2-\delta)(\upsilon-p)-\delta b}{\delta\beta f_e} - \alpha c_r > \omega_o > \frac{\upsilon-p-b+\beta f_m\alpha(l_r-c_r)}{\beta f_e} - \alpha c_r$. Additionally, when $\frac{\delta\beta f_e(s_n+\alpha l_i)-(1-\delta)(\upsilon-p)}{\delta\beta f_e} - \alpha l_r > \hat{s}_{o7}(\omega_o) > \frac{\beta f_e(s_n+\alpha l_i)+(1-\delta)b}{\beta f_e} - \alpha l_r$, $\Pi^{CM}_{r(7)}(p,\omega_o,s_o)$ is also concave on s_o. The corresponding second condition on ω_o is $\frac{\beta f_e(s_n+\alpha l_i)+(1-\delta)b}{\beta f_e} - \alpha c_r > \omega_o > \frac{\beta f_e(2-\delta)(s_n+\alpha l_i)-(1-\delta)(\delta b+2(\upsilon-p))}{\delta\beta f_e} - \alpha c_r$. Then, there are two possible alternatives based on how the objective function value compares at the two candidate points:
    - The maximum is realized on $s_o(p,\omega_o) = \frac{\upsilon-p+b+\beta f_e(\omega_o+\alpha c_r)-\beta f_m\alpha(l_r-c_r)}{2\beta f_e} - \alpha l_r$ if $\Pi^{CM}_{r(2)}(p,\omega_o,\frac{\upsilon-p+b+\beta f_e(\omega_o+\alpha c_r)-\beta f_m\alpha(l_r-c_r)}{2\beta f_e} - \alpha l_r) > \Pi^{CM}_{r(7)}(p,\omega_o,\frac{\beta f_e(s_n+\alpha l_i)+b(1-\delta)}{\beta f_e} - \alpha l_r)$, *i.e.*, $\frac{(\upsilon-p+b-\beta(f_e(\omega_o+\alpha c_r)-f_m\alpha(l_r-c_r)))^2}{4\beta b} > \frac{f_m\alpha(l_r-c_r)(\upsilon-p-\beta f_e(s_n+\alpha l_i)+b\delta)}{\delta b} + \frac{\alpha c_r\beta f_e-b(1-\delta)-\beta f_e(s_n+\alpha l_i-\omega_o)^2}{4\beta b(1-\delta)}$.

- The maximum is realized on $s_o(p, \omega_o) = \frac{b(1-\delta)+\beta f_e(c_o+\alpha c_r+s_n+\alpha l_i)}{2\beta f_e} - \alpha l_r$ if $\Pi^{CM}_{r(2)}(p, \omega_o, \frac{\upsilon-p+b+\beta f_e(\omega_o+\alpha c_r)-\beta f_m \alpha(l_r-c_r)}{2\beta f_e} - \alpha l_r) < \Pi^{CM}_{r(7)}(p, \omega_o, \frac{\beta f_e(s_n+\alpha l_i)+b(1-\delta)}{\beta f_e} - \alpha l_r)$, *i.e.*, $\frac{(\upsilon-p+b-\beta(f_e(\omega_o+\alpha c_r)-f_m\alpha(l_r-c_r)))^2}{4\beta b} < \frac{f_m\alpha(l_r-c_r)(\upsilon-p-\beta f_e(s_n+\alpha l_i)+b\delta)}{\delta b} + \frac{\alpha c_r \beta f_e - b(1-\delta)-\beta f_e(s_n+\alpha l_i-\omega_o)^2}{4\beta b(1-\delta)}$.

A summary of the above items is also provided in Tables D.1-D.2 where the following notation is used for thresholds related to ω_o for brevity:

$$\begin{aligned}
A &= \frac{\upsilon - p - b + \beta f_m \alpha(l_r - c_r)}{\beta f_e} - \alpha c_r \\
B &= \frac{2\beta f_e(s_n + \alpha l_i) + \delta\beta f_m \alpha(l_r - c_r) - (2-\delta)(\upsilon - p) - \delta b}{\delta\beta f_e} - \alpha c_r \\
C &= \frac{\beta f_e(2-\delta)(s_n + \alpha l_i) - (1-\delta)(\delta b + 2(\upsilon - p))}{\delta\beta f_e} - \alpha c_r \\
D &= \frac{\beta f_e(s_n + \alpha l_i) + (1-\delta)b}{\beta f_e} - \alpha c_r
\end{aligned}$$

Also note that $A < B$ and $C < D$ by definition, limiting then the number of potential orderings to six. ■

Proof of Lemma 5. In order to find $\Pi^{CL}_m(p, \omega_o, s^*_o(p, \omega_o))$, we simply substitute s_o with $s^*_o(p, \omega_o)$ in $\Pi^C_m(\omega_o, s_o)$.

Since all three pieces of the function are rational, it is clear that each piece is continuous in ω_o. To complete the proof for continuity, we only need to show continuity in the two transition points $s_n + \alpha l_i - \frac{b(1-\delta)}{\beta f_e} - \alpha c_r$ and $s_n + \alpha l_i + \frac{b(1-\delta)}{\beta f_e} - \alpha c_r$. It is left to the reader to verify that the following equations hold:

$$\begin{aligned}
\Pi^{CL}_{m(-3)}\left(s_n + \alpha l_i - \frac{b(1-\delta)}{\beta f_e} - \alpha c_r\right) &= \Pi^{CL}_{m(6)}\left(s_n + \alpha l_i - \frac{b(1-\delta)}{\beta f_e} - \alpha c_r\right) \\
\Pi^{CL}_{m(6)}\left(s_n + \alpha l_i + \frac{b(1-\delta)}{\beta f_e} - \alpha c_r\right) &= \Pi^{CL}_{m(5)}\left(s_n + \alpha l_i - \frac{b(1-\delta)}{\beta f_e} - \alpha c_r\right)
\end{aligned}$$

Thus, $\Pi^{CL}_m(p, \omega_o, s^*_o(p, \omega_o))$ is continuous in ω_o.

Next, analyze the first and second derivatives of the pieces of the profit function, in order to establish the truth of the other two statements.

$$\begin{aligned}
\frac{\partial \Pi^{CL}_{m(-3)}(p, \omega_o)}{\partial \omega_o} &= f_e > 0 \\
\frac{\partial^2 \Pi^{CL}_{m(-3)}(p, \omega_o)}{\partial \omega_o^2} &= 0
\end{aligned}$$

Table D.1: Valid intervals for ω_o and the corresponding best responses for s_o (Part 1)

Alt.	s_o^*	Shape of Π_r^{DM}	Conditions on ω_o	Cases (Part 1)		
				$A < B < C < D$	$A < C < B < D$	$C < D < A < B$
1	$s_{o1}^* = \frac{\beta f_e(s_n+\alpha l_i)+(1-\delta)b}{\beta f_e} - \alpha l_r$	$\nearrow\nearrow\nearrow\longrightarrow$	$\omega_o > B$ $\omega_o > D$	$\omega_o > D$	$\omega_o > D$	$\omega_o > B$
2	$s_{o2}^* = \frac{\delta\beta f_e(s_n+\alpha l_i)-(1-\delta)(\upsilon-p)}{\delta\beta f_e} - \alpha l_r$	$\nearrow\nearrow\searrow\longrightarrow$	$\omega_o > B$ $\omega_o < C$	$B < \omega_o < C$	infeasible	infeasible
3	$s_{o3}^* = \frac{\beta f_e(\omega_o+\alpha c_r+s_n+\alpha l_i)+(1-\delta)b}{2\beta f_e} - \alpha l_r$	$\nearrow\nearrow\cap\longrightarrow$	$\omega_o > B$ $C < \omega_o < D$	$C < \omega_o < D$	$B < \omega_o < D$	infeasible
4	$s_{o4}^* = \frac{\upsilon-p}{\beta f_e} - \alpha l_r$	$\nearrow\searrow\searrow\longrightarrow$	$\omega_o < A$ $\omega_o < C$	$\omega_o < A$	$\omega_o < A$	$\omega_o < C$
5	$s_{o5}^* = \frac{\upsilon-p+b+\beta f_e(\omega_o+\alpha c_r)-\beta f_m\alpha(l_r-c_r)}{2\beta f_e} - \alpha l_r$	$\nearrow\cap\searrow\longrightarrow$	$A < \omega_o < B$ $\omega_o < C$	$A < \omega_o < B$	$A < \omega_o < C$	infeasible
6	s_{o4}^* or s_{o1}^*	$\nearrow\searrow\nearrow\longrightarrow$	$\omega_o < A$ $\omega_o > D$	infeasible	infeasible	$D < \omega_o < A$
7	s_{o4}^* or s_{o3}^*	$\nearrow\searrow\cap\longrightarrow$	$\omega_o < A$ $C < \omega_o < D$	infeasible	infeasible	$C < \omega_o < D$
8	s_{o5}^* or s_{o1}^*	$\nearrow\cap\nearrow\longrightarrow$	$A < \omega_o < B$ $\omega_o > D$	infeasible	infeasible	$A < \omega_o < B$
9	s_{o5}^* or s_{o3}^*	$\nearrow\cap\cap\longrightarrow$	$A < \omega_o < B$ $C < \omega_o < D$	infeasible	$C < \omega_o < B$	infeasible

Table D.2: Valid intervals for ω_o and the corresponding best responses for s_o (Part 2)

Alt.	s_o^*	Shape of Π_r^{DM}	Conditions on ω_o	Cases (Part 2)		
				$C < A < D < B$	$A < C < D < B$	$C < A < B < D$
1	$s_{o1}^* = \frac{\beta f_e(s_n+\alpha l_i)+(1-\delta)b}{\beta f_e} - \alpha l_r$	$\nearrow\nearrow\nearrow\longrightarrow$	$\omega_o > B$ $\omega_o > D$	$\omega_o > B$	$\omega_o > B$	$\omega_o > D$
2	$s_{o2}^* = \frac{\delta\beta f_e(s_n+\alpha l_i)-(1-\delta)(v-p)}{\delta\beta f_e} - \alpha l_r$	$\nearrow\nearrow\searrow\longrightarrow$	$\omega_o > B$ $\omega_o < C$	infeasible	infeasible	infeasible
3	$s_{o3}^* = \frac{\beta f_e(\omega_o+\alpha c_r+s_n+\alpha l_i)+(1-\delta)b}{2\beta f_e} - \alpha l_r$	$\nearrow\nearrow\cap\longrightarrow$	$\omega_o > B$ $C < \omega_o < D$	infeasible	infeasible	$B < \omega_o < D$
4	$s_{o4}^* = \frac{v-p}{\beta f_e} - \alpha l_r$	$\nearrow\searrow\searrow\longrightarrow$	$\omega_o < A$ $\omega_o < C$	$\omega_o < C$	$\omega_o < A$	$\omega_o < C$
5	$s_{o5}^* = \frac{v-p+b+\beta f_e(\omega_o+\alpha c_r)-\beta f_m\alpha(l_r-c_r)}{2\beta f_e} - \alpha l_r$	$\nearrow\cap\searrow\longrightarrow$	$A < \omega_o < B$ $\omega_o < C$	infeasible	$A < \omega_o < C$	infeasible
6	s_{o4}^* or s_{o1}^*	$\nearrow\searrow\nearrow\longrightarrow$	$\omega_o < A$ $\omega_o > D$	infeasible	infeasible	infeasible
7	s_{o4}^* or s_{o3}^*	$\nearrow\searrow\cap\longrightarrow$	$\omega_o < A$ $C < \omega_o < D$	$C < \omega_o < A$	infeasible	$C < \omega_o < A$
8	s_{o5}^* or s_{o1}^*	$\nearrow\cap\nearrow\longrightarrow$	$A < \omega_o < B$ $\omega_o > D$	$D < \omega_o < B$	$D < \omega_o < B$	infeasible
9	s_{o5}^* or s_{o3}^*	$\nearrow\cap\cap\longrightarrow$	$A < \omega_o < B$ $C < \omega_o < D$	$A < \omega_o < D$	$C < \omega_o < D$	$A < \omega_o < B$

$$\begin{aligned}\frac{\partial \Pi^{CL}_{m(6)}(p,\omega_o)}{\partial \omega_o} &= \frac{f_e}{2} - \frac{\beta f_e^2(\omega_o - c_o + \omega_o + \alpha c_r - s_n - \alpha l_i)}{2b(1-\delta} \\ \frac{\partial^2 \Pi^{CL}_{m(6)}(\omega_o)}{\partial \omega_o^2} &= -\frac{\beta f_e^2}{b(1-\delta)} < 0\end{aligned}$$

■

Proof of Proposition 9. We solve for the equilibrium by using backward induction. Proposition 6 gives the retailer's best response and Lemma 5 gives the corresponding profit function of the manufacturer and relevant properties of this function.

First, find the unconstrained optimizer of $\Pi^{CL}_{m(6)}(p,\omega_o)$, which is given by the FOC:

$$\frac{\partial \Pi^{CL}_{m(6)}(p,\omega_o)}{\partial \omega_o} = 0 \Leftrightarrow \hat{\omega}_o(p) = \frac{b(1-\delta) + \beta f_e(c_o + \alpha c_r + s_n + \alpha l_i)}{2\beta f_e} - \alpha c_r$$

Then, we have three possibilities depending on where $\hat{\omega}_o$ is, as compared to the boundaries of the interval where $\Pi^{CL}_{m(6)}(p,\omega_o)$ is valid, *i.e.,* $\left[s_n + \alpha l_i - \frac{b(1-\delta)}{\beta f_e} - \alpha c_r, s_n + \alpha l_i + \frac{b(1-\delta)}{\beta f_e} - \alpha c_r\right]$. In all three possibilities, we have $\Pi^{CL}_{m(-3)}(p,\omega_o)$ increasing on ω_o on the interval $\left(-\infty, s_n + \alpha l_i - \frac{b(1-\delta)}{\beta f_e} - \alpha c_r\right)$ and $\Pi^{CL}_{m(5)}(p,\omega_o) = p - f_m(c_o + \alpha l_r))$ constant on ω_o on the interval ($s_n + \alpha l_i + \frac{b(1-\delta)}{\beta f_e} - \alpha c_r$, ∞) (as proven in Lemma 5).

- When $\hat{\omega}_o < s_n + \alpha l_i - \frac{b(1-\delta)}{\beta f_e} - \alpha c_r$, $\Pi^{CL}_{m(6)}(p,\omega_o)$ is decreasing on ω_o. Then, the maximum is realized on $\omega_o = s_n + \alpha l_i - \frac{b(1-\delta)}{\beta f_e} - \alpha c_r$.
- When $s_n + \alpha l_i - \frac{b(1-\delta)}{\beta f_e} - \alpha c_r \leq \hat{\omega}_o \leq s_n + \alpha l_i + \frac{b(1-\delta)}{\beta f_e} - \alpha c_r$, $\Pi^{CL}_{m(6)}(p,\omega_o)$ is convex on ω_o. Then, the maximum is realized on $\omega_o = \hat{\omega}_o = \frac{(b(1-\delta)+\beta f_e(c_o+\alpha c_r+sn+\alpha l_i)}{2\beta f_e} - \alpha c_r$.
- When $\hat{\omega}_o > s_n + \alpha l_i + \frac{b(1-\delta)}{\beta f_e} - \alpha c_r$, $\Pi^{CL}_{m(6)}(p,\omega_o)$ is increasing on ω_o. Then, the maximum is realized on $\omega_o \in \left[s_n + \alpha l_i + \frac{b(1-\delta)}{\beta f_e} - \alpha c_r\right.$, $\infty)$.

s_o^* follows from $\omega = \omega_o^*$. ■

Proof of Lemma 6. The profit function follows from plugging in $q_i^C(p,s_o)$ and $q_r^C(p,s_o)$ in Equation 4.7. $\Pi^{CCL}_{(-3)}(p,s_o)$, $\Pi^{CCL}_{(6)}(p,s_o)$ and $\Pi^{CCL}_{(5)}(p,s_o)$ are all continuous in s_o. To complete the proof for continuity, we only need to check the two transition points. It is left to the reader to verify that the following equations hold:

$$\begin{aligned}
\Pi^{CCL}_{(-3)}\left(p, s_n + \alpha l_i - \alpha l_r\right) &= \Pi^{CCL}_{(6)}\left(p_o, s_n + \alpha l_i - \alpha l_r\right) \\
\Pi^{CCL}_{(6)}\left(p, s_n + \alpha l_i + \frac{b(1-\delta)}{\beta f_e} - \alpha l_r\right) &= \Pi^{CCL}_{(5)}\left(p, s_n + \alpha l_i + \frac{b(1-\delta)}{\beta f_e} - \alpha l_r\right)
\end{aligned}$$

Thus, $\Pi^{CCL}(p, s_o)$ is continuous in s_o. Next, analyze the first and second derivatives of $\Pi^{CCL}_{(-3)}(p, s_o)$ and $\Pi^{CCL}_{(6)}(p, s_o)$ to establish points 2-3:

$$\frac{\partial \Pi^{CCL}_{(-3)}(p, s_o)}{\partial s_o} = f_e > 0$$

$$\frac{\partial^2 \Pi^{CCL}_{(6)}(p, s_o)}{\partial s_o^2} = -\frac{2\beta f_e^2}{b(1-\delta)} < 0$$

■

Proof of Proposition 10. First, find the unconstrained maximizer of $\Pi^{CCL}_{(6)}(p, s_o)$, which is given by the FOC (due to concavity as proven in Lemma 6):

$$\frac{\partial \Pi^{CCL}_{(6)}(p, s_o)}{\partial s_o} = 0 \Leftrightarrow \hat{s}_o(p) = \frac{b(1-\delta) + \beta f_e(c_o + \alpha c_r + s_n + \alpha l_i)}{2\beta f_e} - \alpha l_r$$

Then, we have three possibilities depending on where $\hat{s}_o$ is, as compared to the boundaries of the interval where $\Pi^{CCL}_{(6)}(p, s_o)$ is valid, *i.e.,* $\left[s_n + \alpha l_i - \alpha l_r, \frac{b(1-\delta)}{\beta f_e} + s_n + \alpha l_i - \alpha l_r\right]$. In all three possibilities, we have $\Pi^{CCL}_{(-3)}(p, s_o)$ increasing on s_o on the interval $(-\infty, s_n + \alpha l_i - \alpha l_r)$ and $\Pi^{CCL}_{(5)}(p, s_o) = p - c - f_m(c_o + \alpha c_r)$ constant on s_o on the interval $\left(\frac{b(1-\delta)}{\beta f_e} + s_n + \alpha l_i - \alpha l_r\ , \infty\right)$ (as proven in Lemma 6).

- When $\hat{s}_o(p) < s_n + \alpha l_i - \alpha l_r$, $\Pi^{CCL}_{(6)}(p, s_o)$ is decreasing on s_o. Then, the maximum is realized on $s_o(p) = s_n + \alpha l_i - \alpha l_r$.

- When $s_n + \alpha l_i - \alpha l_r \leq \hat{s}_o(p) \leq \frac{b(1-\delta)}{\beta f_e} + s_n + \alpha l_i - \alpha l_r$, $\Pi^{CCL}_{(6)}(p, s_o)$ is convex on s_o. Then, the maximum is realized on $s_o(p) = \hat{s}_o(p) = \frac{b(1-\delta)+\beta f_e(c_o+\alpha c_r+s_n+\alpha l_r)}{2\beta f_e} - \alpha l_r$.

- When $\hat{s}_o(p) > \frac{b(1-\delta)}{\beta f_e} + s_n + \alpha l_i - \alpha l_r$, $\Pi^{CCL}_{(6)}(p, s_o)$ is increasing on s_o. Then, the maximum is realized on $s_o(p) = \left[\frac{b(1-\delta)}{\beta f_e} + s_n + \alpha l_i - \alpha l_r\ , \infty\right)$.

■

Proof of Lemma 17. In order to find $\Pi_m^{DM1}(\omega_o, s_o^*(\omega_o))$, we simply substitute s_o with $s_o^*(\omega_o)$ in $\Pi_m^D(\omega_o, s_o)$.

Since all five pieces of the function are rational, it is clear that each piece is continuous in ω_o. To complete the proof for continuity, we only need to show continuity in the four transition points A, B, C, and D. It is left to the reader to verify that the following equations hold:

$$\begin{aligned}
\Pi_{m1}^{DM1}(A) &= \Pi_{m2}^{DM1}(A) \\
\Pi_{m2}^{DM1}(B) &= \Pi_{m3}^{DM1}(B) \\
\Pi_{m3}^{DM1}(C) &= \Pi_{m4}^{DM1}(C) \\
\Pi_{m4}^{DM1}(D) &= \Pi_{m5}^{DM1}(D)
\end{aligned}$$

Thus, $\Pi_m^{DM1}(\omega_o, s_o^*(\omega_o))$ is continuous in ω_o.

Next, analyze the first and second derivatives of the pieces of the profit function, in order to establish the truth of the other four statements.

$$\begin{aligned}
\frac{\partial \Pi_{m1}^{DM1}(\omega_o)}{\partial \omega_o} &= f_e > 0 \\
\frac{\partial^2 \Pi_{m1}^{DM1}(\omega_o)}{\partial \omega_o^2} &= 0
\end{aligned}$$

$$\frac{\partial^2 \Pi_{m2}^{DM1}(\omega_o)}{\partial \omega_o^2} = -\frac{\beta f_e^2}{b} < 0$$

$$\begin{aligned}
\frac{\partial \Pi_{m3}^{DM1}(\omega_o)}{\partial \omega_o} &= \frac{f_e(\upsilon - p - \beta f_e(s_n + \alpha l_i) + \delta b)}{\delta b} > 0 \\
\frac{\partial^2 \Pi_{m3}^{DM1}(\omega_o)}{\partial \omega_o^2} &= 0
\end{aligned}$$

$$\frac{\partial^2 \Pi_{m4}^{DM1}(\omega_o)}{\partial \omega_o^2} = -\frac{\beta f_e^2}{b(1-\delta)} < 0$$

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CURRICULUM VITAE

PERSONAL INFORMATION

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EDUCATION

Degree	Institution	Year of Graduation
M.S. (Ind. Eng.)	University of Arkansas	2009
Project: Analysis of the Forced-Uniform Put-Away Policy for the Multiple-Copy Product Allocation Problem		
M.S. (Ind. Eng.)	Middle East Technical University	2006
Thesis: Modeling and Analysis of the Facility Layout Problem		
B.S. (Ind. Eng.)	Middle East Technical University	2004
High School	Gazi Anadolu Lisesi	2000

PROFESSIONAL EXPERIENCE

Place	Enrollment	Year
MAN Kamyon ve Otobüs Tic. A.Ş., Ankara/Turkey	Finance Director	2017 - ongoing
MAN Truck & Bus SE, Munich/Germany	Project Lead	2016 - 2017

PROFESSIONAL EXPERIENCE (Contd.)

Place	Enrollment	Year
MAN Kamyon ve Otobüs Tic. A.Ş., Ankara/Turkey	Head of Processes and IT	2014 - 2016
	Sales Controlling Manager	2013 - 2014
	Sales Support Manager	2012 - 2013
	Sales Support Engineer	2011 - 2012
	Project Engineer	2010 - 2011
University of Arkansas, Fayetteville, AR, USA	Research and Teaching Assistant	2006 - 2009
Middle East Technical University, Ankara/Turkey	Research and Teaching Assistant	2004 - 2006

PUBLICATIONS

Kirkizoglu, Z. & Karaer, Ö. (in press). After-sales service and warranty decisions of a durable goods manufacturer. Omega.

Meller, R. D., *Kirkizoglu, Z.*, & Chen, W. (2010). A new optimization model to support a bottom-up approach to facility design. Computers & Operations Research, 37(1), 42-49.

Kirkizoglu, Z. (2005). A Genetic Algorithm Approach for Sales Territory Alignment Problem. In 35th International Conference on Computers and Industrial Engineering, Istanbul.